Question 10 Find The Infinite Sum, If It Exists For This Series: (-2) + (0.5) + (-0.125) +.

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Understanding the Series and Its Components

When confronted with the series (-2) + (0.5) + (-0.125) + ..., the primary goal is to determine whether this infinite series converges to a finite sum and, if so, to find that sum. Infinite series are fundamental in mathematics, especially in calculus and analysis, as they allow us to understand sums of infinitely many terms that follow a specific pattern.

This particular series appears to be a geometric series, characterized by each term being a constant multiple of the previous one. Recognizing the pattern and understanding the properties of geometric series are essential steps in calculating the sum.

What Is an Infinite Geometric Series?

A geometric series is a series where each term is obtained by multiplying the previous term by a fixed, non-zero constant called the common ratio. Formally, a geometric series can be written as:

$$S = a + ar + ar^2 + ar^3 + \dots$$

where:
- a is the first term,
- r is the common ratio.

The key question is whether the series converges (approaches a finite value) or diverges (grows without bound). The geometric series converges if and only if:

$$|r| < 1$$

In such cases, the sum to infinity is given by the formula:

$$S_\infty = \frac{a}{1 - r}$$

provided $|r| < 1$.

Identifying the First Term and Common Ratio

Let's analyze the series:

$$ -2 + 0.5 - 0.125 + \dots $$

Step 1: Determine the first term a.

- The first term a is clearly:

$$ a = -2 $$

Step 2: Find the common ratio r.

- To find r, divide the second term by the first:

$$ r = \frac{\text{second term}}{\text{first term}} = \frac{0.5}{-2} = -0.25 $$

- Confirm this ratio by checking the third term:

$$ \text{third term} = a r^2 = -2 \times (-0.25)^2 = -2 \times 0.0625 = -0.125 $$

which matches the given third term.

Result:

- The common ratio:

$$ r = -0.25 $$

- The first term:

$$ a = -2 $$

Checking for Series Convergence

Since the series is geometric with $|r| = 0.25 < 1$, the series converges. Therefore, the infinite sum exists and can be calculated using the geometric series sum formula:

$$ S_\infty = \frac{a}{1 - r} $$

Note: The convergence condition is satisfied because the magnitude of the common ratio is less than 1.

Calculating the Infinite Sum

Using the identified values:

$$ a = -2 $$
$$ r = -0.25 $$

Plug into the formula:

$$ S_\infty = \frac{-2}{1 - (-0.25)} = \frac{-2}{1 + 0.25} = \frac{-2}{1.25} $$

Simplify:

$$ S_\infty = -2 \div 1.25 = -2 \times \frac{1}{1.25} $$

Recognize that:

$$ \frac{1}{1.25} = \frac{1}{\frac{5}{4}} = \frac{4}{5} $$

Therefore:

$$ S_\infty = -2 \times \frac{4}{5} = -\frac{8}{5} $$

Expressed as a decimal:

$$ -\frac{8}{5} = -1.6 $$

Final Answer:

The infinite sum of the series is:

$$ \boxed{-\frac{8}{5} \quad \text{or} \quad -1.6} $$

Summary and Key Takeaways

- Recognizing the series as geometric is crucial for analyzing its convergence.
- The first term a is -2.
- The common ratio r is -0.25.
- Since $|r| < 1$, the series converges.
- The sum of the infinite series is $-\frac{8}{5}$ or -1.6.

Additional Insights on Infinite Series

Understanding infinite series is foundational in advanced mathematics, physics, and engineering. Here are some key concepts:

- Divergent vs. Convergent Series: Series that grow without bound do not have a finite sum. Recognizing convergence criteria is essential.
- Geometric Series Applications: Used in calculating present value in finance, signal processing, and computer science algorithms.
- Power Series: Infinite series where terms involve powers of a variable, expanding functions into infinite sums (e.g., Taylor series).

Practical Examples of Geometric Series

1. Financial Calculations: Present value of annuities or loans often involves geometric series.
2. Signal Decay: Radioactive decay or capacitor discharge models involve geometric decay.
3. Computer Graphics: Fractal generation and recursive algorithms utilize geometric series.

Conclusion

The series (-2) + (0.5) + (-0.125) + ... is a classic example of a convergent geometric series. By identifying the first term and common ratio, applying the convergence condition, and calculating the sum, we find that the infinite sum exists and equals \(-\frac{8}{5}\) or -1.6. This process exemplifies the power of geometric series analysis and its broad applications across scientific disciplines.

Remember: Always verify the ratio's magnitude to determine convergence before calculating the sum. Infinite series are elegant tools that, when understood correctly, unlock deep insights into the behavior of mathematical and real-world systems.

Frequently Asked Questions

What is the common ratio in the series (-2) + 0.5 + (-0.125) + ...?

The common ratio is -0.25 because each term is multiplied by -0.25 to get the next term.

How do you determine if the infinite sum of a series exists?

An infinite geometric series converges (has a finite sum) if the absolute value of its common ratio is less than 1, i.e., |r| < 1.

What is the formula to find the sum of an infinite

geometric series?

The sum S of an infinite geometric series with first term a and common ratio r (where $|r| < 1$) is $S = a / (1 - r)$.

Calculate the sum of the series (-2) + 0.5 + (-0.125) + ...

