The Mid Point Of The Sides Of A Triangle Are (1,5,-1),(0,4,-2),(2,3,4) Find The Vertices

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Understanding how to find the vertices of a triangle when given the midpoints of its sides is a fundamental problem in coordinate geometry. This problem involves working with the properties of midpoints and the coordinate plane to reverse-engineer the original vertices of the triangle. In this article, we will explore the step-by-step process to determine the vertices of a triangle when the midpoints of its sides are known, specifically focusing on the midpoints (1, 5, -1), (0, 4, -2), and (2, 3, 4).

Introduction to Midpoints and Triangle Vertices

Before diving into the solution, it is essential to understand the concepts of midpoints and how they relate to the vertices of a triangle.

What Are Midpoints?

- A midpoint of a line segment is the point that divides the segment into two equal parts.
- In coordinate geometry, the midpoint M of a segment with endpoints $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is given by:

$$
M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)
$$

Using Midpoints to Find Triangle Vertices

- The problem provides the midpoints of the sides, not the vertices directly.
- Since each midpoint corresponds to a side of the triangle, we can set up equations to find the vertices.
- The key is to understand that each side's midpoint is the average of the coordinates of its endpoints.

Step-by-Step Approach to Find the Vertices

To find the vertices $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$, and $C(x_3, y_3, z_3)$, given the midpoints of the sides, follow these steps:

1. Assign the Midpoints to Corresponding Sides

- Let’s denote the midpoints as:

$$
M_{AB} = (1, 5, -1) \quad \text{(midpoint of side AB)}
$$

$$
M_{BC} = (0, 4, -2) \quad \text{(midpoint of side BC)}
$$

$$
M_{CA} = (2, 3, 4) \quad \text{(midpoint of side CA)}
$$

- Each of these midpoints corresponds to a side of the triangle.

2. Understand the Relationship Between Vertices and Midpoints

- The midpoint of side AB, M_{AB}, is given by:

$$
M_{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)
$$

- Similarly, for other sides:

$$
M_{BC} = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}, \frac{z_2 + z_3}{2} \right)
$$

$$
M_{CA} = \left(\frac{x_3 + x_1}{2}, \frac{y_3 + y_1}{2}, \frac{z_3 + z_1}{2} \right)
$$

3. Set Up Equations for the Vertices

- From the midpoints, we derive the following equations:

$$x_1 + x_2 = 2 \times 1 = 2$$
$$y_1 + y_2 = 2 \times 5 = 10$$
$$z_1 + z_2 = 2 \times (-1) = -2$$

$$x_2 + x_3 = 2 \times 0 = 0$$
$$y_2 + y_3 = 2 \times 4 = 8$$
$$z_2 + z_3 = 2 \times (-2) = -4$$

$$x_3 + x_1 = 2 \times 2 = 4$$
$$y_3 + y_1 = 2 \times 3 = 6$$
$$z_3 + z_1 = 2 \times 4 = 8$$

- These equations form a system to be solved for the six variables $x_1, x_2, x_3, y_1, y_2, y_3, z_1, z_2, z_3$.

Solving the System of Equations

To find the vertices, solve the system step by step:

1. Find x_1, x_2, x_3

- From the equations:

$$x_1 + x_2 = 2$$

\[
x_2 + x_3 = 0
\]

\[
x_3 + x_1 = 4
\]

- Express \(x_2 \) from the second equation:

\[
x_2 = -x_3
\]

- Substitute into the first:

\[
x_1 - x_3 = 2
\]

- From the third:

\[
x_3 + x_1 = 4
\]

- Now, solve for \(x_1 \) and \(x_3 \):

\[
x_1 = 4 - x_3
\]

\[
(4 - x_3) - x_3 = 2
\]

\[
4 - 2x_3 = 2
\]

\[
2x_3 = 2
\]

\[
x_3 = 1
\]

