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Understanding the relationship between rational numbers and their products is fundamental in algebra. When given the product of two rational numbers and one of the numbers, the problem typically involves finding the other number. This type of problem emphasizes the importance of inverse operations, specifically division, to isolate the unknown. In this article, we will explore how to approach and solve the problem: "The product of two rational numbers is -16/9. If one of the numbers is -4/3, find the other." We will delve into the concepts of rational numbers, multiplication, and division, and walk through the step-by-step solution process, complemented by illustrative examples and explanations.

Understanding Rational Numbers and Their Properties

What Are Rational Numbers?

- Rational numbers are numbers that can be expressed as the quotient or fraction of two integers, where the denominator is not zero.
- Examples include: 1/2, -3/4, 7, -5, 0, and -4/3.
- Rational numbers are dense on the number line, meaning between any two rational numbers, there exists another rational number.

Properties of Rational Numbers in Multiplication and Division

- Closure Property: The product or quotient of two rational numbers is always a rational number.
- Commutative Property: a × b = b × a.
- Associative Property: (a × b) × c = a × (b × c).
- Multiplicative Inverse: For any non-zero rational number a/b, its inverse is b/a such that a/b × b/a = 1.

Understanding these properties helps in manipulating and solving equations involving rational numbers.

Formulating the Problem Mathematically

Given Data

- Product of two rational numbers: $P = -\frac{16}{9}$
- One of the numbers: $a = -\frac{4}{3}$

Objective

- Find the other rational number, b.

Mathematical Approach to the Solution

Using the Multiplication Formula

The key relationship is:

$$
a \times b = P
$$

Substituting the known values:

$$
-\frac{4}{3} \times b = -\frac{16}{9}
$$

To find b, we need to isolate it:

$$
b = \frac{P}{a}
$$

Performing the Calculation

- Since both a and P are rational numbers, dividing P by a involves multiplying P by the reciprocal of a:

$$
b = P \times \frac{1}{a}
$$

- Find the reciprocal of $a = -\frac{4}{3}$:

$$
\frac{1}{a} = -\frac{3}{4}
$$

- Now multiply $P = -\frac{16}{9}$ by $\frac{1}{a} = -\frac{3}{4}$:

$$
b = -\frac{16}{9} \times -\frac{3}{4}
$$

- Multiply numerator and denominator:

$$
b = \left(-16 \times -3\right) / \left(9 \times 4\right)
$$

$$
b = (48) / (36)
$$

Simplifying the Result

- Simplify the fraction:

$$
b = \frac{48}{36} = \frac{4 \times 12}{3 \times 12} = \frac{4}{3}
$$

- Since both numerator and denominator are divisible by 12, the simplified form is:

$$
b = \frac{4}{3}
$$

Final Answer and Interpretation

Conclusion

- The other rational number is $\frac{4}{3}$.
- This makes sense because:

$$
a \times b = -\frac{4}{3} \times \frac{4}{3} = -\frac{16}{9}
$$

which matches the original product.

Summary of the Solution Steps

1. Identify the known quantities: $a = -\frac{4}{3}$, $P = -\frac{16}{9}$.

2. Express the unknown b as $b = \frac{P}{a}$.

3. Replace P and a with their fractions and find the reciprocal of a.

4. Multiply P by the reciprocal of a to obtain b.

5. Simplify the resulting fraction to find the value of b.

Additional Examples and Practice Problems

Example 1: Find the other number if the product is 10/7 and one of the numbers is 2/3.

- Solution:

$$
b = \frac{\frac{10}{7}}{\frac{2}{3}} = \frac{10}{7} \times \frac{3}{2} = \frac{10 \times 3}{7 \times 2} = \frac{30}{14} = \frac{15}{7} = \frac{15}{7}
$$

- Final answer: $b = \frac{15}{7}$.

Practice Problems for Readers

- Find the other number if the product is $\frac{25}{16}$ and one of the numbers is $-\frac{5}{4}$.

- Determine the missing rational number if the product is $-3/8$ and one number is $3/2$.

- Given the product $-9/4$ and one number $-3/2$, find the other number.

Key Takeaways

- To find the unknown rational number given the product and one rational factor, divide the product by the known number.
- Remember that dividing by a fraction is equivalent to multiplying by its reciprocal.
- Simplify the resulting fraction to its lowest terms for clarity.
- Always verify the solution by multiplying the two numbers to ensure the product matches the original value.

