.Suppose That A Particle Moves Along A Straight Line With Velocity V(t) = 7 - 5t, Where 0

Suppose That A Particle Moves Along A Straight Line With Velocity V(t) = 7 - 5t, Where 0. Understanding the motion of particles along a straight line is fundamental in physics and calculus, providing insights into how objects move, accelerate, and change position over time. In this article, we will explore the detailed analysis of a particle with the velocity function V(t) = 7 - 5t, focusing on key concepts such as displacement, velocity, acceleration, and the significance of critical points in the motion.

Introduction to Particle Motion Along a Straight Line

Particle motion along a line can be described entirely by its velocity function V(t), which indicates how fast and in what direction the particle moves at any given time t. When the velocity function is known, we can determine several important aspects of the particle's behavior:

- Displacement: The change in position over a time interval.
- Total Distance Traveled: The actual length of the path covered, considering direction changes.
- Acceleration: The rate of change of velocity, indicating how the particle speeds up or slows down.
- Critical Points: Times when the particle changes direction or velocity reaches a maximum or minimum.

Understanding these concepts allows us to analyze the particle's motion comprehensively, predict future positions, and interpret physical phenomena accurately.

Analyzing the Velocity Function $V(t) = 7 - 5t$

The given velocity function $V(t) = 7 - 5t$ is a linear function of time t, where t □ 0. The coefficients provide information about the particle's initial velocity and acceleration:

- Initial Velocity: When $t = 0$, $V(0) = 7 - 5(0) = 7$ units/time.
- Velocity at Time t: $V(t) = 7 - 5t$, which decreases linearly as time increases.

This indicates that the particle starts with a positive velocity of 7 units/time at $t = 0$ and slows down over time at a constant rate of 5 units/time^2.

Understanding the Sign of $V(t)$

The sign of $V(t)$ determines the direction of particle movement:

- If $V(t) > 0$, the particle moves in the positive direction.
- If $V(t) < 0$, the particle moves in the negative direction.
- If $V(t) = 0$, the particle momentarily stops.

By analyzing $V(t)$, we can find critical moments when the particle changes direction, which occur when $V(t) = 0$:

$7 - 5t = 0$
□ $t = 7/5 = 1.4$ units of time.

This point is crucial because it marks the instant when the particle transitions from moving forward to backward or vice versa.

Calculating Displacement and Position

Displacement over a time interval [a, b] is obtained by integrating the velocity function:

$$\text{Displacement} = \int_a^b V(t) \, dt.$$

Suppose the particle's initial position at t=0 is $s(0) = s_0$; then, the position function s(t) can be found by integrating V(t):

$$s(t) = s_0 + \int_0^t V(u) \, du.$$

Example: Find the Displacement from t=0 to t=3

Calculate the displacement:

$$\text{Displacement} = \int_0^3 (7 - 5t) \, dt.$$

Compute the integral:

$$\int (7 - 5t) \, dt = 7t - \frac{5}{2} t^2 + C.$$

Evaluate from 0 to 3:

$$\left(7 \times 3 - \frac{5}{2} \times 3^2\right) - \left(7 \times 0 - \frac{5}{2} \times 0^2\right) = (21 - \frac{5}{2} \times 9) - 0 = 21 - \frac{45}{2} = 21 - 22.5 = -1.5.$$

Thus, the particle's displacement from t=0 to t=3 is -1.5 units, indicating it has moved 1.5 units in the negative direction during this period.

Determining When the Particle Changes Direction

As established, the velocity $V(t) = 7 - 5t$ becomes zero at $t=1.4$. This is the critical point where the particle stops momentarily and then reverses direction.

Behavior Before and After t=1.4

- For $0 \le t < 1.4$: $V(t) > 0$, particle moves in the positive direction.
- At $t=1.4$: $V(t) = 0$, particle is momentarily at rest.
- For $t > 1.4$: $V(t) < 0$, particle moves in the negative direction.

This change significantly impacts the total distance traveled, which accounts for all movement regardless of direction, not just net displacement.

