

In What Ratio Is The Line Segment Joining The Points $(-3, 2)$ And $(6, 1)$ Divided By Y Axis?

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Understanding how a line segment interacts with the coordinate axes is fundamental in coordinate geometry. When dealing with points plotted on the Cartesian plane, a common question involves determining the point where a line segment connecting two points intersects a particular axis, and then finding the ratio in which this point divides the segment. Specifically, the problem: In what ratio is the line segment joining the points $(-3, 2)$ and $(6, 1)$ divided by the y-axis? is a classic example that helps deepen one's understanding of coordinate geometry concepts like section formula, division ratios, and the intersection of lines with axes.

This comprehensive article aims to explore this problem step by step, providing clarity on the methods used to find the ratio and explaining the underlying principles involved. Whether you're a student preparing for exams or an enthusiast seeking to solidify your understanding of coordinate geometry, this guide will serve as an informative resource.

Understanding the Problem

Before diving into calculations, it's essential to interpret what the problem is asking.

- Points involved:
 - Point A: $(-3, 2)$
 - Point B: $(6, 1)$
- Line segment: The straight line connecting A and B.
- Axis of interest: The y-axis, which is the line $x = 0$.
- Question: The line segment AB is intersected by the y-axis at a certain point, say P. The goal is to find the ratio in which P divides the segment AB.

Fundamental Concepts in Coordinate Geometry

To effectively solve this problem, it's crucial to understand some key concepts:

1. Coordinates of Points

Every point in a plane is represented by an ordered pair (x, y) , where x and y are real numbers.

2. The Line Segment Connecting Two Points

Given points $A(x_1, y_1)$ and $B(x_2, y_2)$, the segment AB is the straight line between these points.

3. Intersection with the Y-Axis ($x = 0$)

Any point on the y -axis has an x -coordinate of zero. To find where the segment AB intersects the y -axis, we need to find the point $P(x=0, y)$.

4. Section Formula

The section formula gives the coordinates of a point dividing a line segment in a given ratio.

- If a point P divides the segment AB in the ratio $m:n$, then:

$$\begin{aligned} x_P &= \frac{mx_2 + nx_1}{m + n} \quad \text{and} \quad y_P = \frac{my_2 + ny_1}{m + n} \end{aligned}$$

- Conversely, if we know the coordinates of P and the points A and B , we can find the ratio $m:n$.

Step-by-Step Solution

Now, we'll proceed to find the point of intersection P of the segment AB with the y -axis, and then determine the ratio in which P divides AB .

Step 1: Write down the coordinates of the points

- $A: (-3, 2)$
- $B: (6, 1)$

Step 2: Find the coordinates of the intersection point P with $x=0$

The point P lies on the line segment AB and on the y -axis, which has $x=0$.

The equation of the line passing through A and B can be written parametrically or in slope-intercept form.

Step 3: Find the equation of line AB

- Slope of AB:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 2}{6 - (-3)} = \frac{-1}{9} = -\frac{1}{9}$$

- Equation of line AB:

Using point-slope form with point A(-3, 2):

$$y - 2 = -\frac{1}{9}(x + 3)$$

Simplify:

$$\begin{aligned} y - 2 &= -\frac{1}{9}x - \frac{1}{3} \\ y &= -\frac{1}{9}x - \frac{1}{3} + 2 \\ y &= -\frac{1}{9}x + \frac{5}{3} \end{aligned}$$

Step 4: Find the intersection point P (x=0)

Set x=0 in the line equation:

$$y = -\frac{1}{9}(0) + \frac{5}{3} = \frac{5}{3}$$

Thus, the point of intersection is:

$$P = (0, \frac{5}{3})$$

Determining the Division Ratio

Now, with P(0, 5/3), and points A(-3, 2), B(6, 1), we need to find the ratio m:n in which P divides AB.

Step 5: Apply the section formula in reverse

- We know P divides AB in ratio m:n, and:

$$x_P = \frac{mx_2 + nx_1}{m + n}$$

$$y_P = \frac{my_2 + ny_1}{m + n}$$

- Use the x-coordinate:

$$0 = \frac{m \times 6 + n \times (-3)}{m + n}$$

Multiply both sides by (m + n):

$$0 = 6m - 3n$$

Rearranged:

$$6m = 3n \Rightarrow 2m = n$$

- Use the y-coordinate to find the actual ratio:

$$\frac{5}{3} = \frac{m \times 1 + n \times 2}{m + n}$$

Substitute $n = 2m$:

$$\frac{5}{3} = \frac{m \times 1 + 2m \times 2}{m + 2m} = \frac{m + 4m}{3m} = \frac{5m}{3m} = \frac{5}{3}$$

This confirms the ratio is consistent.

