II. RV X With Pdf F(x)=cx 2,10) 2. P(X>0X<1) 3. P(X<1X<0) 4. E(X) 5. Var(X)

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Understanding random variables (RVs) and their probability density functions (PDFs) is fundamental in probability theory and statistics. In this article, we delve into the properties of a specific continuous random variable X characterized by its PDF $f(x) = c x^2$ over a certain interval, focusing on key concepts such as calculating probabilities, expected value, and variance. We will explore each aspect step-by-step to provide a comprehensive understanding suitable for students, researchers, or anyone interested in stochastic processes.

Defining the Random Variable X and Its PDF

1. The PDF $f(x) = c x^2$

The probability density function (PDF) $f(x)$ describes the likelihood of the random variable X taking on a particular value x. In this case, the PDF is given as:

$$
f(x) = c x^2
$$

where c is a constant ensuring that the total probability over the support of X equals 1. To fully understand this distribution, we need to determine the support (the interval where $f(x)$ is non-zero) and compute the normalization constant c.

Determining the Support and Normalization Constant c

2. Establishing the support of X

Typically, the form $f(x) = c x^2$ suggests that x takes values in an interval $[a, b]$, where the PDF is positive. For illustration, let's assume the support is $[0, 1]$, a common interval for such distributions. If the problem specifies a different interval, the calculation can be adjusted accordingly.

3. Calculating c to satisfy the probability axioms

Since the total probability must be 1, we have:

$$
\int_{a}^{b} f(x) \, dx = 1
$$

Assuming support $[0, 1]$:

$$
\int_0^1 c x^2 \, dx = 1
$$

Calculating the integral:

$$
c \int_0^1 x^2 \, dx = c \left[\frac{x^3}{3} \right]_0^1 = c \times \frac{1}{3}
$$

Setting this equal to 1:

$$
c \times \frac{1}{3} = 1 \Rightarrow c = 3
$$

Thus, the PDF becomes:

$$
f(x) = 3 x^2, \quad 0 \leq x \leq 1
$$

Calculating Probabilities for X

4. Probability that $X > 0 \text{ and } X < 1$: $P(0 < X < 1)$

Since the support is $[0, 1]$, the probability that X falls within this interval is:

$$
P(0 < X < 1) = \int_0^1 f(x) \, dx = 1
$$

which confirms that the entire probability mass is within the support.

5. Probability that $X < 1 \text{ and } X > 0$: $P(X > 0, X < 1)$

This is identical to the previous probability:

$$
P(0 < X < 1) = 1
$$

since the support is exactly $[0, 1]$.

6. Probability that $X < 1 \text{ given } X < 0$: $P(X < 1 \mid X < 0)$

This probability involves conditional probability:

$$
P(X < 1 \mid X < 0) = \frac{P(X < 1, X < 0)}{P(X < 0)}
$$

But since X is supported in $[0, 1]$, the probability $P(X < 0) = 0$. Therefore:

$$
P(X < 1 \mid X < 0) = 0
$$

Calculating the Expected Value $E(X)$

7. Definition of the expectation

The expected value (mean) of a continuous random variable X with PDF $f(x)$ over support $[a, b]$ is:

$$
E(X) = \int_a^b x f(x) \, dx
$$

Assuming support $[0, 1]$ and $f(x) = 3x^2$:

$$
E(X) = \int_0^1 x \times 3x^2 \, dx = 3 \int_0^1 x^3 \, dx
$$

Calculating the integral:

$$
3 \left[\frac{x^4}{4} \right]_0^1 = 3 \times \frac{1}{4} = \frac{3}{4} = 0.75
$$

Thus, the expected value:

$$
E(X) = \frac{3}{4}
$$

Calculating the Variance $Var(X)$

8. Variance formula

Variance measures the spread of the distribution:

$$
Var(X) = E[X^2] - (E[X])^2
$$

We already have $E[X]$, so we need $E[X^2]$.

9. Computing $E[X^2]$

$$
E[X^2] = \int_0^1 x^2 f(x) \, dx = 3 \int_0^1 x^4 \, dx
$$

Calculating:

$$
3 \left[\frac{x^5}{5} \right]_0^1 = 3 \times \frac{1}{5} = \frac{3}{5} = 0.6
$$

Now, the variance:

$$
Var(X) = E[X^2] - (E[X])^2 = \frac{3}{5} - \left(\frac{3}{4}\right)^2
$$

Calculating:

$$
Var(X) = \frac{3}{5} - \frac{9}{16} = \frac{48}{80} - \frac{45}{80} = \frac{3}{80} = 0.0375
$$

Summary of Key Results

- Support of X: $[0, 1]$
- PDF: $f(x) = 3 x^2$
- Probability X lies between 0 and 1: 1
- Conditional probability $P(X < 1 \mid X < 0)$: 0 (since support is $[0, 1]$)
- Expected value $E(X)$: $\frac{3}{4} = 0.75$
- Variance $Var(X)$: $\frac{3}{80} = 0.0375$

Conclusion

Analyzing the properties of a random variable with a specific PDF like $f(x) = c x^2$ provides critical insights into its behavior. By determining the normalization constant, calculating probabilities, expectation, and variance, we gain a comprehensive understanding of X. This process underscores the importance of integrating the PDF over the support and applying fundamental formulas in probability theory. Whether in theoretical research or practical applications, mastering these calculations enables better modeling and interpretation of stochastic phenomena.

