

Let θ Be An Angle Such That $\cot \theta = 83$ and $\sin \theta > 0$. Find The Exact Values Of $\sec \theta$ And $\cos \theta$.

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Understanding the relationships between trigonometric functions is fundamental in mathematics, especially in the study of angles and their properties. In this article, we will explore how to determine the exact values of $\sec \theta$ and $\cos \theta$ given that $\cot \theta = 83$ and $\sin \theta > 0$. This problem involves leveraging core identities and properties of trigonometric functions, particularly within specific quadrants.

Introduction to the Problem

Given:

- $\cot \theta = 83$
- $\sin \theta > 0$

Our goal:

- Find the exact values of $\sec \theta$ (secant of θ)
- Find the exact values of $\cos \theta$ (cosine of θ)

This problem requires a solid understanding of the definitions and relationships between cotangent, sine, cosine, and secant functions.

Understanding the Trigonometric Functions Involved

Definitions and Basic Relationships

- Cotangent: $\cot \theta = \cos \theta / \sin \theta$
- Sine: $\sin \theta$
- Cosine: $\cos \theta$
- Secant: $\sec \theta = 1 / \cos \theta$

Given $\cot \theta = 83$, which is a very large value, indicates that the angle θ is located in a quadrant where sine is positive, i.e., the first or second quadrant.

Since it's given that $\sin \theta > 0$, θ must be in Quadrant I ($0 < \theta < 90^\circ$) or Quadrant II ($90^\circ < \theta < 180^\circ$).

But because cotangent is positive (83), which is a very large positive number, and cotangent is positive in Quadrants I and III, the fact that $\sin \theta > 0$ implies that θ is in Quadrant I.

Step-by-Step Solution Approach

To find $\sec \theta$ and $\cos \theta$, we need to:

1. Use the given cotangent value to find $\sin \theta$ and $\cos \theta$.
2. Calculate $\sec \theta = 1 / \cos \theta$.
3. Confirm the signs based on the quadrant information.

Step 1: Expressing $\sin \theta$ and $\cos \theta$ Using $\cot \theta$

Recall that:

- $\cot \theta = \cos \theta / \sin \theta$
- Given $\cot \theta = 83$

Let's denote:

- $\sin \theta = s$
- $\cos \theta = c$

Then:

$$c / s = 83$$

$$\Rightarrow c = 83s$$

Since $\sin \theta > 0$ and θ is in Quadrant I, both s and c are positive.

Step 2: Use Pythagorean Identity to Find s and c

We know the fundamental Pythagorean identity:

$$s^2 + c^2 = 1$$

Substitute $c = 83s$:

$$s^2 + (83s)^2 = 1$$

$$\Rightarrow s^2 + 6889s^2 = 1$$

$$\Rightarrow (1 + 6889)s^2 = 1$$

$$\Rightarrow 6890s^2 = 1$$

$$\Rightarrow s^2 = 1 / 6890$$

Since $\sin \theta > 0$:

$$s = \sqrt{1 / 6890} = 1 / \sqrt{6890}$$

$$\text{Similarly, } c = 83s = 83 / \sqrt{6890}$$

Step 3: Find $\sec \theta$ and $\cos \theta$

- Cosine of θ :

$$c = 83 / \sqrt{6890}$$

- Secant of θ :

$$\sec \theta = 1 / c = 1 / (83 / \sqrt{6890}) = \sqrt{6890} / 83$$

Final Exact Values

- $\cos \theta$:

$$\boxed{\cos \theta = \frac{83}{\sqrt{6890}}}$$

- $\sec \theta$:

$$\boxed{\sec \theta = \frac{\sqrt{6890}}{83}}$$

$$\sec \theta = \frac{\sqrt{6890}}{83}$$

Additional Notes and Simplifications

While the radical in the denominator of $\cos \theta$ can be rationalized for some applications, often in trigonometry, leaving the radical in the denominator is acceptable unless rationalization is explicitly required.

The value of $\sqrt{6890}$ can be approximated but is typically left in radical form for exactness. If needed, you can approximate:

- $\sqrt{6890} \approx 83.00$ (since $83^2 = 6889$), so:

$$\cos \theta \approx \frac{83}{83} = 1$$

but that is an approximation; the exact radical form remains more precise.

Summary of Results

Function	Exact Value
$\cos \theta$	$\frac{83}{\sqrt{6890}}$
$\sec \theta$	$\frac{\sqrt{6890}}{83}$

These values satisfy the given conditions and are derived using fundamental identities and properties of the trigonometric functions.

Conclusion

By carefully analyzing the relationships between cotangent, sine, cosine, and secant, and leveraging the Pythagorean identity, we were able to determine the exact values of $\sec \theta$ and $\cos \theta$ given $\cot \theta = 83$ with $\sin \theta > 0$. This approach highlights the importance of understanding the fundamental identities and the significance of the quadrant in determining the signs and values of trigonometric functions. Mastery of these concepts is crucial for solving complex trigonometric problems accurately.

and efficiently.

