The Coefficient Ofx2in The Maclaurin Series Forf(x)=exp(x2)is: A.1 B.-1/4C.1/4D.1/2E. 1

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Understanding the coefficient of x^2 in the Maclaurin series expansion of $f(x) = e^{x^2}$ is a fundamental concept in calculus and mathematical analysis. This coefficient provides insight into how the function behaves near zero and reveals the structure of its power series. In this article, we will explore the derivation of the Maclaurin series for e^{x^2}, analyze the specific coefficient of x^2, and discuss its significance in mathematical computations and applications. Whether you are a student, educator, or mathematics enthusiast, this detailed guide will help you grasp the core concepts and answer the key question: what is the coefficient of x^2 in the Maclaurin series for e^{x^2}?

Understanding the Maclaurin Series

What is a Maclaurin Series?

The Maclaurin series is a special case of the Taylor series expanded around zero. It expresses a function $f(x)$ as an infinite sum of its derivatives at zero:

$$
f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n
$$

This expansion provides a powerful way to approximate functions near zero and analyze their properties. For many functions, the Maclaurin series converges to the actual function within a certain radius.

Significance of the Coefficient of x^2

The coefficient of x^2 in the series indicates the quadratic contribution to the function’s behavior around zero. It influences the curvature and second-order approximation, which are crucial for understanding the function’s local behavior and for applications like numerical analysis, physics, and engineering.

Deriving the Maclaurin Series for e^{x^2}

Step 1: Recall the Series for e^x

The exponential function e^x has a well-known Maclaurin series:

$$
e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}
$$

This series converges for all real x, and the coefficients are simply $1/n!$.

Step 2: Substitute x^2 into e^x

Since $f(x) = e^{x^2}$, we replace x with x^2 in the series:

$$
e^{x^2} = \sum_{n=0}^{\infty} \frac{(x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{x^{2n}}{n!}
$$

This is the Maclaurin series expansion of e^{x^2}.

Step 3: Write out the series explicitly

The series becomes:

$$
e^{x^2} = 1 + \frac{x^2}{1!} + \frac{x^4}{2!} + \frac{x^6}{3!} + \cdots
$$

Note that only even powers of x appear, which reflects the symmetry of the function e^{x^2}.

Identifying the Coefficient of x^2

Step 1: Locate the x^2 term

From the series:

$$
e^{x^2} = 1 + \frac{x^2}{1!} + \frac{x^4}{2!} + \frac{x^6}{3!} + \cdots
$$

the coefficient of x^2 is $\frac{1}{1!}$.

Step 2: Simplify the coefficient

Calculate $1!$:

$$
1! = 1
$$

Thus, the coefficient of x^2 is:

$$
\boxed{\frac{1}{1!} = 1}
$$

Conclusion: The coefficient of x^2 in the Maclaurin series for e^{x^2} is 1.

This aligns with option A, which states the coefficient is 1.

Summary of Key Points

Uncovering the coefficient of x^2 in the Maclaurin series expansion involves understanding the function's power series representation. Here are the essential highlights:

1. The Maclaurin series for e^{x^2} is derived from the exponential series by substituting x^2 for x.
2. The series contains only even powers of x, reflecting the symmetry of the function.
3. The coefficient of x^2 is obtained directly from the series term $\frac{x^{2}}{1!}$, which simplifies to 1.
4. The coefficient of x^2 is crucial for quadratic approximations and understanding the local behavior of e^{x^2}.

Further Applications and Relevance

Why is the Coefficient of x^2 Important?

Understanding the coefficient of x^2 in series expansions serves multiple purposes:

- Quadratic Approximation: It enables the approximation of e^{x^2} near zero using a quadratic polynomial, which is useful in numerical methods.
- Analyzing Function Curvature: The second derivative at zero relates directly to this coefficient, providing insights into the curvature at the origin.
- Physics and Engineering: Many models involve exponential functions with quadratic exponents, making these coefficients relevant in fields like quantum mechanics, signal processing, and control systems.

