The Curved Surface Area Of A Cone Is 108cm^2 And The Radius Is 8cm. Find The Slant Height

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Understanding the geometry of a cone is fundamental in many mathematical and real-world applications, from designing conical structures to calculating material requirements. When given specific measurements such as the curved surface area and the radius, it becomes an interesting challenge to determine the missing dimension — in this case, the slant height. This article aims to guide you through the process of calculating the slant height of a cone when the curved surface area and radius are provided, emphasizing clarity, step-by-step procedures, and practical insights.

Understanding the Components of a Cone

Before diving into calculations, it’s essential to familiarize yourself with the key parts of a cone:

1. Radius (r)

- The distance from the center of the base to its edge.
- In our problem: r = 8 cm.

2. Height (h)

- The perpendicular distance from the base to the apex.
- Not directly given in the problem but can be derived once the slant height is known.

3. Slant Height (l)

- The hypotenuse of the triangle formed by the radius and height.
- It is the length from the apex of the cone to a point on the edge of the base.

4. Curved Surface Area (CSA)

- The lateral surface area of the cone.
- Given as 108 cm^2 in this problem.

Understanding how these components relate helps us set up the correct formulas to find the unknowns.

Key Formulas for a Cone

To solve for the slant height, we need to understand the relevant formulas:

1. Curved Surface Area (CSA)

- Formula: $\text{CSA} = \pi r l$
- Where:
- r = radius
- l = slant height
- $\pi \approx 3.1416$

2. Slant Height (l)

- Relationship with height and radius:
- $l = \sqrt{h^2 + r^2}$

3. Total Surface Area (if needed)

- Not necessary for this problem but useful to know:
- $\text{Surface Area} = \pi r (l + r)$

With these formulas in mind, we can proceed to find the slant height.

Step-by-Step Calculation of the Slant Height

Given:
- CSA = 108 cm²
- Radius $r = 8$ cm

Our goal:
- Find the slant height l.

Step 1: Write down the CSA formula with known values

$$
\text{CSA} = \pi r l
$$

Substituting the knowns:

$$
108 = \pi \times 8 \times l
$$

Step 2: Isolate l
$$
l = \frac{108}{\pi \times 8}
$$

Step 3: Calculate the denominator
$$
\pi \times 8 \approx 3.1416 \times 8 = 25.1328
$$

Step 4: Compute l
$$
l \approx \frac{108}{25.1328} \approx 4.295 \text{ cm}
$$

Result:
$$
\boxed{l \approx 4.3 \text{ cm}}
$$

This is the slant height of the cone based on the given data.

Practical Insights and Additional Calculations

Knowing the slant height enables further exploration, such as determining the height of the cone or verifying the calculations through different methods.

Calculating the Height of the Cone

Using the relationship:
$$
l = \sqrt{h^2 + r^2}
$$
Rearranged as:
$$
h = \sqrt{l^2 - r^2}
$$

Substitute the known values:
$$
h = \sqrt{(4.3)^2 - 8^2} = \sqrt{18.49 - 64} = \sqrt{-45.51}
$$

Since the square root of a negative number isn't real, this indicates an inconsistency. This suggests that the initial calculation of the slant height may have an error or that the given data are incompatible for a real cone with these dimensions.

Note: The earlier calculation of l appears inconsistent because the

slant height cannot be less than the radius in a real cone. Typically, the slant height is greater than or equal to the radius. Let's verify this step-by-step.

Re-evaluating the Calculation

The mistake arises from the approximation in calculating l. Let's perform the calculation again with more precision:

$$
l = \frac{108}{\pi \times 8} = \frac{108}{25.1328} \approx 4.295 \text{ cm}
$$

But as per the geometric relationship, the slant height must satisfy:

$$
l \geq r
$$
and since the slant height must be longer than the radius, l should be at least 8 cm.

This inconsistency indicates that the initial assumption or calculation might be flawed, or the provided data are inconsistent with the geometry of a cone.

Clarifying the Discrepancy

Given the formula for CSA:

$$
\text{CSA} = \pi r l
$$

and the values:

$$
\text{CSA} = 108\,cm^2,\quad r=8\,cm
$$

Let's check the expected l:

$$
l = \frac{\text{CSA}}{\pi r} = \frac{108}{3.1416 \times 8} \approx \frac{108}{25.1328} \approx 4.3\,cm
$$

But since the slant height l should be at least equal to the radius, this indicates inconsistency. The problem might have been incorrectly posed

or the data are incompatible.

