

The Sum Of Two Numbers Is 24.The Difference Of Their Square Is 144.what Are The Two Numbers?

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Understanding how to solve algebraic problems involving sums and differences of squares is fundamental in mathematics. When faced with a problem stating that the sum of two numbers equals 24 and the difference of their squares is 144, students and math enthusiasts alike need a systematic approach. This article explores the problem comprehensively, providing step-by-step solutions, explanations, and insights into the underlying mathematical principles. Whether you're a student preparing for exams or someone interested in enhancing your problem-solving skills, this guide offers valuable information to help you find the two numbers accurately.

Understanding the Problem Statement

Before diving into solving the problem, it's essential to clearly understand what is being asked and what information is provided.

Problem Recap:

- The sum of two numbers is 24.
- The difference of their squares is 144.
- Find the two numbers.

Key Concepts Involved:

- Sum of two numbers
- Difference of squares
- Algebraic equations

Mathematical Notation:

Let the two unknown numbers be x and y .

Given:

$$x + y = 24$$

$$x^2 - y^2 = 144$$

The goal is to find the values of x and y .

Breaking Down the Problem

To solve this problem efficiently, it's crucial to understand the relationships between the given expressions and how to manipulate them algebraically.

The Sum Equation

$$x + y = 24$$

This is a straightforward linear equation representing the sum of two numbers.

The Difference of Squares Equation

$$x^2 - y^2 = 144$$

Recall the algebraic identity:

$$x^2 - y^2 = (x + y)(x - y)$$

Using this identity simplifies the problem significantly.

Transforming the Problem Using Algebraic Identities

Applying the difference of squares identity:

$$(x + y)(x - y) = 144$$

Since $x + y = 24$, substitute this into the equation:

$$24(x - y) = 144$$

Now, solve for $x - y$:

$$x - y = \frac{144}{24} = 6$$

Key Results:

- $x + y = 24$

- $x - y = 6$

These two equations now form a simple system of linear equations.

Solving the System of Equations

With the two equations:

1. $x + y = 24$

2. $x - y = 6$

we can find the values of x and y using addition or subtraction.

Method 1: Addition

Add the two equations:

$$(x + y) + (x - y) = 24 + 6$$

Simplify:

$$2x = 30$$

$$x = \frac{30}{2} = 15$$

Method 2: Substitution

Now, substitute $x = 15$ into the first equation:

$$15 + y = 24$$

$$y = 24 - 15 = 9$$

Solution:

- $x = 15$

- $y = 9$

Alternatively, reversing the roles:

$$x = 9$$

$$y = 15$$

Both pairs satisfy the original conditions.

Verifying the Solution

Verification ensures the solution is correct.

Check the sum:

$$\boxed{15 + 9 = 24} \checkmark$$

Check the difference of squares:

$$\boxed{15^2 - 9^2 = 225 - 81 = 144} \checkmark$$

The calculations confirm that the solutions satisfy both conditions.

Alternative Approaches to the Problem

While the above method is straightforward, other methods can also be employed.

1. Graphical Method

Plotting the equations:

- $(x + y = 24)$: a straight line.
- $(x^2 - y^2 = 144)$: a hyperbola.

The intersection points of these graphs give the solutions.

2. Substitution Method

Express (y) in terms of (x) :

$$\boxed{y = 24 - x}$$

Substitute into the difference of squares:

$$\boxed{x^2 - (24 - x)^2 = 144}$$

Expand:

$$\boxed{x^2 - (576 - 48x + x^2) = 144}$$

Simplify:

$$\boxed{x^2 - 576 + 48x - x^2 = 144}$$

$$\boxed{48x - 576 = 144}$$

$$\boxed{48x = 720}$$

$$\boxed{x = 15}$$

Correspondingly, $(y = 24 - 15 = 9)$.

This aligns with previous solutions.

Understanding the Mathematical Principles

This problem exemplifies fundamental algebraic concepts:

- Use of algebraic identities: Recognizing $(x^2 - y^2 = (x + y)(x - y))$ simplifies the problem.
- System of equations: Combining equations to find unknown variables.
- Substitution: Replacing variables to reduce the problem to a single variable.
- Verification: Confirming solutions satisfy initial conditions.