Using a = -2 and r = -0.25, the sum is $S = -2 / (1 - (-0.25)) = -2 / 1.25 = -1.6$.

Does the series (-2) + 0.5 + (-0.125) + ... converge or diverge?

The series converges because the common ratio r = -0.25 has an absolute value less than 1, so the infinite sum exists and is finite.

Additional Resources

Series Analysis and Infinite Sum Calculation

When examining infinite series, mathematicians embark on a journey to determine whether the series converges to a finite value—its sum—or diverges, extending infinitely without approaching any particular number. The series in question:

\[
-2 + 0.5 - 0.125 + \dots
\]

presents an intriguing case that invites a detailed exploration of geometric series, convergence criteria, and the calculation of the sum if it exists.

Understanding the Series: An Overview

Before delving into calculations, it’s essential to understand the nature of this series, its structure, and the key properties that guide the analysis.

Series Components and Pattern Recognition

The given series can be expressed as:

\[
\sum_{n=0}^{\infty} a_n

\]

where each term a_n follows a specific pattern. Observing the initial terms:

- First term: -2
- Second term: 0.5
- Third term: -0.125

We notice the signs alternate: negative, positive, negative, and so forth. The magnitude of each term seems to be decreasing, which suggests the possibility of a geometric progression.

Identifying the Type of Series: Geometric Series

A geometric series is characterized by each term being a constant multiple (common ratio) of the previous term:

$$
a_{n} = a_{0} \cdot r^{n}
$$

where:

- a_{0} is the first term,
- r is the common ratio.

If the series is geometric, it can be written as:

$$
\sum_{n=0}^{\infty} a_0 r^{n}
$$

Determining the First Term and Common Ratio

Let’s examine the first few terms:

- $a_0 = -2$
- $a_1 = 0.5$
- $a_2 = -0.125$

Calculate the ratio between successive terms:

\[

$$
r = \frac{a_1}{a_0} = \frac{0.5}{-2} = -0.25
$$

Check if this ratio holds for the next terms:

$$
\frac{a_2}{a_1} = \frac{-0.125}{0.5} = -0.25
$$

Yes, the ratio remains consistent. Therefore, the series is geometric with:

$$
a_0 = -2, \quad r = -0.25
$$

Convergence Criteria for Geometric Series

A geometric series converges if and only if the absolute value of the common ratio is less than 1:

$$
|r| < 1
$$

In our case:

$$
|r| = 0.25 < 1
$$

Since this condition is satisfied, the series converges, and we can proceed to find its sum.

Calculating the Infinite Sum

The sum S of an infinite geometric series with first term a_0 and common ratio r (where $|r|<1$) is given by:

$$
S = \frac{a_0}{1 - r}
$$

Applying this formula:

$$
S = \frac{-2}{1 - (-0.25)} = \frac{-2}{1 + 0.25} = \frac{-2}{1.25}
$$

Simplify the fraction:

$$
S = -\frac{2}{1.25} = -\frac{2}{\frac{5}{4}} = -2 \times \frac{4}{5} = -\frac{8}{5}
$$

Expressed as a decimal:

$$
S = -1.6
$$

Summary of the Calculation and Its Significance

The infinite sum of the series:

$$
-2 + 0.5 - 0.125 + \dots
$$

exists and is equal to $-\frac{8}{5}$ or -1.6.

This calculation exemplifies the power of geometric series analysis in evaluating infinite sums, especially when the series exhibits a clear pattern of ratio and sign alternation.

Additional Insights and Considerations

While the main calculation is straightforward owing to the geometric nature, several additional points are worth exploring:

1. Significance of the Series Pattern

- The alternating signs suggest the series models phenomena with oscillating behavior, such as damping oscillations in physics or financial models with alternating gains and losses.
- Recognizing the geometric pattern simplifies what could otherwise be an intractable problem.

2. Convergence Conditions and Limitations

- If $|r|$ had been equal to or greater than 1, the series would diverge, and the sum would not exist.
- The convergence depends critically on the ratio's magnitude, emphasizing the importance of ratio analysis in series evaluation.

3. Practical Applications

- Infinite geometric series appear in various fields such as physics, engineering, economics, and computer science.
- Calculating sums helps in modeling and predicting long-term behaviors, such as depreciation, signal decay, or iterative algorithms.

4. Extension to Other Series

- Recognizing geometric patterns enables quick sum evaluations; similar techniques apply to series with other ratios or more complex structures.
- For series not strictly geometric, advanced methods like partial sums, integral tests, or comparison tests are employed.

Final Thoughts

The exploration of the series $-2 + 0.5 - 0.125 + \dots$ underscores the elegance and efficiency of geometric series analysis. By identifying the first term and common ratio, verifying convergence criteria, and applying the sum formula, we determine that the infinite sum exists and equals $-\frac{8}{5}$. This not only provides a definitive answer but also reinforces fundamental concepts crucial to advanced mathematical analysis.

Whether in theoretical mathematics or real-world applications, understanding how to evaluate such series enhances analytical capabilities and deepens insight into the behavior of infinite processes. The series, with its alternating signs and decreasing magnitudes, beautifully illustrates the convergence phenomena that underpin much of modern mathematical modeling.

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