Reconciling the Data

Possible causes:

- The given CSA might be the total surface area, not just the lateral surface area.
- The data provided might be approximate or intended to test understanding.

If the CSA is total surface area:

$$
\text{Total Surface Area} = \pi r (l + r)
$$
Substitute knowns:
$$
\text{Total Surface Area} = 108\,cm^2
$$

Solve for l:

$$
108 = \pi \times 8 \times (l + 8)
$$
$$
l + 8 = \frac{108}{\pi \times 8} \approx 4.3
$$

Again, $l + 8 \approx 4.3$, which implies $l \approx -3.7$, impossible.

Conclusion:

- The initial data seem inconsistent with the properties of a cone.
- For a realistic cone with radius 8 cm, the lateral surface area $\pi r l$ should be at least:

$$
l \geq r \Rightarrow \text{CSA} \geq \pi r^2 \approx 3.1416 \times 8^2 = 3.1416 \times 64 \approx 201\,cm^2
$$

Since the given CSA is 108 cm², which is less than 201 cm², this suggests that the surface area is only part of the total surface area or the data are inconsistent.

Final Clarification and Correct Approach

Given the typical properties:

- The lateral surface area $\pi r l$ is always greater than or equal to πr^2.
- For $r=8\,cm$:
$$
\pi r^2 \approx 201\,cm^2
$$
- But the problem states CSA = 108 cm², which is less than this.

Therefore:

- The initial statement might be intended differently, perhaps the total surface area is 108 cm², or the problem involves different dimensions.

Summary and Practical Steps

Despite the inconsistencies in the data, the general process to find the slant height when the curved surface area and radius are known remains:

1. Use the formula:
$$
l = \frac{\text{CSA}}{\pi r}
$$
2. Plug in known values:
$$
l = \frac{\text{given CSA}}{\pi \times r}
$$
3. Calculate l using precise values of π and the given data.
4. Use the Pythagorean theorem to find the height:
$$
h = \sqrt{l^2 - r^2}
$$

In real-world scenarios, ensure the given data are consistent with the geometric properties of a cone.

Conclusion

Calculating the slant height of a cone from the curved surface area and radius involves applying the fundamental formula $\text{CSA} = \pi r l$. When the data are consistent, this process is straightforward; however, as demonstrated, inconsistent data can lead to impossible or contradictory results. Always verify the provided measurements and understand

Frequently Asked Questions

What is the formula for the curved surface area of a cone?

The formula for the curved surface area (CSA) of a cone is $2\pi rh$, where r is the radius and h is the height.

Given the curved surface area of a cone is 108 cm² and radius is 8 cm, how do you find the slant height?

Use the formula $CSA = 2\pi rL$, where L is the slant height. Plug in the known values: $108 = 2\pi \times 8 \times L$. Simplify to find L.

What is the value of the slant height of the cone when the CSA is 108 cm² and radius is 8 cm?

The slant height L is 2.55 cm. Calculated by $L = CSA / (2\pi r) = 108 / (2 \times 3.1416 \times 8) \approx 2.55$ cm.

Why is the slant height important in calculating the surface area of a cone?

The slant height is essential because it directly relates to the curved surface area; the CSA depends on the slant height and radius.

Can the slant height be greater than the height of the cone? Why or why not?

Yes, the slant height can be greater than the height because it is the hypotenuse of the right triangle formed by the radius and the height; it is always equal to or longer than the vertical height.

What steps would you take to verify the calculated slant height for accuracy?

First, calculate the slant height using the formula $L = CSA / (2\pi r)$. Then, verify by plugging L back into the CSA formula to see if it equals 108 cm². Check for calculation errors.

Additional Resources

The Curved Surface Area Of A Cone Is 108cm² And The Radius Is 8cm. Find The Slant Height – this problem encapsulates fundamental concepts of geometry related to cones, specifically focusing on the curved surface area, radius, and slant height. Understanding how these elements interconnect allows us to solve for unknown dimensions and deepen our grasp of three-dimensional shapes. In this comprehensive guide, we will explore the properties of a cone, review relevant formulas, and step through the process of calculating the slant height from given data.

Understanding the Cone and Its Surface Area

Before diving into calculations, let's clarify what a cone is and what the curved surface area signifies.

What Is a Cone?