Mastering these principles enhances problem-solving skills for more complex algebraic problems.

Common Mistakes and How to Avoid Them

When solving similar problems, be cautious of the following pitfalls:

- Forgetting to verify solutions: Always check if the found numbers satisfy all conditions.
- Incorrect algebraic manipulation: Be precise with expanding and simplifying expressions.
- Mixing up variables: Keep track of which variable represents which number.
- Ignoring the possibility of multiple solutions: Some problems may have more than one solution; verify all.

By paying attention to these details, you can ensure accurate and reliable solutions.

Practical Applications of the Problem

Understanding how to solve such algebraic problems has real-world applications:

- Financial calculations: Determining unknown investments or profits based on sums and differences.
- Physics: Calculating variables in motion or energy problems.
- Engineering: Analyzing systems where sums and differences of quantities are involved.
- Computer Science: Algorithm development involving equations and data analysis.

Mastering these concepts enhances analytical skills across various disciplines.

Conclusion

The problem involving the sum of two numbers being 24 and the difference of their squares being 144 illustrates the power of algebraic identities and systematic problem-solving techniques. By recognizing the difference of squares identity and applying simple algebraic methods, we found that the two numbers are 15 and 9. This approach not only provides the solution but also reinforces foundational math principles essential for tackling more advanced problems.

Remember, always verify your solutions, understand the underlying concepts, and explore alternative methods to deepen your grasp of algebra. With practice, solving such problems becomes intuitive, enabling you to approach similar challenges with confidence and accuracy.

Keywords: algebra, sum of two numbers, difference of squares, solving equations, algebraic identities, system of equations, mathematical problem-solving, quadratic equations, educational math resources

Frequently Asked Questions

What are the two numbers if their sum is 24 and the difference of their squares is 144?

The two numbers are 12 and 12.

How can we find two numbers given their sum and the difference of their squares?

By setting up equations based on the sum and difference, then solving simultaneously, typically using substitution or factoring methods.

Are the two numbers in the problem equal or different?

In this case, the two numbers are equal, both being 12.

Can the two numbers be different if their sum is 24 and the difference of their squares is 144?

No, in this specific problem, the numbers must be equal, so they are both 12.

What equation represents the sum of the two numbers in this problem?

If the numbers are x and y , the equation is $x + y = 24$.

What equation represents the difference of their squares?

It is $x^2 - y^2 = 144$.

How do you solve for the two numbers given these conditions?

Use the equations $x + y = 24$ and $x^2 - y^2 = 144$, then factor the second as $(x - y)(x + y) = 144$, substitute $x + y = 24$, and solve for $x - y$.

What is the value of $(x - y)$ in this problem?

Since $(x - y)(x + y) = 144$ and $x + y = 24$, then $(x - y) = 144 / 24 = 6$.

What are the individual values of x and y ?

Solving the equations $x + y = 24$ and $x - y = 6$ yields $x = 15$ and $y = 9$.

Is there more than one solution to this problem?

No, the solution is unique: the two numbers are 15 and 9.

Additional Resources

The sum of two numbers is 24. The difference of their square is 144. What are the two numbers? This classic algebraic problem challenges students and math enthusiasts alike to employ their problem-solving skills, algebraic manipulation, and understanding of quadratic equations. By breaking down the problem systematically, we can uncover the two numbers that satisfy both conditions.

Introduction: Understanding the Problem

In many algebraic problems, translating words into mathematical expressions is the key to finding solutions. Here, we're given two main pieces of information:

1. The sum of two numbers is 24.
2. The difference of their squares is 144.

Our goal is to determine the two numbers that satisfy these conditions.

Step 1: Define the Variables

To approach the problem systematically, assign variables to the unknowns:

- Let x be the first number.

- Let y be the second number.

With these variables, the problem can now be expressed as equations:

- Equation 1: $x + y = 24$

- Equation 2: $x^2 - y^2 = 144$

Step 2: Express One Variable in Terms of the Other

From Equation 1, express y in terms of x :

- $y = 24 - x$

This substitution simplifies the problem to a single variable, enabling us to solve more straightforwardly.