Conclusion

In conclusion, solving for an unknown rational number when the product and one number are known involves straightforward algebraic manipulation rooted in the properties of rational numbers. The main steps include expressing the unknown as a division problem, finding the reciprocal, and simplifying the result. In the specific problem discussed, the other rational number is $\frac{4}{3}$, which, when multiplied by $-\frac{4}{3}$, yields the original product of $-\frac{16}{9}$. Mastery of these concepts enhances problem-solving skills in algebra and provides a strong foundation for more complex mathematical topics involving rational expressions and equations.

Frequently Asked Questions

What is the problem asking for in the given question?

The problem is asking to find the other rational number when the product of two rational numbers is -16/9, and one of the numbers is -4/3.

How do you set up the equation to find the unknown rational number?

Let the unknown number be x. Since the product of the two numbers is -16/9, the equation is (-4/3) x = -16/9.

What is the method to solve for x in the equation (-4/3) x = -16/9?

Divide both sides of the equation by -4/3 or multiply both sides by its reciprocal, which is -3/4, to isolate x.

What is the reciprocal of -4/3?

The reciprocal of -4/3 is -3/4.

What is the step-by-step solution to find the other rational number?

Multiply both sides by -3/4: x = (-16/9) (-3/4). Simplify the multiplication to find x.

What is the value of x after performing the multiplication?

x = (-16/9) (-3/4) = (16/9) (3/4) = (16 3) / (9 4) = 48 / 36 = 4 / 3.

What is the final answer to the problem?

The other rational number is 4/3.

Why is the answer 4/3 and not -4/3?

Because multiplying -4/3 by 4/3 yields -16/9, which matches the given product. Using the reciprocal of -4/3 ensures the product is negative, resulting in 4/3 as the other number.

Can this method be used for other similar problems

involving rational numbers?

Yes, the method of setting up an equation, isolating the unknown, and using reciprocals to solve is applicable for any problem involving the product of two rational numbers where one is known.

Additional Resources

The Product Of Two Rational Numbers Is -16/9. If One Of The Numbers Is -4/3 Find The Other

Understanding the relationship between rational numbers and their products is fundamental in algebra and number theory. The problem stating that the product of two rational numbers is $-\frac{16}{9}$, and one of these numbers is $-\frac{4}{3}$, provides an excellent opportunity to explore concepts such as multiplicative relationships, inverse operations, and solving for unknowns within the realm of rational numbers. This article delves into the details of this problem, breaking down the steps involved, and examining related mathematical principles, offering a comprehensive review of the topic.

Understanding Rational Numbers and Their Properties

What Are Rational Numbers?

Rational numbers are numbers that can be expressed as the quotient or fraction $\frac{p}{q}$, where p and q are integers, and $q \neq 0$. They include integers, fractions, and finite or repeating decimals. For example, $\frac{2}{5}$, $-\frac{7}{3}$, and -4 (which can be written as $-\frac{4}{1}$) are all rational numbers.

Features of Rational Numbers:
- Closure under Addition and Multiplication: The sum or product of two rational numbers is always rational.
- Existence of Additive and Multiplicative Inverses: For any non-zero rational number, there exists an inverse such that their product is 1.
- Density: Rational numbers are densely packed on the number line; between any two rational numbers, there exists another rational number.

Pros:
- They are easy to manipulate algebraically.
- They form a field, allowing for a rich set of operations.

Cons:
- Rational numbers can be non-terminating or repeating decimals.
- Some problems involve complex fractions, which can be cumbersome.

Analyzing the Given Problem

Restating the Problem

The core question is: If the product of two rational numbers is $-\frac{16}{9}$, and one of these numbers is $-\frac{4}{3}$, what is the other number?

Mathematically, this can be represented as:

$$
\text{Number}_1 \times \text{Number}_2 = -\frac{16}{9}
$$

Given:

$$
\text{Number}_1 = -\frac{4}{3}
$$

Find:

$$
\text{Number}_2
$$

Step-by-Step Solution Approach

Using Algebra to Find the Unknown

To find the other rational number, we utilize the property of multiplication and the concept of multiplicative inverse:

$$
\text{Number}_2 = \frac{\text{Product}}{\text{Number}_1}
$$

Plugging in the known values:

$$
\text{Number}_2 = \frac{-\frac{16}{9}}{-\frac{4}{3}}
$$

Note: Dividing by a fraction is equivalent to multiplying by its reciprocal.