- Back to find \(x_1 \) and \(x_2 \):

\[

$$
x_1 = 4 - 1 = 3
$$

$$
x_2 = -1
$$

2. Find y_1, y_2, y_3

- Similar process:

$$
y_1 + y_2 = 10
$$

$$
y_2 + y_3 = 8
$$

$$
y_3 + y_1 = 6
$$

- From the second:

$$
y_2 = 8 - y_3
$$

- Substitute into the first:

$$
y_1 + (8 - y_3) = 10
$$

$$
y_1 = 10 - 8 + y_3 = 2 + y_3
$$

- From the third:

$$
y_3 + y_1 = 6
$$

$$
y_3 + (2 + y_3) = 6
$$

\[

$$
2 y_3 + 2 = 6
$$

$$
2 y_3 = 4
$$

$$
y_3 = 2
$$

- Find y_1 and y_2:

$$
y_1 = 2 + 2 = 4
$$

$$
y_2 = 8 - 2 = 6
$$

3. Find z_1, z_2, z_3

- Equations:

$$
z_1 + z_2 = -2
$$

$$
z_2 + z_3 = -4
$$

$$
z_3 + z_1 = 8
$$

- From the second:

$$
z_2 = -4 - z_3
$$

- Substitute into the first:

$$
z_1 + (-4 - z_3) = -2
$$

\[

z_1 = -2 + 4 + z_3 = 2 + z_3
\]

- From the third:

Frequently Asked Questions

What are the given midpoints of the sides of the triangle?

The midpoints are (1, 5, -1), (0, 4, -2), and (2, 3, 4).

How do you find the vertices of a triangle given the midpoints of its sides?

You can find the vertices by using the midpoint formula in reverse, setting up equations for the midpoints as averages of the vertices, and solving the resulting system.

What is the midpoint formula in 3D coordinate geometry?

The midpoint M of segment AB with points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is given by $M = ((x_1 + x_2)/2, (y_1 + y_2)/2, (z_1 + z_2)/2)$.

How can the midpoints be used to determine the original vertices of the triangle?

By expressing each vertex in terms of the other two vertices and the midpoints, and solving the system of equations, you can find the original vertices.

What is the importance of the midpoints in reconstructing a triangle?

Midpoints help in reconstructing the triangle because they are directly related to the vertices through the midpoint formula, enabling the calculation of the vertices when the midpoints are known.

Can you demonstrate the steps to find the vertices from the given midpoints?

Yes, by setting up equations based on the midpoints and solving for the vertices, you can determine the original vertices. For example, if the midpoints are between vertices A, B, and C, equations can be created and solved systematically.

What is the general formula to find vertices given side midpoints in 3D?

The general approach involves solving the equations derived from the midpoint formulas for each side, often using substitution or linear algebra techniques.

What are the final steps to verify the vertices once found?

Once the vertices are calculated, verify by checking that the midpoints between the computed vertices match the given midpoints.

Additional Resources

The Mid Point Of The Sides Of A Triangle Are (1, 5, -1), (0, 4, -2), (2, 3, 4): Find The Vertices is a classic problem in coordinate geometry that combines understanding of midpoints, vectors, and the properties of triangles in three-dimensional space. This type of question is fundamental for students and professionals alike, as it tests one's ability to work backward from given geometric data to determine the original vertices of a triangle. In this comprehensive guide, we will delve into the process step by step, offering insights into the underlying principles and techniques necessary to solve such problems efficiently.

Introduction to the Problem

Given the midpoints of the sides of a triangle in 3D space, we are asked to find the original vertices of that triangle. The problem provides three midpoints:

- $M_{AB} = (1, 5, -1)$
- $M_{BC} = (0, 4, -2)$
- $M_{CA} = (2, 3, 4)$

where each midpoint corresponds to the midpoint of side AB, BC, and CA respectively.

Why is this problem important?

Understanding how to reverse-engineer the vertices from midpoints is vital in various applications, including computer graphics, physics (e.g., centroid calculations), and advanced geometry. It also reinforces key concepts such as midpoint formulas, vector addition, and coordinate transformations.

Fundamental Concepts and Preliminaries

Before diving into the solution, let's review some essential concepts:

1. Midpoint Formula in 3D

For two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$, the midpoint M is given by:

$$
M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)
$$

2. Coordinates of Vertices from Midpoints

If we denote the vertices of the triangle as $A(x_A, y_A, z_A)$, $B(x_B, y_B, z_B)$, and $C(x_C, y_C, z_C)$, then each midpoint is related to the vertices as:

- $M_{AB} = \text{midpoint of } A \text{ and } B$
- $M_{BC} = \text{midpoint of } B \text{ and } C$
- $M_{CA} = \text{midpoint of } C \text{ and } A$

From these relations, we can work backwards to find the vertices.

3. Symmetry and Coordinate Relations

Since the midpoints are averages of the endpoints, the vertices can be expressed in terms of the midpoints by setting up equations based on the midpoint formulas.

Step-by-Step Solution Approach

Now, let's approach this problem methodically.

