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Understanding geometric problems involving algebraic expressions often requires a systematic approach to translating given information into equations, solving for unknown variables, and applying geometric principles. In this particular problem, we are given expressions for two segments, AB and BC, in terms of an unknown variable x, along with the total length of the segment AC. Our goal is to find the length of segment AB by determining the value of x and then calculating AB's length. This process involves setting up an algebraic equation based on segment relationships and solving it step-by-step.

Identifying the Geometric Setup and Known Data

Understanding the Segments and Their Relationships

- The problem references a figure with points A, B, and C, forming segments AB, BC, and AC.
- The segments AB and BC are expressed as algebraic functions of x:
- AB = 4x + 6
- BC = 7x + 15
- The total length of segment AC is given as 120.

Interpreting the Given Data

- Since AC is the sum of segments AB and BC when points A, B, and C are collinear in that order, we assume:

AC = AB + BC

- Given that AC = 120, the relationship simplifies to:

120 = (4x + 6) + (7x + 15)

Setting Up the Algebraic Equation

Expressing the Total Length

- The total length AC is the sum of the parts AB and BC:

AC = AB + BC

- Substituting the expressions:

120 = (4x + 6) + (7x + 15)

Formulating the Equation

- Simplify the right side:

120 = 4x + 6 + 7x + 15

- Combine like terms:

120 = (4x + 7x) + (6 + 15)

120 = 11x + 21

Solving for the Variable x

Isolating x

- Subtract 21 from both sides to isolate the term with x:

120 - 21 = 11x

99 = 11x

- Divide both sides by 11:

x = 99 / 11

x = 9

Verifying the Solution

- It's good practice to verify the solution by plugging x back into the expressions for AB and BC:

- AB = 4(9) + 6 = 36 + 6 = 42
- BC = 7(9) + 15 = 63 + 15 = 78

- Check that their sum equals AC:

42 + 78 = 120

- The sum matches the given total length, confirming the solution is correct.

Calculating the Length of Segment AB

Applying the Value of x

- Recall the expression for AB:

AB = 4x + 6

- Substitute x = 9:

AB = 4(9) + 6 = 36 + 6 = 42

Final Answer

- The length of segment AB is 42 units.

Additional Considerations and Related Concepts

Understanding the Role of Algebra in Geometry

- Algebraic expressions enable precise calculation of segment lengths when direct measurement isn't possible.
- Setting up equations based on segment relationships is a fundamental skill in solving geometric problems.

Possible Variations of the Problem

- If the points were not collinear, the problem would involve different geometric principles, such as triangle inequalities.
- If additional segments or angles were involved, the problem could extend into trigonometry or coordinate geometry.

Common Mistakes to Avoid

- Assuming segments are connected in a different order without verification.
- Forgetting to verify the solution by back-substituting into the original expressions.
- Ignoring the possibility of extraneous solutions if the algebra involves more complex equations.

Conclusion

This problem exemplifies how algebraic expressions can be used to model geometric segments and how setting up and solving equations is essential in determining unknown lengths. By recognizing that AC is the sum of AB and BC, translating the given expressions into an algebraic equation, and solving for x, we obtained the value necessary to find the length of segment AB. The process underscores the importance of systematic problem-solving in geometry, integrating algebra and geometric reasoning seamlessly. The length of segment AB, based on our calculations, is 42 units, illustrating how algebra simplifies complex geometric relationships into manageable calculations.

Frequently Asked Questions

Given AB = 4x + 6, BC = 7x + 15, and AC = 120, how do you find the length of segment AB?

Since AC is the sum of AB and BC, set up the equation: (4x + 6) + (7x + 15) = 120. Simplify to 11x + 21 = 120. Subtract 21 from both sides: 11x = 99. Divide both sides by 11: x = 9. Find AB by substituting x back: AB = 4(9) + 6 = 36 + 6 = 42. So, the length of AB is 42 units.

What is the first step to find AB given the expressions for AB and BC and the total AC length?

The first step is to set up an equation summing AB and BC to equal AC: (4x + 6) + (7x + 15) = 120, because AC is the total length of segments AB and BC.

How do you verify that the segments AB and BC sum up to AC in this problem?

By adding the expressions for AB and BC: (4x + 6) + (7x + 15), simplifying to 11x + 21, and then equating to AC's length of 120 to find x. Once x is found, substitute back into AB to verify the segment length.

If AC is 120 units and AB and BC are expressed in terms of x, what algebraic method is used to find the segment lengths?

You use algebra to set up an equation: (AB) + (BC) = AC, then solve for x, and substitute x back into the expressions for AB or BC to find their lengths.

Can the length of segment AB be determined without knowing x explicitly?

No, because AB is expressed as 4x + 6, and without solving for x, you cannot determine its exact length. Solving for x allows you to find the precise length of AB.

What is the importance of setting up the correct equation in this problem?