Additional Resources

- Trigonometric Identities: A comprehensive understanding of identities like Pythagorean, reciprocal, and quotient identities is essential.
- Quadrant Sign Rules: Remember which functions are positive or negative in each quadrant.
- Rationalizing Denominators: Practice simplifying radical expressions for cleaner exact values.

If you want to explore more advanced trigonometry problems or need further explanations, numerous online resources and textbooks can deepen your understanding of these fundamental concepts.

Frequently Asked Questions

Given that $\cot \theta = 83$ and $\sin \theta > 0$, what is the value of $\tan \theta$?

Since $\cot \theta = 83$, then $\tan \theta = 1/83$.

How do you find $\sin \theta$ when $\cot \theta = 83$ and $\sin \theta > 0$?

Using $\cot \theta = \cos \theta / \sin \theta$, and knowing $\cot \theta = 83$, we can set up a right triangle or use Pythagoras to find $\sin \theta = 1 / \sqrt{1 + \cot^2 \theta} = 1 / \sqrt{1 + 83^2} = 1 / \sqrt{1 + 6889} = 1 / \sqrt{6890}$.

What is the exact value of $\sin \theta$ if $\cot \theta = 83$ and $\sin \theta > 0$?

$\sin \theta = 1 / \sqrt{6890}$, which can be written as $\sqrt{6890} / 6890$ after rationalizing the denominator.

How do you find $\cos \theta$ given $\cot \theta = 83$ and $\sin \theta > 0$?

Using the identity $\cos \theta = \cot \theta \times \sin \theta$, so $\cos \theta = 83 \times (1 / \sqrt{6890}) = 83 / \sqrt{6890}$.

What is the exact value of $\sec \theta$ in this scenario?

$\sec \theta = 1 / \cos \theta = \sqrt{6890} / 83$.

How can we verify that the values of $\sin \theta$ and $\cos \theta$ are correct given $\cot \theta = 83$?

Check that $\sin^2 \theta + \cos^2 \theta = 1$. Using $\sin \theta = 1 / \sqrt{6890}$ and $\cos \theta = 83 / \sqrt{6890}$, then $(1 / \sqrt{6890})^2 + (83 / \sqrt{6890})^2 = 1/6890 + 6889/6890 = (1 + 6889)/6890 = 6890/6890 = 1$, confirming correctness.

Why is $\sin \theta > 0$ in this problem, and how does it affect the values?

Since $\sin \theta > 0$, θ is in the first or second quadrant, ensuring that cosine and secant values are positive or negative accordingly; here, both are positive because $\cot \theta$ is positive and $\sin \theta > 0$.

Can you express the exact values of $\sec \theta$ and $\cos \theta$ in simplest radical form?

Yes, $\sec \theta = \sqrt{6890} / 83$ and $\cos \theta = 83 / \sqrt{6890}$. Rationalizing the denominator for $\cos \theta$ gives $\cos \theta = (83\sqrt{6890}) / 6890$.

How do the given values of $\cot \theta$ influence the values of $\sec \theta$ and $\cos \theta$?

Since $\cot \theta = 83$, a large value, it indicates θ is close to 0° , making $\sin \theta$ small and $\cos \theta$ large; this directly affects $\sec \theta$, which becomes large, and $\cos \theta$, which is proportional to $1 / \sqrt{6890}$.

What is the significance of the condition $\sin \theta > 0$ in determining the exact values?

It ensures θ is in the first quadrant, where both $\sin \theta$ and $\cos \theta$ are positive, guiding the correct signs for the trigonometric functions and their reciprocals.

Additional Resources

Understanding the Problem:

Given an angle θ such that $\cot \theta = 83$ and $\sin \theta > 0$, our goal is to find the exact values of $\sec \theta$ and $\cos \theta$.

This problem involves fundamental trigonometric identities and the relationships between the various trigonometric functions. To approach this systematically, we need to understand the given information, interpret it correctly, and then apply the appropriate mathematical tools to derive the exact values.

1. Basic Concepts and Notation

Before diving into the solution, let's clarify the key trigonometric functions involved and their interrelationships:

- $\sin \theta$: The ratio of the length of the side opposite to θ to the hypotenuse in a right-angled triangle.

- $\cos \theta$: The ratio of the length of the adjacent side to the hypotenuse.
- $\tan \theta$: The ratio $\frac{\sin \theta}{\cos \theta}$.
- $\cot \theta$: The reciprocal of $\tan \theta$, i.e., $\frac{\cos \theta}{\sin \theta}$.
- $\sec \theta$: The reciprocal of $\cos \theta$, i.e., $\frac{1}{\cos \theta}$.
- $\csc \theta$: The reciprocal of $\sin \theta$, i.e., $\frac{1}{\sin \theta}$.

Given that $\cot \theta = 83$ and $\sin \theta > 0$, we are working in a quadrant where sine is positive, specifically the first or second quadrant.

2. Analyzing the Given Data

Let's interpret the information:

- $\cot \theta = 83$: This is a very large value, indicating that $\tan \theta$ is very small because $\cot \theta = \frac{1}{\tan \theta}$.
- $\sin \theta > 0$: The sine being positive restricts θ to quadrants I and II, where sine is positive.