Additional Notes

- If the support of X differs from $[0, 1]$, the calculations for c, $E(X)$, and $Var(X)$ should be adjusted accordingly.
- The process demonstrated here can be extended to other forms of PDFs, such as exponential, normal, or beta distributions, each with their specific properties and formulas.
- Conditional probabilities involving support outside the defined interval are typically zero, simplifying some calculations.

This comprehensive guide aims to serve as a foundational resource for understanding and working with continuous random variables characterized by polynomial PDFs.

Frequently Asked Questions

What is the probability density function (PDF) for the RV X defined as

$F(x) = cx^2$ over the interval (0,10)?

The PDF is given by $f(x) = cx^2$ for x in (0,10), where c is a constant determined by the condition that the total probability integrates to 1 over (0,10).

How do you determine the normalization constant c for the PDF $f(x) = cx^2$ over (0,10)?

To find c, set the integral of f(x) from 0 to 10 equal to 1: $\int_0^{10} c x^2 \, dx = 1$. Solving yields $c = 3/100$.

What is the probability that X is greater than 0 and less than 1, i.e., $P(0 < X < 1)$?

Calculate $P(0 < X < 1)$ by integrating the PDF over (0,1): $P = \int_0^1 c x^2 \, dx = c/3 \, (1^3 - 0) = (3/100)\,(1/3) = 1/100$.

How do you find the probability $P(X < 1 \mid X < 0)$?

Since the support of X is (0,10), $P(X < 1 \mid X < 0)$ is undefined or zero because X cannot be less than 0. If the question intends $P(X < 1 \mid X > 0)$, it is calculated as $P(0 < X < 1)/P(X > 0)$.

What is the expected value E(X) for the given PDF?

$E(X) = \int_0^{10} x f(x) \, dx = \int_0^{10} x c x^2 \, dx = c \int_0^{10} x^3 \, dx$. Calculating gives $E(X) = (3/100)\,(10^4/4) = 75$.

How do you compute the variance Var(X) for this distribution?

$Var(X) = E(X^2) - [E(X)]^2$. First, find $E(X^2) = \int_0^{10} x^2 f(x) \, dx = c \int_0^{10} x^4 \, dx$, then subtract $[E(X)]^2$ from it.

What are the key steps to analyze the properties of the RV X with the given PDF?

Key steps include determining the normalization constant c, computing probabilities over intervals via integration, calculating expected value E(X), and variance Var(X) using the moments of the distribution.

Are there any assumptions or limitations in analyzing this distribution with the given PDF?

Yes, assumptions include that the PDF is properly normalized and that the support is (0,10). Limitations may arise if the given function does not satisfy the properties of a valid PDF or if the interval changes.

Additional Resources

Comprehensive Analysis of the RV X with PDF $F(x) = c x^2$ for $0 < x < 1$

Understanding the properties and behavior of a random variable (RV) is fundamental in probability theory and statistical analysis. In this discussion, we analyze a specific continuous random variable X, characterized by its probability density function (PDF) $F(x) = c x^2$ over the interval $0 < x < 1$. We will explore key probabilistic measures such as the normalization constant c, various probability calculations, expected value, variance, and interpret their implications in practical contexts.

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1. Defining the Random Variable and Its PDF

Understanding the PDF $F(x) = c x^2$

The function $F(x)$ as given appears to be the probability density function of the random variable X. Typically, the notation $F(x)$ is used for the cumulative distribution function (CDF), but given the form $c x^2$, and the context of the questions, it is more aligned with the PDF $f(x)$. Clarifying this:

- Assumption: $f(x) = c x^2$, for $0 < x < 1$, and $f(x) = 0$ otherwise.

- Purpose: To verify whether this function is a valid PDF, find the constant c, and analyze the distribution.

Key properties of a PDF:

- Non-negativity: $f(x) \geq 0$ for all x.

- Normalization: $\int_{-\infty}^{\infty} f(x) dx = 1$.

Given the support $(0,1)$, the normalization condition simplifies to:

$$
\int_0^1 c x^2 dx = 1
$$

---

2. Determining the Normalization Constant c

Calculation of c

To find c, perform the integral:

$$
\int_0^1 c x^2 dx = c \int_0^1 x^2 dx
$$

Calculating the integral:

$$
\int_0^1 x^2 dx = \left[\frac{x^3}{3} \right]_0^1 = \frac{1^3}{3} - 0 = \frac{1}{3}
$$

Set the integral equal to 1:

$$
c \times \frac{1}{3} = 1 \Rightarrow c = 3
$$

Result:

$$
\boxed{
c = 3
}
$$

Implication: The PDF of X is

\[

$$f(x) = 3 x^2, \quad 0 < x < 1$$

and zero elsewhere.

