The Function Of F(x)=x^3-7x+6 Is Shown In The Figure Which Is A Factor Of The Function Shown

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Understanding the behavior of polynomial functions is fundamental in algebra and calculus, as it provides insights into their roots, factors, and overall shape. The function $f(x) = x^3 - 7x + 6$ is a cubic polynomial that exhibits interesting characteristics, especially when analyzed through its factors and roots. In this article, we will explore the nature of this function, examine its factors, interpret the significance of the figure associated with it, and understand how the factors relate to the overall behavior of the polynomial.

Introduction to the Function f(x) = x^3 - 7x + 6

Basic Characteristics of the Cubic Function

The cubic function $f(x) = x^3 - 7x + 6$ is a polynomial of degree three. Its general form allows for up to three real roots and two turning points—local maximum and minimum. The leading coefficient (1) is positive, which influences the end behavior of the function: as x approaches positive or negative infinity, f(x) also approaches positive or negative infinity, respectively.

Significance of Studying the Function

Examining this particular cubic function helps in understanding:

- How to factor cubic polynomials.
- The location and nature of roots.
- The shape of the graph, including intercepts and turning points.
- The relationship between factors and roots.

Factoring the Cubic Function f(x) = x^3 - 7x + 6

Identifying Rational Roots

To factor the cubic polynomial, the Rational Root Theorem is a helpful tool. It states that any rational root, expressed as a fraction p/q in lowest terms, must have p as a factor of the constant term (6), and q as a factor of the leading coefficient (1).

Factors of 6: ±1, ±2, ±3, ±6

Factors of 1: ±1

Possible rational roots: ±1, ±2, ±3, ±6

Testing Possible Roots

Substitute these candidates into f(x):

- f(1) = 1^3 - 7(1) + 6 = 1 - 7 + 6 = 0 → Root at x=1
- f(-1) = -1 + 7 + 6 = 12 ≠ 0
- f(2) = 8 - 14 + 6 = 0 → Root at x=2
- f(-2) = -8 + 14 + 6 = 12 ≠ 0
- f(3) = 27 - 21 + 6 = 12 ≠ 0
- f(-3) = -27 + 21 + 6 = 0 → Root at x=-3
- f(6) = 216 - 42 + 6 = 180 ≠ 0
- f(-6) = -216 + 42 + 6 = -168 ≠ 0

Thus, the roots are x=1, x=2, and x=-3.

Expressing the Factorization

Using the roots, the cubic can be factored as:

f(x) = (x - 1)(x - 2)(x + 3)

This factorization confirms the roots and allows for easier analysis of the function's behavior.

Graphical Interpretation and the Role of Factors

Understanding the Graph in the Context of Factors

The graph of f(x) = (x - 1)(x - 2)(x + 3) will intersect the x-axis at the roots:

- x = 1
- x = 2
- x = -3

The figure associated with the function, as mentioned, illustrates these roots and demonstrates the factorization visually. The factors correspond to the points where the graph crosses the x-axis, and the multiplicity of roots affects the shape at those points.

Features of the Graph

- Intercepts: The x-intercepts are at the roots; the y-intercept is at $f(0) = 0^3 - 7(0) + 6 = 6$.
- Turning Points: Since the degree is three, the graph has at most two turning points. Calculus can be used to find these extremums.
- End Behavior: As $x \to \pm\infty$, $f(x) \to \infty$ (because the leading coefficient is positive).

Analyzing the Graph: Critical Points and Shape

Finding Critical Points

To identify local maxima and minima, compute the first derivative:

$f'(x) = 3x^2 - 7$

Set $f'(x) = 0$:

$3x^2 - 7 = 0 \to x^2 = 7/3 \to x = \pm\sqrt{7/3}$

Calculate the corresponding y-values:

- At $x = \sqrt{7/3}$:

$f(\sqrt{7/3}) = (\sqrt{7/3})^3 - 7(\sqrt{7/3}) + 6$

- At $x = -\sqrt{7/3}$:

$f(-\sqrt{7/3}) = (-\sqrt{7/3})^3 - 7(-\sqrt{7/3}) + 6$

These points mark the local maximum and minimum, shaping the graph's curvature.

Sketching the Graph

Using the roots, critical points, and end behavior, one can sketch an accurate graph:

- The graph crosses the x-axis at -3, 1, 2.
- It reaches local extremums at the critical points.
- The function's shape is "S-shaped," typical of cubics with three real roots.

Applications and Significance of the Factorization

Solving Polynomial Equations

Knowing the factors allows for straightforward solving of the equation $f(x) = 0$, which is essential in various mathematical and real-world applications.