Final Answer and Interpretation

From the calculations:

$$\begin{aligned} & \backslash[\\ & n = 2m \\ & \backslash] \end{aligned}$$

Dividing both sides by m:

$$\begin{aligned} & \backslash[\\ & \frac{n}{m} = 2 \\ & \backslash] \end{aligned}$$

Therefore, the point P divides the segment AB in the ratio:

$$\begin{aligned} & \backslash[\\ & \boxed{\text{Ratio } \frac{AP}{PB} = \frac{m}{n} = \frac{1}{2}} \\ & \backslash] \end{aligned}$$

Alternatively, the segment is divided in the ratio 1:2, with:

$$\text{- AP : PB = 1 : 2}$$

This means that the point P, where the segment AB intersects the y-axis, divides the segment into two parts, with AP being one-third and PB being two-thirds of the total length of AB.

Summary of Key Steps

1. Identify points and axes: Recognize the points involved and the line of intersection (y-axis, $x=0$).
2. Find the equation of the line AB: Calculate the slope and derive the line's equation.
3. Calculate the intersection point: Plug $x=0$ into the line equation to find P.
4. Apply the section formula: Use the coordinates of A, B, and P to find the ratio $m:n$.
5. Interpret the ratio: Understand what the ratio indicates about how P divides AB.

Additional Tips and Concepts

- Line equations: Always check the slope before writing the line equation.
- Intersection points: To find where a line intersects an axis, substitute the axis coordinate ($x=0$ or $y=0$).
- Section formula: Useful for dividing a segment internally in a given ratio.
- Ratios and distances: The ratio $m:n$ can also be interpreted in terms of distances from P to A and B, especially when considering the division of the segment.

Practical Applications

Understanding how to find the division ratio of a segment by a line is important in various fields such as:

- Engineering: Structural analysis and design.
- Computer Graphics: Rendering and object positioning.
- Navigation: Dividing routes or paths proportionally.
- Mathematics Education: Building foundational knowledge in coordinate geometry.

Conclusion

In conclusion, the line segment joining the points $(-3, 2)$ and $(6, 1)$ is divided by the y-axis at a point that splits the segment in a ratio of 1:2. This detailed exploration demonstrates the application of the slope-intercept form, intersection calculation, and the section formula to solve real-world problems in coordinate geometry. Mastering these techniques enhances problem-solving skills and provides a strong foundation for more advanced mathematical concepts.

Keywords: coordinate geometry, division ratio, line segment, intersection with y-axis, section formula, points, slope, equations of lines, mathematical problem-solving

Frequently Asked Questions

In what ratio is the line segment joining the points $(-3, 2)$ and $(6, 1)$ divided by the y-axis?

The line segment is divided by the y-axis in the ratio 1:2.

How do you find the ratio in which the y-axis divides the segment joining $(-3, 2)$ and $(6, 1)$?

Use the section formula: If the y-axis divides the segment, then the x-coordinate of the dividing point is zero. Solving for the ratio gives the division as 1:2.

What is the coordinate of the point where the line segment joining $(-3, 2)$ and $(6, 1)$ is divided by the y-axis?

The point of division on the y-axis is at $(0, 1.33)$, dividing the segment in the ratio 1:2.

Does the y-axis divide the segment joining $(-3, 2)$ and $(6, 1)$ equally?

No, the y-axis divides the segment in the ratio 1:2, not equally.

What is the significance of the ratio in which the y-axis divides the segment between these points?

The ratio indicates how the segment is partitioned by the y-axis, showing that the point divides the segment closer to the point $(-3, 2)$ than to $(6, 1)$ in a 1:2 ratio.

Additional Resources

In What Ratio Is The Line Segment Joining The Points $(-3, 2)$ And $(6, 1)$ Divided By Y Axis?

Understanding the precise division of a line segment by a specific axis is fundamental in coordinate geometry, a branch of mathematics that helps us analyze and interpret spatial relationships. One common question posed by students and professionals alike involves determining the ratio in which a given line segment is divided by the Y-axis, especially when the segment connects two points with different coordinates. In this article, we'll explore this problem in detail, focusing on the line segment joining the points $(-3, 2)$ and $(6, 1)$, and analyze how the Y-axis divides this segment.

The Foundations of Coordinate Geometry and Line Segments

Before diving into the specific problem, it's essential to understand some foundational concepts:

- Coordinate Plane: A two-dimensional plane where every point is identified by an ordered pair (x, y) .
- Line Segment: The part of a line connecting two points, denoted as $A(x_1, y_1)$ and $B(x_2, y_2)$.
- Division of a Line Segment: When a point P lies on the line segment AB , it divides the segment into two parts with a certain ratio, often expressed as $AP:PB$.

In our problem, we are given two points:

- Point A: $(-3, 2)$
- Point B: $(6, 1)$

We are asked to find the ratio in which the Y-axis (the vertical line $x=0$) divides the segment AB .

Visualizing the Problem: Plotting the Points

To better understand the problem, visualizing the points on the coordinate plane helps:

- Point A is located at (-3, 2), which lies 3 units to the left of the Y-axis and 2 units above the x-axis.
- Point B is at (6, 1), which is 6 units to the right of the Y-axis and 1 unit above the x-axis.

Connecting these points, the segment runs from the left side of the Y-axis to the right side, crossing some point on the way. The key question: at what point along this segment does it intersect the Y-axis, and what is the ratio in which this point divides the segment?

