Let X Be N (u, A^2) So That P(X < 62.8) = 0.90 And P(X < 66.5) = 0.95. Find U And A^2

Let X Be N (u, A^2) So That P(X < 62.8) = 0.90 And P(X < 66.5) = 0.95. Find U And A^2

Understanding the properties of the normal distribution is fundamental in statistics, especially when analyzing data and making inferences about population parameters. In this article, we will explore how to determine the mean (μ) and variance (σ^2) of a normal distribution given specific probability statements. We will systematically approach this problem, explaining the underlying concepts and demonstrating the step-by-step calculations involved.

Introduction to Normal Distribution

The normal distribution, often called the bell curve, is a continuous probability distribution characterized by its symmetric shape centered around the mean. It is defined by two parameters:
- The mean (μ), which indicates the center of the distribution.
- The variance (σ^2), which measures the spread or dispersion of the data.

Mathematically, the probability density function (pdf) of a normal distribution is expressed as:

$$
f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x - \mu)^2}{2\sigma^2} \right)
$$

Understanding how to work with this distribution involves familiarity with standardization and the standard normal distribution.

Standard Normal Distribution and Z-Score

The standard normal distribution, denoted as $Z \sim N(0,1)$, is a special case of the normal distribution with a mean of 0 and variance of 1. To relate any normal distribution $N(\mu, \sigma^2)$ to the standard normal, we use the Z-score transformation:

$$
Z = \frac{X - \mu}{\sigma}
$$

\]

This transformation allows us to find probabilities associated with X by referring to the standard normal distribution.

Given Probabilities and Their Significance

In our problem, we are given two probability statements:

- $P(X < 62.8) = 0.90$
- $P(X < 66.5) = 0.95$

These statements tell us that:
- The 90th percentile of the distribution is 62.8.
- The 95th percentile of the distribution is 66.5.

Our goal is to determine the mean (μ) and variance (σ^2) based on these percentiles.

Step-by-Step Solution Approach

To find μ and σ^2, follow these steps:
1. Convert the given probabilities into Z-scores using standard normal distribution tables or inverse functions.
2. Set up equations linking the Z-scores to the unknown μ and σ.
3. Solve the system of equations to find μ and σ.
4. Calculate the variance as σ^2.

Let's delve into each step in detail.

Step 1: Find Corresponding Z-Scores

The probabilities correspond to specific Z-scores in the standard normal distribution:

- For $P(Z < z_1) = 0.90$
- For $P(Z < z_2) = 0.95$

Using standard normal distribution tables or statistical software:
- $z_{0.90} \approx 1.2816$
- $z_{0.95} \approx 1.6449$

These Z-scores indicate the number of standard deviations the data points are from the mean in the standard normal distribution.

Step 2: Relate Z-Scores to X Values

Using the Z-score formula:

$$
Z = \frac{X - \mu}{\sigma}
$$

we can set up two equations:

$$
\frac{62.8 - \mu}{\sigma} = 1.2816
$$

$$
\frac{66.5 - \mu}{\sigma} = 1.6449
$$

These equations relate the unknown parameters μ and σ to known quantities.

Step 3: Solve the Equations for μ and σ

Subtract the first equation from the second:

$$
\frac{66.5 - \mu}{\sigma} - \frac{62.8 - \mu}{\sigma} = 1.6449 - 1.2816
$$

Simplify:

$$
\frac{(66.5 - \mu) - (62.8 - \mu)}{\sigma} = 0.3633
$$

$$
\frac{66.5 - \mu - 62.8 + \mu}{\sigma} = 0.3633
$$

$$
\frac{3.7}{\sigma} = 0.3633
$$

Solve for σ:

$$
\sigma = \frac{3.7}{0.3633} \approx 10.18
$$

Now, substitute σ back into one of the earlier equations to find μ:

$$
\frac{62.8 - \mu}{10.18} = 1.2816
$$

Multiply both sides by 10.18:

$$
62.8 - \mu = 1.2816 \times 10.18 \approx 13.05
$$

Solve for μ:

$$
\mu = 62.8 - 13.05 \approx 49.75
$$

Step 4: Calculate Variance

Since the variance is the square of the standard deviation:

$$
\sigma^2 = (10.18)^2 \approx 103.63
$$

Final Results:

- The mean (μ) is approximately 49.75.
- The variance (σ^2) is approximately 103.63.

Summary of Findings

Parameter	Estimated Value
Mean (μ)	49.75
Variance (σ^2)	103.63

These parameters characterize the normal distribution that satisfies the given probability conditions.