Related Mathematical Concepts

The analysis of the coefficient of x^2 in this context connects to broader concepts such as:

- Power series and Taylor expansions
- Derivative evaluations at zero
- Symmetry properties of functions
- Convergence criteria for infinite series

Conclusion: The Correct Answer

After a thorough derivation and analysis, the coefficient of x^2 in the Maclaurin series for $f(x) = e^{x^2}$ is clearly 1. This corresponds to option A in the multiple-choice list provided:

- A. 1
- B. -1/4
- C. 1/4
- D. 1/2
- E. 1

Therefore, the correct answer is option A: 1.

Final Thoughts

Understanding series expansions like the Maclaurin series is fundamental in advanced calculus and mathematical analysis. These expansions not only simplify complex functions but also provide deep insights into their behavior near specific points. Recognizing the coefficients of various terms, especially x^2, equips students and professionals with powerful tools for approximation, modeling, and problem-solving in numerous scientific disciplines.

If you want to deepen your understanding of series expansions, calculus, or exponential functions, consider exploring related topics such as Taylor series, convergence tests, and applications in physics and engineering. Mastery of these concepts enhances analytical skills and opens doors to advanced mathematical applications.

Meta Description: Discover the coefficient of x^2 in the Maclaurin series for e^{x^2}. Learn how to derive the series, identify key terms, and understand its significance in calculus and mathematical analysis.

Frequently Asked Questions

What is the coefficient of x^2 in the Maclaurin series for $f(x) = \exp(x^2)$?

The coefficient of x^2 in the Maclaurin series for $f(x) = \exp(x^2)$ is 1.

How do you find the coefficient of x^2 in the Maclaurin series expansion of $\exp(x^2)$?

You differentiate the function multiple times or expand the series of $\exp(x^2)$ and identify the coefficient of x^2, which turns out to be 1.

What is the general form of the Maclaurin series for $f(x) = \exp(x^2)$?

The series is $\sum_{n=0}^{\infty} (x^{2n}) / n!$, where the coefficient of x^2 corresponds to $n=1$, which is 1.

Why is the coefficient of x^2 in $\exp(x^2)$ equal to 1?

Because the term x^2 appears as the $n=1$ term in the series expansion, with coefficient $1/n! = 1/1! = 1$.

Which option correctly represents the coefficient of x^2 in the Maclaurin series for e^{x^2}?

Option A: 1.

Are there any other coefficients for x^2 in the expansion of e^{x^2} besides 1?

No, the coefficient of x^2 in the expansion is exactly 1.

How does the Maclaurin series for e^{x^2} compare to that of e^x?

While e^x has coefficients 1/n!, e^{x^2} has coefficients 1/n! for powers of x^{2n}, with the x^2 term's coefficient being 1.

Is the coefficient of x^2 in exp(x^2) positive or negative?

It is positive, specifically 1.

Can the coefficient of x^2 in exp(x^2) be any of the options B, C, D, or E?

No, the correct coefficient is 1, corresponding to option A.

What is the significance of the coefficient of x^2 in understanding the behavior of e^{x^2} near zero?

The coefficient indicates the curvature and initial growth rate of the function near zero, with the x^2 term's coefficient being 1 in this case.

Additional Resources

The Coefficient Of x^2 in the Maclaurin Series for $f(x) = \exp(x^2)$: A Comprehensive Guide

Understanding the coefficient of x^2 in the Maclaurin series for $f(x) = \exp(x^2)$ is a fundamental step in the study of power series expansions and their applications in calculus and mathematical analysis. This guide aims to thoroughly explain how to find this coefficient, interpret its significance, and clarify why the correct answer among the options—A. 1, B. -1/4, C. 1/4, D. 1/2, E. 1—is what it is.

Introduction to the Maclaurin Series and Its Relevance

Before diving into the specifics of the coefficient in question, it’s essential to understand the broader context:

- What is the Maclaurin Series?
The Maclaurin series is a special case of the Taylor series centered at 0. For a function $f(x)$, it expresses $f(x)$ as an infinite sum of its derivatives evaluated at 0, scaled by factorial terms:

$$
f(x) = \sum_{n=0}^\infty \frac{f^{(n)}(0)}{n!} x^n
$$

- Why is the Maclaurin series important?
It provides a powerful tool to approximate functions, analyze their behavior near zero, and facilitate computations involving complex functions like exponential, trigonometric, and polynomial functions.