Calculating Total Distance Traveled

Total distance traveled over an interval [a, b] requires integrating the absolute value of the velocity:

$$\text{Total Distance} = \int_a^b |V(t)| \, dt.$$

Given the velocity's sign change at $t=1.4$, we split the integral:

$$
\text{Total Distance} = \int_0^{1.4} V(t) \, dt + \int_{1.4}^{b} |V(t)| \, dt.
$$

Since $V(t)$ is positive on [0, 1.4], and negative on [1.4, b], for the second integral, we take the negative of $V(t)$:

$$
\text{Total Distance} = \int_0^{1.4} (7 - 5t) \, dt - \int_{1.4}^{b} (7 - 5t) \, dt.
$$

This calculation provides a comprehensive measure of how much ground the particle covers during the entire period, regardless of direction.

Example: Total Distance from t=0 to t=3

Calculate each integral:

- From 0 to 1.4:

$$ \int_0^{1.4} (7 - 5t) \, dt = \left[7t - \frac{5}{2} t^2 \right]_0^{1.4} = (7 \times 1.4 - \frac{5}{2} \times 1.4^2) - 0. $$

Calculate:

$$ 7 \times 1.4 = 9.8, $$
$$ 1.4^2 = 1.96, $$
$$ \frac{5}{2} \times 1.96 = 2.5 \times 1.96 = 4.9. $$

Total:

$$ 9.8 - 4.9 = 4.9. $$

- From 1.4 to 3:

$$ \int_{1.4}^3 (7 - 5t) \, dt = \left[7t - \frac{5}{2} t^2 \right]_{1.4}^3. $$

Calculate at t=3:

$$7 \times 3 = 21,$$

$$3^2 = 9,$$

$$\frac{5}{2} \times 9 = 22.5,$$

$$21 - 22.5 = -1.5.$$

At t=1.4 (from earlier):

$$9.8 - 4.9 = 4.9.$$

Since the velocity is negative in this interval, we take the absolute value:

$$\text{Distance} = | -1.5 | = 1.5.$$

Total distance traveled:

$$4.9 + 1.5 = 6.4 \text{ units}.$$

This calculation shows the particle moves a total of 6.4 units during the 0 to 3 time interval.

Understanding Acceleration and Its Role

The acceleration function a(t) is the derivative of V(t):

$$a(t) = V'(t).$$

Given V(t) = 7 - 5t,

$$a(t) = -5.$$

This constant negative acceleration indicates the particle's velocity decreases at a uniform rate of 5 units/time^2, which explains the continuous slowing down until it reaches zero velocity at $t=1.4$.

Implications of Constant Acceleration

- The particle experiences uniform deceleration.
- The velocity decreases linearly over time.
- The crossing point at $V(t)=0$ corresponds to the instant when the particle's initial forward motion ceases.

Position Function s(t) and Its Significance

To find the position at any time t, integrate the velocity function:

$$ s(t) = s_0 + \int_0^t V(u) \, du. $$

Assuming the initial position $s_0 = 0$ for simplicity, the position function becomes:

\[s(t) = 7t - \frac

Frequently Asked Questions

What is the initial velocity of the particle at time t = 0?

At $t = 0$, the velocity $V(0) = 7 - 5(0) = 7$ units.

At what time does the particle come to rest?

The particle comes to rest when V(t) = 0, so 7 - 5t = 0 □ t = ¼.

What is the particle's velocity at t = 2 seconds?

At t = 2, V(2) = 7 - 5(2) = 7 - 10 = -3 units.

How do you find the position function s(t) given the velocity V(t)?

You integrate the velocity function V(t) with respect to t: s(t) = □ V(t) dt + C, where C is the initial position.

What is the total displacement of the particle from t = 0 to t = 1?

Displacement = □_{0}^{1} V(t) dt = □_{0}^{1} (7 - 5t) dt = [7t - rac{5}{2} t^2]_{0}^{1} = (7 imes 1 - rac{5}{2} imes 1^2) - 0 = 7 - 2.5 = 4.5 units.