Calculating the Other Number

Expressed explicitly:

$$
\text{Number}_2 = -\frac{16}{9} \times \left(-\frac{3}{4}\right)
$$

Multiplying the numerators and denominators:

$$
\text{Number}_2 = \left(-\frac{16}{9}\right) \times \left(-\frac{3}{4}\right) = \frac{16 \times 3}{9 \times 4}
$$

Since multiplying two negatives yields a positive, the negatives cancel out:

$$
\text{Number}_2 = \frac{48}{36}
$$

Simplify the fraction:

$$
\frac{48}{36} = \frac{4 \times 12}{3 \times 12} = \frac{4}{3}
$$

Result:

$$
\boxed{\text{Number}_2 = \frac{4}{3}}
$$

Thus, the other rational number is $\frac{4}{3}$.

Verification of the Result

Checking the Product

To ensure accuracy, multiply the two numbers:

$$
-\frac{4}{3} \times \frac{4}{3} = -\frac{4 \times 4}{3 \times 3} = -\frac{16}{9}
$$

This confirms that the calculations are correct, as the product matches the given value.

Deeper Insights and Related Concepts

Understanding the Role of Inverses

In this problem, the operation involved dividing the product by one of the numbers, which is equivalent to multiplying by its reciprocal or inverse:

$$
\text{Number}_2 = \frac{\text{Product}}{\text{Number}_1}
$$

The reciprocal of $-\frac{4}{3}$ is $-\frac{3}{4}$. The negative signs cancel out during multiplication, which is key to arriving at the correct solution.

Features of Inverses:
- The inverse of a rational number $\frac{p}{q}$ is $\frac{q}{p}$, provided $p \neq 0$.
- Multiplying a number by its inverse yields 1.

Pros:
- Simplifies solving for unknowns in algebraic equations.
- Facilitates division of rational expressions.

Cons:
- Care must be taken with negative signs.
- Zero does not have an inverse, so division by zero is undefined.

Application in Real-World Contexts

While this problem is mathematical in nature, the principles have practical applications:
- Scaling problems: When adjusting quantities proportionally.
- Financial calculations: Determining unknown ratios or rates.
- Physics and Engineering: Calculations involving inverse relationships.

Additional Practice and Variations

Changing the Known Values

Suppose instead of $-\frac{4}{3}$, the known number was $\frac{2}{5}$, and the product remained $-\frac{16}{9}$. How would we find the other number? The process remains similar:

$$
\text{Unknown} = \frac{-\frac{16}{9}}{\frac{2}{5}} = -\frac{16}{9} \times \frac{5}{2} = -\frac{16 \times 5}{9 \times 2} = -\frac{80}{18} = -\frac{40}{9}
$$

This variation demonstrates the flexibility of the division and multiplication approach for solving similar problems.

Pros and Cons of the Solution Method

Pros:
- Straightforward approach using fundamental algebraic principles.
- Clear steps ensure accuracy and understanding.
- Applicable to a wide range of similar problems involving rational numbers.

Cons:
- Requires understanding of fractions and their properties.
- Mistakes can occur with sign handling or simplification.
- Not suitable if the known number is zero (division by zero is undefined).

Conclusion

The problem involving the product of two rational numbers, with one known value, is a classic example of applying algebraic principles to rational expressions. By understanding the properties of rational numbers, especially their multiplicative inverses, we efficiently arrive at the unknown number. In this case, the other rational number is $\frac{4}{3}$, which perfectly satisfies the original condition. Such problems highlight the importance of fundamental algebraic skills and provide insight into the structure and behavior of rational numbers. The approach outlined here is not only useful for this specific problem but also serves as a template for solving a wide array of similar mathematical challenges.

Key Takeaways:
- Rational numbers can be manipulated using basic algebra, including division and multiplication.
- The inverse of a rational number plays a crucial role in solving for unknowns.
- Always verify your solutions by substituting back into the original problem.
- Mastery of these concepts enhances problem-solving skills across mathematics and applied sciences.

In summary, understanding and applying the properties of rational numbers allows for efficient problem-solving and deepens comprehension of algebraic relationships. The specific scenario discussed—finding the unknown rational number given a product and one factor—is a fundamental example demonstrating the practical use of inverses and fraction arithmetic in mathematics.

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