Given that $\cot \theta = 83$, and $\cot \theta = \frac{\cos \theta}{\sin \theta}$, we can infer:

$$\frac{\cos \theta}{\sin \theta} = 83$$

This relation is crucial because it links $\cos \theta$ and $\sin \theta$.

3. Expressing $\cos \theta$ and $\sin \theta$ in Terms of a Single Variable

Since $\cot \theta = 83$, we can set:

$$\frac{\cos \theta}{\sin \theta} = 83$$

This suggests:

$$\cos \theta = 83 \sin \theta$$

Now, because $(\sin \theta > 0)$, and $(\cos \theta)$ is related via the Pythagorean identity:

$$\sin^2 \theta + \cos^2 \theta = 1$$

Substituting $(\cos \theta = 83 \sin \theta)$:

$$\sin^2 \theta + (83 \sin \theta)^2 = 1$$

$$\sin^2 \theta + 83^2 \sin^2 \theta = 1$$

$$\sin^2 \theta (1 + 83^2) = 1$$

Calculating (83^2) :

$$83^2 = 6889$$

Thus,

$$\sin^2 \theta (1 + 6889) = 1$$

$$\sin^2 \theta \times 6890 = 1$$

$$\sin^2 \theta = \frac{1}{6890}$$

Since $(\sin \theta > 0)$:

$$\sin \theta = \frac{1}{\sqrt{6890}}$$

Similarly, to find $(\cos \theta)$:

$$\cos \theta = 83 \sin \theta = 83 \times \frac{1}{\sqrt{6890}} = \frac{83}{\sqrt{6890}}$$

4. Calculating $\sec \theta$ and $\cos \theta$ Exactly

Having found $\cos \theta$, we can now compute the required functions:

a) $\cos \theta$:

$$\boxed{\cos \theta = \frac{83}{\sqrt{6890}}}$$

b) $\sec \theta$:

Since $\sec \theta = \frac{1}{\cos \theta}$:

$$\sec \theta = \frac{1}{\frac{83}{\sqrt{6890}}} = \frac{\sqrt{6890}}{83}$$

Thus:

$$\boxed{\sec \theta = \frac{\sqrt{6890}}{83}}$$

5. Simplification and Exact Values

To present the exact values clearly, it's often preferable to rationalize denominators or express the radical in simplified form if possible.

Simplify $\sqrt{6890}$:

- Prime factorization of 6890:

Let's factor 6890:

First, note that:

$\sqrt{}$

$$6890 = 2 \times 3445$$

\]

Now, factor 3445:

\[

$$3445 = 5 \times 689$$

\]

Factor 689:

\[

$$689 = 13 \times 53$$

\]

Putting it all together:

\[

$$6890 = 2 \times 5 \times 13 \times 53$$

\]

Since these are all prime factors, the radical form is:

\[

$$\sqrt{6890} = \sqrt{2 \times 5 \times 13 \times 53}$$

\]

None of these are perfect squares, so the radical cannot be simplified further.

Therefore:

\[

\boxed{

$$\cos \theta = \frac{83}{\sqrt{6890}}, \quad \sec \theta = \frac{\sqrt{6890}}{83}$$

}

\]

6. Final Remarks and Summary of Results

- The exact value of $\cos \theta$:

\[

\boxed{

$$\cos \theta = \frac{83}{\sqrt{6890}}$$

}

\]

- The exact value of $\sec \theta$:

$$\boxed{\sec \theta = \frac{\sqrt{6890}}{83}}$$

These values are consistent with the initial conditions, including $\sin \theta > 0$, and the large value of $\cot \theta$.

7. Additional Insights and Geometrical Interpretation

Understanding the geometric implications can deepen comprehension:

- In a right-angled triangle, if $\cot \theta = 83$, then the adjacent side to θ (say, A) and the opposite side (O) relate as:

$$\frac{A}{O} = 83$$

- Assigning:

$$A = 83k, \quad O = k$$

- The hypotenuse H is:

$$H = \sqrt{A^2 + O^2} = \sqrt{(83k)^2 + k^2} = \sqrt{6890k^2} = k \sqrt{6890}$$

- The trigonometric functions:

$$\sin \theta = \frac{O}{H} = \frac{k}{k \sqrt{6890}} = \frac{1}{\sqrt{6890}}$$

$$\cos \theta = \frac{A}{H} = \frac{83k}{k \sqrt{6890}} = \frac{83}{\sqrt{6890}}$$

- The sign of $\cos \theta$ depends on the quadrant. Since $\sin \theta > 0$, θ is in either Quadrant I or II. Given $\cot \theta = 83 > 0$, $\cot \theta$ is positive in Quadrants I and III, but since $\sin \theta > 0$, it restricts θ to Quadrant I (both sine and cotangent positive) or

Quadrant II (sine positive, cotangent negative). Here, because $\cot \theta = 83 > 0$, θ must be in Quadrant I.

Thus, both $\sin \theta$ and $\cos \theta$ are positive in this context, making the computed values consistent with the geometric interpretation.

8. Summary of Key Results

Function	Exact Value
$\sin \theta$	$\frac{1}{\sqrt{6890}}$
$\cos \theta$	

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