1. Assign Variables to Vertices

Let:

$$
A(x_A, y_A, z_A), \quad B(x_B, y_B, z_B), \quad C(x_C, y_C, z_C)
$$

2. Write Equations from Midpoints

Using the midpoint formula:

- For side AB:

\[
M_{AB} = \left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}, \frac{z_A + z_B}{2} \right) = (1, 5, -1)
\]

- For side BC:

\[
M_{BC} = \left(\frac{x_B + x_C}{2}, \frac{y_B + y_C}{2}, \frac{z_B + z_C}{2} \right) = (0, 4, -2)
\]

- For side CA:

\[
M_{CA} = \left(\frac{x_C + x_A}{2}, \frac{y_C + y_A}{2}, \frac{z_C + z_A}{2} \right) = (2, 3, 4)
\]

3. Set Up the System of Equations

From the midpoint equations, we get:

1. \(x_A + x_B = 2 \times 1 = 2 \)

2. \(y_A + y_B = 2 \times 5 = 10 \)

3. \(z_A + z_B = 2 \times (-1) = -2 \)

4. \(x_B + x_C = 2 \times 0 = 0 \)

5. \(y_B + y_C = 2 \times 4 = 8 \)

6. \(z_B + z_C = 2 \times (-2) = -4 \)

7. \(x_C + x_A = 2 \times 2 = 4 \)

8. \(y_C + y_A = 2 \times 3 = 6 \)

9. \(z_C + z_A = 2 \times 4 = 8 \)

Solving the System of Equations

Now, we have nine equations with six unknowns:

\[
x_A, x_B, x_C, y_A, y_B, y_C, z_A, z_B, z_C

$$

Since the system is symmetric, we can find solutions for the vertices systematically.

1. Solve for x-coordinates

From equations (1), (4), and (7):

- $x_A + x_B = 2$ ...(1)

- $x_B + x_C = 0$ ...(4)

- $x_C + x_A = 4$ ...(7)

Using (4):

$$
x_C = -x_B
$$

Substitute into (7):

$$
(-x_B) + x_A = 4 \Rightarrow x_A - x_B = 4
$$

From (1):

$$
x_A + x_B = 2
$$

Adding the two equations:

$$
(x_A - x_B) + (x_A + x_B) = 4 + 2 \Rightarrow 2x_A = 6 \Rightarrow x_A = 3
$$

Using $x_A = 3$ in (1):

$$
3 + x_B = 2 \Rightarrow x_B = -1
$$

And from earlier, $x_C = -x_B = 1$.

Summary for x-coordinates:

$$
x_A = 3, \quad x_B = -1, \quad x_C = 1

\]

---

2. Solve for y-coordinates

From equations (2), (5), and (8):

- $y_A + y_B = 10$ ...(2)

- $y_B + y_C = 8$ ...(5)

- $y_C + y_A = 6$ ...(8)

From (5):

$$
y_C = 8 - y_B
$$

Substitute into (8):

$$
(8 - y_B) + y_A = 6 \Rightarrow y_A - y_B = -2
$$

From (2):

$$
y_A + y_B = 10
$$

Adding the two:

$$
(y_A - y_B) + (y_A + y_B) = -2 + 10 \Rightarrow 2 y_A = 8 \Rightarrow y_A = 4
$$

Using $y_A = 4$ in (2):

$$
4 + y_B = 10 \Rightarrow y_B = 6
$$

And from earlier, $y_C = 8 - y_B = 8 - 6 = 2$.

Summary for y-coordinates:

$$
y_A = 4, \quad y_B = 6, \quad y_C = 2
$$

---

3. Solve for z-coordinates

From equations (3), (6), and (9):

- $z_A + z_B = -2$ ...(3)

- $z_B + z_C = -4$ ...(6)

- $z_C + z_A = 8$ ...(9)

From (6):

$$
z_C = -4 - z_B
$$

Substitute into (9):

$$
(-4 - z_B) + z_A = 8 \Rightarrow z_A - z_B = 12
$$

From (3):

$$
z_A + z_B = -2
$$

Adding the two:

$$
(z_A - z_B) + (z_A + z_B) = 12 + (-2) \Rightarrow 2 z_A = 10 \Rightarrow z_A = 5
$$

Using $z_A = 5$ in (3):

$$
5 + z_B = -2 \Rightarrow z_B = -7
$$

And from earlier, $z_C = -4 - z_B = -4 - (-7) = 3$.

Final Vertices Coordinates

Compiling all the solutions:

```
\[
A(x_A, y_A, z_A) = (3, 4, 5)
\]
\[
B(x_B, y_B, z_B) =
```

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