Step 3: Rewrite the Difference of Squares

Recall that $x^2 - y^2$ can be factored using the difference of squares formula:

- $x^2 - y^2 = (x - y)(x + y)$

Given that $x + y = 24$ (from Equation 1), substitute this into the factored form:

- $(x - y) 24 = 144$

Now, solve for $(x - y)$:

- $x - y = 144 / 24$

- $x - y = 6$

Step 4: Set Up a System of Equations

At this point, we have two equations involving x and y :

- Equation A: $x + y = 24$

- Equation B: $x - y = 6$

This is a simple system of linear equations that can be solved using addition or subtraction.

Step 5: Solve for the Variables

Add Equation A and Equation B:

- $(x + y) + (x - y) = 24 + 6$
- $2x = 30$
- $x = 15$

Now, substitute $x = 15$ back into Equation A:

- $15 + y = 24$
- $y = 24 - 15$
- $y = 9$

Thus, the two numbers are 15 and 9.

Step 6: Verify the Solution

Always verify solutions by plugging the values back into the original conditions:

- Sum check: $15 + 9 = 24$ ✓
- Difference of squares check: $15^2 - 9^2 = 225 - 81 = 144$ ✓

Both conditions are satisfied, confirming our solution.

Alternative Approach: Using Quadratic Equations

While the previous method is straightforward, an alternative approach involves directly working with quadratic expressions.

Step 1: Express y in terms of x

As before, $y = 24 - x$.

Step 2: Write the difference of squares as a quadratic

Using the difference of squares:

- $x^2 - y^2 = 144$

Substitute y :

- $x^2 - (24 - x)^2 = 144$

Expand $(24 - x)^2$:

- $x^2 - [(24)^2 - 224x + x^2] = 144$

Simplify:

- $x^2 - (576 - 48x + x^2) = 144$

Distribute the negative:

$$-x^2 - 576 + 48x - x^2 = 144$$

Combine like terms:

$$-(x^2 - x^2) + 48x - 576 = 144$$

$$-0 + 48x - 576 = 144$$

Solve for x:

$$-48x = 144 + 576$$

$$-48x = 720$$

$$-x = 720 / 48$$

$$-x = 15$$

Substitute x back to find y:

$$-y = 24 - 15 = 9$$

Again, confirming the previous solution.

Additional Insights and Variations

What if the difference of their squares was negative?

Suppose the problem stated that the difference of their squares was -144. The same process applies, but with a sign change, resulting in different solutions.

Can the numbers be negative?

In the context of the problem, negative numbers are possible if the conditions are rephrased accordingly. For instance, if the sum is 24, and the difference of squares is 144, the numbers could be:

$$-x = 15, y = 9 \text{ (positive solutions)}$$

$$- \text{or } x = -15, y = -9 \text{ (if negatives are considered)}$$

Check:

- Sum: $-15 + (-9) = -24$ (which does not satisfy the sum condition of 24), so negative solutions are invalid unless the problem explicitly states so.

Real-world applications

Problems like this can model various real-world situations, such as:

- Distributing resources with certain constraints
- Engineering problems involving forces or energy differences
- Financial calculations involving sums and differences

Summary

The sum of two numbers is 24. The difference of their squares is 144. what are the two numbers? Through algebraic manipulation, substitution, and verification, we find that the two numbers are 15 and 9. The key steps involve:

- Assigning variables
- Expressing one variable in terms of the other
- Applying the difference of squares formula
- Solving the resulting system of equations
- Verifying the solutions

This problem exemplifies the elegance of algebra in solving seemingly complex word problems by translating them into manageable mathematical expressions.

Final Notes

- Always check your solutions in the original conditions.
- Practice similar problems to strengthen your algebraic reasoning.
- Remember the difference of squares formula: $a^2 - b^2 = (a - b)(a + b)$.

By mastering these techniques, you'll be well-equipped to tackle a wide range of algebraic problems with confidence.

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