Setting up the correct equation (AB + BC = AC) is crucial because it relates all known and unknown quantities, allowing you to solve for x and subsequently find the length of segment AB.

If AB = 42 units, what is the value of x?

Since AB = 4x + 6 and AB = 42, set 4x + 6 = 42. Subtract 6: 4x = 36. Divide by 4: x = 9.

Why is it necessary to substitute the value of x back into the expression for AB?

Substituting x back into AB's expression (4x + 6) provides the actual length of segment AB, which is the goal of the problem.

What does the variable x represent in this problem?

The variable x is an unknown value used to express the lengths of segments AB and BC, which can be found by solving the equations involving these expressions.

What are the key steps to solve similar segment length problems involving algebraic expressions?

The key steps include: 1) Write an equation relating the segments; 2) Simplify and solve for the unknown variable; 3) Substitute the value back into the expressions to find the segment lengths; 4) Verify the solution if necessary.

Additional Resources

In The Figure Above AB = 4x + 6 And BC = 7x + 15. If AC = 120, Find The Length Of Segment AB is a classic example of a problem involving algebraic expressions and geometric relationships. Such problems are fundamental in developing a deeper understanding of how algebra and geometry intertwine, especially when dealing with segments within a triangle or a line segment divided into parts. This type of question not only tests your ability to translate a diagram into algebraic expressions but also your skill in solving equations systematically. In this comprehensive guide, we will explore how to approach this problem step-by-step, highlighting key concepts, strategies, and tips to ensure clarity and mastery.

Understanding the Problem and Its Context

Before diving into calculations, it’s essential to carefully interpret the problem statement:

- Given Data:
- Segment AB is expressed as $4x + 6$.
- Segment BC is expressed as $7x + 15$.
- The total length of segment AC is 120 units.

- What is asked?
- To find the length of segment AB.

The problem implies that the points A, B, and C lie on a line (probably in order A-B-C), with segments AB and BC forming parts of AC. Recognizing this is crucial because it allows us to set up a relationship between the segments using algebra.

Decoding the Geometric Setup

Key assumptions:

- The points A, B, and C are collinear, with B between A and C.
- The segments AB and BC together make up the entire segment AC.
- The expressions $4x + 6$ and $7x + 15$ are the lengths of segments AB and BC, respectively.

This is a common scenario in geometry problems involving segments divided into parts with algebraic expressions. Understanding this helps us translate the problem into an algebraic equation.

Step-by-Step Approach to Solve the Problem

Step 1: Write the relationship between segments

Since A, B, and C are collinear with B between A and C, the length of AC is the sum of AB and BC:

$$
AC = AB + BC
$$

Substituting the given expressions:

$$
120 = (4x + 6) + (7x + 15)
$$

Step 2: Simplify the equation

Combine like terms:

$$
120 = 4x + 6 + 7x + 15
$$

$$
120 = (4x + 7x) + (6 + 15)
$$

$$
120 = 11x + 21
$$

Step 3: Solve for x

Isolate x:

$$
11x = 120 - 21
$$

$$
11x = 99
$$

$$
x = \frac{99}{11} = 9
$$

Step 4: Find the length of AB

Recall the expression for AB:

$$
AB = 4x + 6
$$

Substitute $x = 9$:

$$
AB = 4(9) + 6 = 36 + 6 = 42
$$

Therefore, the length of segment AB is 42 units.

Additional Insights and Tips

Understanding the process:

- Always verify the assumptions about the geometry before setting up equations.
- Recognize that segments on a straight line can be added directly if they are contiguous.
- Algebraic expressions often represent variable parts; solving for the variable helps find specific segment lengths.

Common pitfalls:

- Forgetting to combine like terms correctly.
- Misinterpreting the problem setup (e.g., assuming segments are in different arrangements).
- Overlooking the importance of the total length when setting up the equation.

Practical tips:

- Draw a clear diagram with labeled segments and expressions.
- Write down what is known and what needs to be found.
- Check your solution by substituting back into the original expressions.

Extensions and Related Problems

Once comfortable with this type of problem, you can explore related challenges:

- Segment division problems with ratios (e.g., point B divides AC in a specific ratio).
- Coordinate geometry approaches, where points are assigned coordinates, and distances are calculated using the distance formula.
- Applying the segment addition postulate in more complex figures such as polygons or intersecting lines.

Conclusion

In The Figure Above AB = 4x + 6 And BC = 7x + 15. If AC = 120, Find The Length Of Segment AB exemplifies the power of algebraic reasoning in geometry. By carefully translating the problem into an algebraic equation, simplifying, and solving systematically, we obtained that the length of AB is 42 units. Mastering such problems enhances your problem-solving skills, boosts confidence in algebra, and deepens your geometric understanding. Keep practicing similar problems to build a strong foundation in these essential mathematical concepts!

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