A cone is a three-dimensional geometric figure characterized by:

- A circular base
- A vertex (apex) not in the plane of the base
- A slant height that measures the length from the apex to a point on the edge of the base along the lateral surface

Visualize an ice cream cone or a traffic cone—these are practical examples of cones.

Key Dimensions of a Cone

- Radius (r): The distance from the center of the base to its edge.
- Height (h): The perpendicular distance from the base to the apex.
- Slant Height (l): The length of the sloping side from the apex to the edge of the base.
- Curved Surface Area (CSA): The area of the lateral (curved) surface, excluding the base.

Fundamental Formulas for a Cone

To solve problems related to a cone's surface areas and dimensions, certain formulas are essential:

1. Curved Surface Area (CSA)

$$
\text{CSA} = \pi r l
$$

where:
- r = radius
- l = slant height
- π ≈ 3.1416

2. Total Surface Area (TSA)

$$
\text{TSA} = \pi r (r + l)
$$

3. Volume (V)

$$
V = \frac{1}{3} \pi r^2 h
$$

In our current problem, we're focused on the curved surface area and how to find the slant height given the CSA and radius.

The Problem at Hand

> The curved surface area of a cone is 108cm² and the radius is 8cm. Find the slant height.

This is a straightforward application of the CSA formula, rearranged to solve for l.

Step-by-Step Solution

Step 1: Write down the known values

- $\text{CSA} = 108\, \text{cm}^2$
- $r = 8\, \text{cm}$

Step 2: Recall the formula for CSA

$$
\text{CSA} = \pi r l
$$

Step 3: Rearrange the formula to solve for l:

$$
l = \frac{\text{CSA}}{\pi r}
$$

Step 4: Substitute the known values

$$
l = \frac{108}{\pi \times 8}
$$

Step 5: Calculate the denominator

$$
\pi \times 8 \approx 3.1416 \times 8 = 25.1328
$$

Step 6: Calculate l

$$
l = \frac{108}{25.1328} \approx 4.297\, \text{cm}
$$

Answer: The slant height l is approximately 4.3 cm.

Additional Insights and Considerations

While the above calculation directly finds the slant height based on the given data, understanding the broader context can be helpful.

Is the Slant Height Less Than the Height of the Cone?

In a right circular cone, the slant height l, the radius r, and the vertical height h are related through the Pythagorean theorem:

$$
l = \sqrt{h^2 + r^2}
$$

Knowing l and r, you can find the height h:

$$
h = \sqrt{l^2 - r^2}
$$

In our case:

$$
h = \sqrt{(4.3)^2 - 8^2} = \sqrt{18.49 - 64} = \sqrt{-45.51}
$$

Since this results in a negative value under the square root, it suggests inconsistency with the initial assumptions, indicating that the cone might not be a right cone, or the problem parameters need to be re-examined.

However, in the context of the problem, the calculated slant height appears shorter than the radius, which is geometrically impossible for a right cone. This discrepancy indicates that either:

- The given CSA is incorrect for a typical cone with radius 8cm, or
- The cone is oblique (not right-angled), which complicates the geometry.

In most standard problems, the slant height should be greater than or equal to the radius.

Common Mistakes and Clarifications

- Misapplying the formula: Remember that CSA = $\pi r l$; do not confuse this with total surface area.
- Incorrect units: Ensure all measurements are in centimeters and consistent.
- Assuming the cone is right-angled: For most standard problems, the cone is right-angled unless specified otherwise.
- Neglecting the geometric constraints: The slant height cannot be less than the radius in a typical cone.

Summary and Final Remarks

The problem of finding the slant height based on the curved surface area and radius is a classic application of basic geometry formulas. Using:

$$
l = \frac{\text{CSA}}{\pi r}
$$

and substituting the known values, we obtain an approximate slant height of 4.3 cm.

Understanding the relationships between the dimensions of a cone enhances problem-solving skills and fosters geometric intuition. Always verify the reasonableness of your results within the context of geometric constraints. In more complex scenarios, additional data such as the height or total surface area might be necessary for comprehensive analysis.

Final Tips for Solving Cone Problems

- Always write down knowns and unknowns clearly.
- Recall the relevant formulas and understand their derivations.
- Check units and convert if necessary.
- Use approximate values for π unless specified otherwise.
- Verify your results for geometric consistency.

By mastering these steps, you'll confidently approach a wide array of problems involving cones and their dimensions.

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