When does the particle change direction?

The particle changes direction when the velocity changes sign. Since V(t) = 7 - 5t, it changes direction at t = ¼, where V(t) = 0.

What is the significance of the velocity function V(t) in analyzing the particle's motion?

The velocity function V(t) indicates the speed and direction of the particle at any time t, helping determine when it moves forward, backward, or is stationary.

How can you determine the total distance traveled by the particle over a time interval?

Total distance traveled is the integral of the absolute value of V(t) over the interval, accounting for changes in direction by integrating □bs(V(t)) dt.

If the initial position s(0) is known, how do you find the position s(t)?

Integrate V(t) to find s(t): s(t) = s(0) + □_{0}^{t} V(au) d au. For V(t) = 7 - 5t, s(t) = s(0) + 7t - rac{5}{2} t^2.

Additional Resources

Particle Motion Along a Line with Velocity V(t) = 7 - 5t

Understanding the motion of particles along a line is fundamental in physics and calculus, offering insights into how objects move, accelerate, and change direction over time. In this article, we examine a specific case where the velocity of a particle is given by the function V(t) = 7 - 5t, with the time interval starting from t = 0. This scenario serves as an excellent example to explore concepts such as displacement, acceleration, and the particle's overall behavior over time.

Introduction to Particle Motion and Velocity Functions

Particle motion along a straight line can be characterized entirely by its velocity function V(t), which describes how fast and in which direction the particle moves at any given time t. When V(t) is known, critical aspects such as displacement, average velocity, and the time of motion can be deduced.

The velocity function V(t) = 7 - 5t is a linear function that decreases over time, indicating that the particle initially moves in a positive direction but eventually slows down, possibly changing direction depending on the time interval considered.

Analyzing the Velocity Function V(t) = 7 - 5t

1. Graphical Interpretation

Plotting V(t) = 7 - 5t provides a straight line with a y-intercept at 7 and a slope of -5. This indicates:

- The particle starts with an initial velocity of 7 units per second at t = 0.
- The velocity decreases linearly as time increases, with a slope of -5.
- The zero velocity point occurs when V(t) = 0, at t = 7/5 = 1.4 seconds.

Pros:
- Straightforward to analyze and graph.
- Provides clear insight into how velocity changes over time.

Cons:
- Assumes a uniform rate of change, which may not be realistic for all physical systems.

2. Physical Interpretation

Initially, the particle moves forward with positive velocity. As time progresses, the velocity diminishes steadily, reaching zero at t = 1.4 seconds, after which the particle begins to move in the opposite direction. This behavior can model scenarios such as a thrown object decelerating due to resistance or a vehicle braking.

Displacement and Position Function

1. Calculating Displacement

Displacement over a time interval [a, b] is given by the integral of V(t):

$$ s(b) - s(a) = \int_a^b V(t) \, dt $$

Assuming the initial position at t = 0 is s(0) = 0, the position at any time t is:

$$ s(t) = \int_0^t V(u) \, du $$

Calculating:

$$ s(t) = \int_0^t (7 - 5u) \, du = [7u - \frac{5u^2}{2}]_0^t = 7t - \frac{5t^2}{2} $$

Features:

- The position function is quadratic, opening downward due to the negative coefficient of t^2.
- The maximum displacement occurs at the vertex of this parabola (which can be found by calculus).

Pros:

- Provides a direct way to compute the particle's position over time.

Cons:

- Assumes initial position s(0) = 0; different initial positions require adjustment.

2. Displacement at Specific Times

- At t = 1.4 seconds (when velocity reaches zero):

$$ s(1.4) = 7 \times 1.4 - \frac{5 \times (1.4)^2}{2} = 9.8 - \frac{5 \times 1.96}{2} = 9.8 - 4.9 = 4.9 \text{ units} $$

- At t = 2 seconds:

$$ s(2) = 7 \times 2 - \frac{5 \times 4}{2} = 14 - 10 = 4 \text{ units} $$

This shows that after t=1.4 seconds, the particle moves back toward the initial position as the velocity becomes negative.