---

3. Probabilistic Calculations

With the PDF defined, we can now proceed to compute various probabilities and expectations.

3.1 Probability $P(X > 0 \,|\, X < 1)$

Interpretation:

- Since X is supported only on $(0,1)$, the probability that X lies between 0 and 1 is:

$$P(0 < X < 1) = \int_0^1 f(x) dx$$

which, by normalization, equals 1:

$$P(0 < X < 1) = 1$$

Conditional probability:

- The question $P(X > 0 \,|\, X < 1)$ is essentially asking: What is the probability that X exceeds 0 given it is less than 1? Since the support is only on $(0,1)$, and $X > 0$ within this support, the probability is:

$$
P(X > 0 \,|\, X < 1) = \frac{P(0 < X < 1)}{P(X < 1)} = \frac{1}{1} = 1
$$

Conclusion:

$$
\boxed{
P(X > 0 \,|\, X < 1) = 1
}
$$

- Note: This is trivial given the support, but it confirms that the entire distribution is within $(0,1)$.

3.2 Probability $P(X < 1 \,|\, X < 0)$

Analysis:

- Since X is supported on $(0,1)$, the event $X < 0$ is impossible, so:

$$
P(X < 0) = 0
$$

- Therefore, the conditional probability $P(X < 1 \,|\, X < 0)$ is undefined as division by zero occurs, or more precisely, zero probability:

$$
P(X < 1 \,|\, X < 0) = \frac{P(X < 1 \cap X < 0)}{P(X < 0)} = \frac{0}{0}
$$

which is indeterminate.

Summary:

- This question seems to be more of a theoretical check or perhaps a typo. Since $P(X < 0) = 0$, the probability $P(X < 1 \,|\, X < 0)$ is undefined or zero, as the conditioning event has zero probability.

---

4. Calculating the Expected Value $E(X)$

Formula and Calculation

The expected value $E(X)$ of a continuous random variable is given by:

$$
E(X) = \int_{-\infty}^\infty x f(x) dx
$$

Given $f(x) = 3 x^2$ for $0 < x < 1$, and zero elsewhere:

$$
E(X) = \int_0^1 x \times 3 x^2 dx = 3 \int_0^1 x^3 dx
$$

Calculating the integral:

$$
\int_0^1 x^3 dx = \left[\frac{x^4}{4} \right]_0^1 = \frac{1}{4}
$$

Thus,

$$
E(X) = 3 \times \frac{1}{4} = \frac{3}{4} = 0.75
$$

Result:

$$
\boxed{
E(X) = \frac{3}{4}
}
$$

Interpretation:

- The mean of the distribution is 0.75, indicating that on average, the value of X is closer to 1 within its support.

---

5. Variance $\operatorname{Var}(X)$

Calculation of $\operatorname{Var}(X)$

Variance measures the spread or dispersion of a random variable around its mean:

$$
\operatorname{Var}(X) = E(X^2) - [E(X)]^2
$$

First, compute $E(X^2)$:

$$
E(X^2) = \int_0^1 x^2 \times 3 x^2 dx = 3 \int_0^1 x^4 dx
$$

Calculating the integral:

$$
\int_0^1 x^4 dx = \left[\frac{x^5}{5} \right]_0^1 = \frac{1}{5}
$$

Therefore,

$$
E(X^2) = 3 \times \frac{1}{5} = \frac{3}{5} = 0.6
$$

Now, apply the variance formula:

$$
\operatorname{Var}(X) = \frac{3}{5} - \left(\frac{3}{4} \right)^2 = \frac{3}{5} - \frac{9}{16}
$$

Express both fractions with common denominator 80:

$$
\frac{3}{5} = \frac{48}{80}
$$

$$
\frac{9}{16} = \frac{45}{80}
$$

Subtract:

$$
\operatorname{Var}(X) = \frac{48}{80} - \frac{45}{80} = \frac{3}{80}
$$

Final result:

$$
\boxed{
\operatorname{Var}(X) = \frac{3}{80} = 0.0375
}
$$

Implications:

- The variance is relatively small, indicating that the values of X are tightly clustered around the mean of 0.75.

---

6. Additional Insights and Practical Applications

6.1 Distribution Shape and Characteristics

- The PDF $f(x) = 3 x^2$ over $(0,1)$ describes a distribution skewed towards higher values within the interval.

- The shape is "right-skewed" in the sense that higher values of x have larger densities, given the quadratic nature x^2.

- This distribution resembles a Beta

Ii Rv X With Pdf F X Cx 2 10 2 P X Gt 0x Lt 1 3 P X Lt 1x Lt 0 4 E X 5 Var X

Ii Rv X With Pdf F X Cx 2 10 2 P X Gt 0x Lt 1 3 P X Lt 1x Lt 0 4 E X 5 Var X

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