Step 1: Determining the Equation of the Line Segment

To find where the segment intersects the Y-axis, it's essential to first establish the equation of the line passing through points A and B.

Calculating the slope (m):

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 2}{6 - (-3)} = \frac{-1}{9} = -\frac{1}{9}$$

Equation of the line:

Using point-slope form with point A(-3, 2):

$$y - y_1 = m(x - x_1)$$
$$y - 2 = -\frac{1}{9}(x + 3)$$

Simplify:

$$y = -\frac{1}{9}(x + 3) + 2$$

Expressed explicitly:

$$y = -\frac{1}{9}x - \frac{1}{3} + 2$$
$$y = -\frac{1}{9}x + \frac{5}{3}$$

This is the equation of the line passing through points A and B.

Step 2: Finding the Intersection with the Y-Axis

The Y-axis corresponds to the line $x=0$. To find the point where our segment intersects the Y-axis, substitute $x=0$ into the line's equation:

$$y = -\frac{1}{9}(0) + \frac{5}{3} = 0 + \frac{5}{3} = \frac{5}{3}$$

Thus, the point of intersection is at:

$$(0, \frac{5}{3}) \approx (0, 1.666...)$$

This point lies somewhere between the two original points, as it is on the segment connecting A and B.

Step 3: Determining the Ratio in Which the Y-Axis Divides the Segment

Now that we know the intersection point, the next step involves calculating the ratio in which this point divides the segment AB.

Methodology:

The key idea is to use the concept of section formula (also known as the internal division formula). If a point P divides the segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m:n$, then:

$$x_p = \frac{mx_2 + nx_1}{m + n}$$
$$y_p = \frac{my_2 + ny_1}{m + n}$$

Given that P lies on the segment and we are looking for the ratio $m:n$, we can use the coordinates of P, A, and B.

Applying the section formula:

$$\begin{aligned} - \quad & x_p = 0 \\ - \quad & y_p = \frac{5}{3} \\ - \quad & x_1 = -3, \quad y_1 = 2 \\ - \quad & x_2 = 6, \quad y_2 = 1 \end{aligned}$$

From the x-coordinate:

$$0 = \frac{m \times 6 + n \times (-3)}{m + n}$$

\]

Cross-multiplied:

\[

$$0 \times (m + n) = 6m - 3n$$

\]

\[

$$0 = 6m - 3n$$

\]

Simplify:

\[

$$6m = 3n$$

\]

\[

$$2m = n$$

\]

From the y-coordinate:

\[

$$\frac{5}{3} = \frac{m \times 1 + n \times 2}{m + n}$$

\]

Express numerator:

\[

$$m + 2n$$

\]

Express denominator:

\[

$$m + n$$

\]

So:

\[

$$\frac{5}{3} = \frac{m + 2n}{m + n}$$

\]

Using the relation $(n = 2m)$, substitute:

\[

$$\frac{5}{3} = \frac{m + 2 \times 2m}{m + 2m} = \frac{m + 4m}{3m} = \frac{5m}{3m} =$$

\]

This confirms the ratio is consistent.

Final Ratio:

Since:

$$\begin{aligned} &\backslash \\ n &= 2m \\ &\backslash \end{aligned}$$

The ratio in which the Y-axis divides the segment is:

$$\begin{aligned} &\backslash \\ AP : PB &= m : n = 1 : 2 \\ &\backslash \end{aligned}$$

In other words, the point where the segment intersects the Y-axis divides the segment in the ratio 1:2, with the division closer to point A and extending toward point B.

Interpretation of the Result

The division ratio of 1:2 indicates that:

- The intersection point divides the segment into two parts, with the segment closer to point A being one-third of the entire length, and the part closer to point B being two-thirds.
- The point on the Y-axis at $(0, 5/3)$ is closer to point A than to point B, consistent with the ratio.

This division ratio is crucial in various applications such as:

- Geometric constructions
- Analyzing the centroid or other special points
- Engineering designs involving coordinate systems

Summary and Practical Implications

Understanding how a line segment is divided by a specific axis or line is a vital skill in coordinate geometry, with numerous practical applications:

- In navigation and mapping: Determining exact points along paths.
- In engineering: Designing components that must meet specific spatial criteria.
- In computer graphics: Calculating intersections and divisions for rendering.

The process involves deriving the equation of the line, locating the intersection point, and then applying the section formula to find the ratio of division. This approach combines algebraic manipulation with geometric intuition, providing a comprehensive method to analyze spatial

relationships.

Conclusion

Through detailed calculation and geometric reasoning, we have established that the line segment joining the points $(-3, 2)$ and $(6, 1)$ is divided by the Y-axis in the ratio 1:2. This means that the intersection point at $(0, 5/3)$ divides the segment into two parts: one-third and two-thirds of the total length, with the division closer to point A. Such insights are fundamental in the study of coordinate geometry and serve as a foundation for more complex spatial analyses.

Mastering these concepts enhances not only mathematical proficiency but also the ability to solve real-world problems involving spatial relationships and geometric constructions.

In What Ratio Is The Line Segment Joining The Points 3 2 And 6 1 Is Divided By Y Axis

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