Understanding Acceleration and Changes in Motion

1. Calculating Acceleration

Since velocity is V(t) = 7 - 5t, acceleration a(t) is its derivative:

$$ a(t) = V'(t) = -5 $$

This constant negative acceleration indicates the particle is uniformly decelerating. The negative sign signifies a force acting opposite to the initial direction of motion.

Features:

- Uniform acceleration simplifies analysis.

- The particle's velocity decreases linearly over time.

Pros:

- Easy to predict when the particle stops and changes direction.

Cons:

- Assumes no external factors altering acceleration.

2. Implications of Constant Acceleration

- The particle slows down at a rate of 5 units/sec².

- It reaches zero velocity at t=1.4 seconds.

- After this point, the particle moves in the negative direction, with velocity becoming negative.

Behavior Over Time and Critical Points

1. Time of Direction Change

Since the velocity becomes zero at t = 1.4 seconds, the particle's motion changes direction at this point. Prior to t=1.4, it moves forward; afterward, it moves backward.

Pros:

- Identifies when the particle reverses direction.

- Critical for understanding total displacement and path.

Cons:

- Assumes motion continues smoothly; in real systems, abrupt changes may occur.

2. Total Displacement Over a Time Interval

To compute total displacement from t=0 to t=2 seconds:

$$ s(2) = 14 - 10 = 4 \text{ units} $$

But because the particle moves forward initially and then backward, the net displacement is less than the total distance traveled.

If we want to find the total distance traveled:

- From t=0 to t=1.4 seconds:

$$ \text{Distance} = s(1.4) = 4.9 \text{ units} $$

- From t=1.4 to t=2 seconds, velocity is negative, so the particle moves back:

$$ s(2) - s(1.4) = (8 - 10) - 4.9 = -2 + 4.9 = 2.9 \text{ units} $$

Total distance traveled:

$$ 4.9 + 2.9 = 7.8 \text{ units} $$

Applications and Real-World Examples

The analysis of V(t) = 7 - 5t has practical applications across various fields:

- Physics: Modeling objects thrown upward with initial velocity and subject to uniform deceleration due to gravity.
- Engineering: Designing systems with linearly decreasing speeds, such as decelerating vehicles or machinery.
- Biology: Describing the rate of movement of organisms or particles under controlled conditions.

Limitations and Assumptions

While the model provides valuable insights, it relies on certain assumptions:

- Linearity: Assumes velocity decreases linearly, which may not hold in complex real-world situations.
- Constant acceleration: Simplifies calculations but may not reflect variable forces acting on the particle.
- Initial conditions: Assumes initial position s(0) = 0; different initial positions require adjustments.

Summary of Key Features and Insights

- The velocity function V(t) = 7 - 5t indicates a uniformly decelerating particle starting at 7 units/sec.
- The particle's position over time is quadratic, with a maximum displacement at t = 1.4 seconds.

- The particle reverses direction at t = 1.4 seconds, moving backward afterward.
- Constant acceleration of -5 units/sec² simplifies analysis and predicts critical points like zero velocity.
- Total distance traveled exceeds net displacement due to back-and-forth motion.

Conclusion

The detailed examination of a particle moving along a line with velocity $V(t) = 7 - 5t$ illustrates how calculus tools like integration and differentiation facilitate a comprehensive understanding of motion. From analyzing velocity and acceleration to calculating displacement and identifying critical points of motion, such models are foundational in physics and engineering. Despite its simplicity, this linear model captures essential dynamics, offering a stepping stone toward more complex and realistic motion analyses.

Pros:
- Clear mathematical framework.
- Demonstrates fundamental concepts of kinematics.

Cons:
- Simplifies real-world complexities.
- Assumes ideal conditions like constant acceleration.

Overall, studying such velocity functions deepens our understanding of motion and enhances our ability to interpret and predict real-world phenomena involving moving particles.

Note: For practical applications, always consider initial conditions, external forces, and potential nonlinearities in real systems.

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