Conclusion

In this comprehensive analysis, we demonstrated how to determine the mean and variance of a normal distribution based on specific percentile probabilities. The key steps involved converting probabilities into Z-scores, relating these

Z-scores to the original data points, and solving the resulting equations. Mastery of these techniques is essential for statisticians and data analysts working with normally distributed data, enabling them to infer distribution parameters precisely and efficiently.

Understanding this process enhances your ability to interpret statistical data, perform hypothesis testing, and construct confidence intervals — all fundamental tools in the arsenal of modern statistics.

Frequently Asked Questions

Given $X \sim N(\mu, \sigma^2)$ with $P(X < 62.8) = 0.90$ and $P(X < 66.5) = 0.95$, how do you find the mean μ?

To find μ, identify the z-scores corresponding to 0.90 and 0.95, then set up equations using the inverse normal function: $\mu = x - z\sigma$. Using the z-scores for 0.90 (~1.28) and 0.95 (~1.64), solve the system to find μ.

What is the significance of the probabilities $P(X < 62.8) = 0.90$ and $P(X < 66.5) = 0.95$ in this problem?

They specify the cumulative distribution function (CDF) values at specific points, allowing us to find the corresponding z-scores and subsequently the mean and variance.

How do you determine the z-scores associated with the given probabilities?

Use the standard normal table or inverse normal function: for 0.90, $z \approx 1.28$; for 0.95, $z \approx 1.64$.

Once the z-scores are known, how do you compute the mean μ?

Use the formula $x = \mu + z\sigma$. Rearranged, $\mu = x - z\sigma$. Plug in the x-values and the z-scores to solve for μ once σ is known or expressed in terms of μ.

How do you find the variance σ^2 in this problem?

Use the two points and their corresponding z-scores to set up two equations. Solving these equations simultaneously yields σ, and squaring it gives σ^2.

Can you explain the step-by-step process to find μ

and σ^2 from the given probabilities?

Yes. First, identify z-scores for the given probabilities (1.28 and 1.64). Then, set up equations: $62.8 = \mu + 1.28\sigma$ and $66.5 = \mu + 1.64\sigma$. Solve these simultaneously to find μ and σ. Finally, compute σ^2.

What assumptions are made when solving for μ and σ^2 in this problem?

The main assumption is that X follows a normal distribution $N(\mu, \sigma^2)$, which allows us to use z-scores and properties of the normal distribution to find parameters.

Is there a shortcut to find μ and σ^2 without solving complex equations directly?

Typically, you need to set up and solve the equations using the z-scores and the given x-values. No significant shortcut exists beyond using the standard normal table and algebraic manipulation.

What is the final step after calculating μ and σ^2 for this problem?

Verify the results by checking if the calculated μ and σ^2 satisfy the original probability statements, ensuring consistency.

Additional Resources

Understanding the Problem:

The problem involves a normal distribution, denoted as $X \sim N(\mu, \sigma^2)$, where:

- μ is the mean of the distribution,
- σ^2 is the variance,
- σ is the standard deviation.

Given:

- $P(X < 62.8) = 0.90$,
- $P(X < 66.5) = 0.95$.

Our objective is to determine the parameters μ and σ^2.

Introduction to Normal Distribution Parameters

The normal distribution is one of the most fundamental distributions in statistics, characterized entirely by its mean μ and variance σ^2. These parameters govern the shape and position of the bell curve:

- The mean μ determines the center or the expected value of the distribution.
- The variance σ^2, or the square of the standard deviation σ, measures the spread or dispersion.

In practice, the key to solving problems like this is understanding how probabilities relate to the standard normal distribution, often denoted as $Z \sim N(0, 1)$.

Standard Normal Distribution and Z-Scores

To find μ and σ^2, we leverage the properties of the standard normal distribution. The process involves:

1. Transforming the original variable X to a standard normal variable Z:

$$
Z = \frac{X - \mu}{\sigma}
$$

2. Using the given probabilities to find corresponding Z-scores:

$$
P(X < x) = P\left(Z < \frac{x - \mu}{\sigma} \right)
$$

This transformation allows us to use standard normal distribution tables or computational tools to find Z-values corresponding to given probabilities.

Step-by-Step Solution Approach

Step 1: Find the Z-scores corresponding to the given probabilities

- For $P(X < 62.8) = 0.90$,
- For $P(X < 66.5) = 0.95$.

Using standard normal distribution tables or software:

- $Z_{0.90} \approx 1.2816$,
- $Z_{0.95} \approx 1.6449$.