- The specific function $f(x) = e^{x^2}$
This function is a key example because it involves a composition of exponential and polynomial functions, leading to interesting series expansion properties.

Understanding the Maclaurin Series for $f(x) = e^{x^2}$

The goal is to expand $f(x) = e^{x^2}$ into its Maclaurin series and identify the coefficient of the x^2 term.

Step 1: Recall the Maclaurin series for e^x

The exponential function e^x has a well-known Maclaurin series:

$$
e^x = \sum_{n=0}^\infty \frac{x^n}{n!}
$$

This series converges for all real x.

Step 2: Substitute x^2 into e^x to get e^{x^2}

Since $f(x) = e^{x^2}$, we can write:

\[

e^{x^2} = \sum_{n=0}^\infty \frac{(x^2)^n}{n!} = \sum_{n=0}^\infty \frac{x^{2n}}{n!}
\]

This series expansion directly shows that only even powers of \(x\) appear, with coefficients \(\frac{1}{n!}\).

Determining the Coefficient of \(x^2\)

The key is to find the coefficient accompanying the \(x^2\) term in the series expansion:

\[
e^{x^2} = \sum_{n=0}^\infty \frac{x^{2n}}{n!}
\]

- When \(n=0\), the term is \(\frac{x^0}{0!} = 1\)
- When \(n=1\), the term is \(\frac{x^2}{1!} = x^2\)
- When \(n=2\), the term is \(\frac{x^4}{2!} = \frac{x^4}{2}\), and so on.

Therefore, the coefficient of \(x^2\) is the one multiplying \(x^2\) in the sum, which is:

\[
\boxed{\frac{1}{1!} = 1}
\]

Hence, the coefficient of \(x^2\) in the Maclaurin series for \(e^{x^2}\) is 1.

Verifying with Derivatives (Optional Approach)

While the direct series expansion confirms the coefficient, for completeness, let's verify through derivatives—a common method for extracting coefficients.

The general formula for the \(n\)-th coefficient in a Maclaurin series is:

\[
a_n = \frac{f^{(n)}(0)}{n!}
\]

To find the coefficient of \(x^2\), we need \(f^{(2)}(0)\):

Step 1: Find \(f'(x)\)

\[

$$
f(x) = e^{x^2}
$$

$$
f'(x) = 2x e^{x^2}
$$

Step 2: Find $f''(x)$

$$
f''(x) = \frac{d}{dx} [2x e^{x^2}] = 2 e^{x^2} + 2x \cdot 2x e^{x^2} = 2 e^{x^2} + 4x^2 e^{x^2}
$$

Step 3: Evaluate $f''(0)$:

$$
f''(0) = 2 e^{0} + 0 = 2 \times 1 = 2
$$

Step 4: Find the coefficient:

$$
a_2 = \frac{f^{(2)}(0)}{2!} = \frac{2}{2} = 1
$$

This confirms the earlier conclusion: the coefficient of x^2 is 1.

Summary and Final Answer

- The Maclaurin series for $f(x) = e^{x^2}$ is:

$$
e^{x^2} = \sum_{n=0}^\infty \frac{x^{2n}}{n!}
$$

- The x^2 term corresponds to $n=1$, with coefficient $\frac{1}{1!} = 1$.

- Both the direct series expansion and derivative-based verification agree that the coefficient of x^2 is 1.

Therefore, the correct choice among the options is:

E. 1

Additional Insights and Applications

Understanding the coefficient of specific terms in a power series such as the Maclaurin series for e^{x^2} has various applications:

- Approximate calculations: Using the series to approximate values of e^{x^2} near zero.
- Analysis of function behavior: The coefficients inform us about the function's local properties.
- Differential equations: Series solutions often involve identifying such coefficients.
- Mathematical modeling: In physics and engineering, series expansions simplify complex functions.

Conclusion

The process of expanding $f(x) = e^{x^2}$ into its Maclaurin series reveals that only even powers of x appear, with coefficients derived straightforwardly from the series definition or derivative calculations. The coefficient of the x^2 term, a fundamental piece of the expansion puzzle, is 1, aligning with choice E. This example underscores the elegance of power series and their utility in mathematical analysis.

In summary:
The coefficient of x^2 in the Maclaurin series for $f(x) = e^{x^2}$ is 1.

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