Integration and Area Calculations

Factoring simplifies integration processes involving the polynomial, enabling calculations of areas under the curve and other integral-based analyses.

Modeling Real-World Phenomena

Cubic functions often model phenomena such as population growth, physics systems, or economic trends, where understanding roots and turning points is crucial.

Conclusion

The function $f(x) = x^3 - 7x + 6$, when factored into $(x - 1)(x - 2)(x + 3)$, reveals the deep connection between the algebraic structure of the polynomial and its graphical representation. The roots at $x=-3$, 1, and 2 not only identify the points where the graph intersects the x-axis but also guide us in understanding the shape and behavior of the cubic curve. The figure associated with the function illustrates these roots and the local extrema, providing a visual comprehension of the polynomial's nature. Recognizing the factorization is fundamental for solving equations, analyzing the graph, and applying the function in practical contexts. Overall, the study of $f(x) = x^3 - 7x + 6$ exemplifies the elegance and utility of polynomial factorization in mathematics.

Frequently Asked Questions

What are the roots of the function $f(x) = x^3 - 7x + 6$?

The roots of the function are the values of x where the function equals zero. Factoring or using the Rational Root Theorem reveals the roots are $x = 1$, $x = 2$, and $x = -3$.

How does the factorization of $f(x) = x^3 - 7x + 6$ relate to its roots?

The factorization corresponds to the roots of the function, expressed as $f(x) = (x - 1)(x - 2)(x + 3)$, indicating these are the zeros of the polynomial.

What is the significance of the function's factors in understanding its graph?

The factors (x - 1), (x - 2), and (x + 3) indicate the x-intercepts of the graph, showing where the function crosses the x-axis at x=1, x=2, and x=-3.

How can the factorization of f(x) help in solving equations involving the function?

By factoring f(x), you can set each factor equal to zero to find the roots quickly, simplifying the process of solving f(x) = 0.

What does the figure showing the factors of f(x) = x^3 - 7x + 6 illustrate about the polynomial's behavior?

The figure demonstrates the polynomial's x-intercepts at the roots, and helps visualize how the factors relate to the shape and roots of the cubic function.

Additional Resources

Function Analysis

Understanding the behavior and properties of mathematical functions is fundamental in algebra and calculus. In this article, we will delve deeply into the function:

$$ f(x) = x^3 - 7x + 6 $$

and explore how its factors relate to its graph and roots. We will examine the function's structure, roots, factors, and the significance of its graphical representation, providing a comprehensive review suitable for students, educators, and enthusiasts alike.

Introduction to the Function $f(x) = x^3 - 7x + 6$

The cubic polynomial $f(x) = x^3 - 7x + 6$ is a classic example used to demonstrate fundamental concepts in polynomial functions. Its degree is three, making it a cubic function, which is inherently interesting due to the potential for multiple real roots, inflection points, and complex behavior.

Key Characteristics

- Degree: 3 (cubic)
- Leading Coefficient: 1 (positive)

- Constant term: 6
- Standard form: $ax^3 + bx^2 + cx + d$

In this case, the quadratic term (bx^2) is missing, which simplifies some aspects of analysis but still offers rich behavior.

Analyzing the Function: Roots and Factors

A core part of understanding any polynomial function is determining its roots—the values of x where $f(x) = 0$. These roots reveal the x-intercepts on the graph and are directly related to the factors of the polynomial.

Finding Roots of $f(x) = x^3 - 7x + 6$

Since the polynomial is cubic, it can have up to three real roots, which may be rational or irrational. To find roots systematically, we often start with rational root testing.

Rational Root Theorem

Potential rational roots are factors of the constant term divided by factors of the leading coefficient. Here:

- Factors of 6: $\pm1, \pm2, \pm3, \pm6$
- Factors of 1: ±1

Thus, possible rational roots are:

$$
\pm1, \pm2, \pm3, \pm6
$$

Testing Rational Roots

Using synthetic division or direct substitution:

- $f(1) = 1^3 - 7(1) + 6 = 1 - 7 + 6 = 0$

Since $f(1) = 0$, $x=1$ is a root.