Step 2: Set up equations using the Z-score formula

From the definition:

$$
Z = \frac{x - \mu}{\sigma}
$$

we get:

$$
\frac{62.8 - \mu}{\sigma} = 1.2816
$$
$$
\frac{66.5 - \mu}{\sigma} = 1.6449
$$

Step 3: Solve for μ and σ

Subtract the first equation from the second:

$$
\frac{66.5 - \mu}{\sigma} - \frac{62.8 - \mu}{\sigma} = 1.6449 - 1.2816
$$

Simplify numerator:

$$
\frac{(66.5 - \mu) - (62.8 - \mu)}{\sigma} = 0.3633
$$

$$
\frac{66.5 - \mu - 62.8 + \mu}{\sigma} = 0.3633
$$

$$
\frac{3.7}{\sigma} = 0.3633
$$

Solve for σ:

$$
\sigma = \frac{3.7}{0.3633} \approx 10.18
$$

\]

Now, substitute \(\sigma \) back into one of the earlier equations:

\[
\frac{62.8 - \mu}{10.18} = 1.2816
\]

Solve for \(\mu \):

\[
62.8 - \mu = 1.2816 \times 10.18 \approx 13.05
\]

\[
\mu = 62.8 - 13.05 \approx 49.75
\]

Step 4: Find the variance \(\sigma^2 \):

\[
\sigma^2 = (10.18)^2 \approx 103.63
\]

Summary of Findings and Final Values

Based on the calculations:

\[
\boxed{
\text{Mean } \mu \approx 49.75
}
\]

\[
\boxed{
\text{Variance } \sigma^2 \approx 103.63
}
\]

The normal distribution parameters are:

\[
X \sim N(49.75, 103.63)
\]

Interpretation and Practical Implications

Understanding the parameters:

- The mean of approximately 49.75 indicates that, on average, the variable X centers around this value.
- The variance of about 103.63 suggests a spread around the mean, with a standard deviation:

$$
\sigma = \sqrt{103.63} \approx 10.18
$$

This spread indicates that most data points lie within roughly 10 units around the mean, following the empirical rule that about 95% of data in a normal distribution falls within two standard deviations.

Further Validation of the Results

To ensure the accuracy of the derived parameters, one can:

- Plug the values back into the Z-score formulas and verify that the computed probabilities match the original values.
- Use computational tools or software (like R, Python's SciPy library, or online calculators) to confirm the probabilities for the derived μ and σ.

Verification:

Calculate $P(X < 62.8)$ with $\mu = 49.75$ and $\sigma = 10.18$:

$$
Z = \frac{62.8 - 49.75}{10.18} \approx 1.28
$$

which corresponds to approximately 90%, consistent with the initial condition.

Similarly, for $P(X < 66.5)$:

$$
Z = \frac{66.5 - 49.75}{10.18} \approx 1.65
$$

which corresponds to approximately 95%, again confirming the consistency.

Broader Context and Applications

Why is this problem important?

- In real-world scenarios, knowing distribution parameters allows for better decision-making, risk assessment, and hypothesis testing.
- Fields like quality control, finance, environmental science, and engineering often rely on such calculations.

Potential applications:

- Estimating the average time between failures in manufacturing.
- Predicting the distribution of scores or measurements.
- Setting control limits in process control charts.

Additional Insights and Considerations

- Confidence Intervals: Once parameters are estimated, confidence intervals can be constructed to quantify the uncertainty.
- Assumptions: The calculation presumes the data follows a perfect normal distribution, which may not always be true in practice.
- Limitations: Small sample sizes or skewed data can violate the assumptions underlying the normal model.

Conclusion

In summary, given the probabilities at specific points, we have successfully deduced the parameters of the normal distribution:

$$
\boxed{
X \sim N(49.75, 103.63)
}
$$

This comprehensive process showcases how probability statements can be translated into distribution parameters, illustrating the power and utility of the normal distribution in statistical analysis. Mastery of such techniques is essential for statisticians, data analysts, and researchers who engage with probabilistic models and real-world data interpretation.

Note: For precise calculations or applications in sensitive contexts, always verify results with statistical software or detailed tables, considering the potential for rounding errors in Z-score approximations.

Let X Be N U A 2 So That P X Lt 62 8 0 90 And P X Lt 66 5 0 95 Find U And A 2

Let X Be N U A 2 So That P X Lt 62 8 0 90 And P X Lt 66 5 0 95 Find U And A 2

Related Articles

- How Do You Say "some Female Cousins" In Spanish?A. Unas PrimasB. Una PrimaC. Unos Primos
- A Logo Is 15 Cm Wide. If The Logo Is Dilated By A Factor Of 7/4, Find The Width Of The Logo
- If East St. Intersects Both North St. And South St.Are North St. And South St. Parallel?

Back to Home