- $f(-1) = -1 -7(-1)+6 = -1 + 7 +6= 12 \neq 0$

- $f(2) = 8 - 14 + 6= 0$

- $f(-2) = -8 +14 +6= 12 \neq 0$

- $f(3) = 27 - 21 + 6= 12 \neq 0$

- $f(-3) = -27 + 21 +6= 0$

- $f(6) = 216 - 42 +6= 180 \neq 0$

- $f(-6) = -216 + 42 +6= -168 \neq 0$

Roots identified:

$$
x=1,\quad x=2,\quad x=-3
$$

Factoring the Polynomial

Since the roots are $1, 2, -3$, the polynomial factors as:

$$
f(x) = (x - 1)(x - 2)(x + 3)
$$

Expanding to verify:

$$
(x - 1)(x - 2)(x + 3)
$$

First, multiply $(x - 1)(x - 2)$:

$$
x^2 - 2x - x + 2 = x^2 - 3x + 2
$$

Next, multiply $(x^2 - 3x + 2)$ by $(x + 3)$:

$$
x^2(x + 3) - 3x(x + 3) + 2(x + 3)
$$

$$
= x^3 + 3x^2 - 3x^2 - 9x + 2x + 6
$$

$$
= x^3 + (3x^2 - 3x^2) + (-9x + 2x) + 6
$$

$$
= x^3 - 7x + 6
$$

which confirms the factorization.

Graphical Representation and Significance of Factors

The factorization provides critical insights into the graph of $f(x)$:

- X-intercepts (roots): The points where the graph crosses the x-axis are at $x=1, 2, -3$.
- Multiplicity: Each root appears once, indicating simple zeros and smooth crossings.
- Behavior at roots:

- At $x=1$, the graph crosses the x-axis going from positive to negative or vice versa.
- At $x=2$, similar crossing behavior.
- At $x=-3$, the graph crosses from negative to positive.

Visual Features of the Graph

The cubic nature implies the graph:

- Extends to $\pm \infty$ as $x \to \pm \infty$.
- Has an inflection point where the concavity changes.
- Crosses the x-axis at the roots, with the graph passing through these points smoothly.

Critical Points and Inflection Point

- Critical points: Found by taking the first derivative:

$$
f'(x) = 3x^2 - 7
$$

Set $f'(x) = 0$:

$$
3x^2 - 7 = 0 \Rightarrow x^2 = \frac{7}{3} \Rightarrow x = \pm \sqrt{\frac{7}{3}}
$$

- Concavity change: The second derivative:

$$
f''(x) = 6x
$$

- At $x=0$, the second derivative is zero; the inflection point occurs at $x=0$:

$$
f(0) = 0 - 0 + 6 = 6
$$

This point indicates a change in concavity without a maximum or minimum.

Implications of the Factors and Roots

Understanding the factors of $f(x)$ allows for multiple practical applications:

Solving Equations

Knowing the roots simplifies solving equations involving $f(x)$, such as:

- Finding intersections with other functions.
- Solving inequalities.

Polynomial Behavior

The factorization emphasizes the nature of the cubic:

- Real roots at $x=-3, 1, 2$.
- The polynomial's sign changes at these roots, dictating the graph's shape.

Sign Chart Analysis

Analyzing the sign of $f(x)$ across intervals determined by roots:

Interval	Sign of $f(x)$	Description
$x < -3$	Negative	Polynomial tends to $-\infty$ as $x \to -\infty$
$-3 < x < 1$	Positive	Between roots, the graph is above the x-axis
$1 < x < 2$	Negative	The graph crosses below the x-axis
$x > 2$	Positive	Polynomial tends to $+\infty$ as $x \to +\infty$

This sign analysis helps in understanding the overall shape and behavior of the graph.

Applications and Real-World Relevance

While at first glance, $f(x) = x^3 - 7x + 6$ appears as a purely mathematical construct, its principles extend to various fields:

- Physics: Modeling systems with cubic relationships, such as certain force equations.
- Economics: Representing profit or cost functions with cubic terms.
- Engineering: Analyzing system stability where polynomial equations govern dynamics.

The clear factorization and root identification streamline these applications, enabling precise solutions and predictions.

Conclusion: The Power of Factorization in Polynomial Functions

The analysis of \(f(x) = x^3 - 7x + 6\) exemplifies the elegance and utility of polynomial factoring. By identifying roots at \(x=-3, 1, 2\), we unlock a comprehensive understanding of the function's behavior, graph, and real-world applications.

This polynomial's factorization not only simplifies solving equations but also illuminates the structure of its graph, the nature of its roots, and the transitions between positive and negative values. It showcases how algebraic techniques intertwine with graphical insights, providing a holistic view of the function's characteristics.

In essence, the function's factors act as a roadmap to its behavior, confirming that mastery of polynomial factorization remains a cornerstone of mathematical literacy and problem-solving prowess.

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