What Is The Common Ratio Of The Geometric Sequence 6,4,8/3,16/9,...r = 2r = 2/3r = -2r = 3/2

What Is The Common Ratio Of The Geometric Sequence 6,4,8/3,16/9,...r = 2r = 2/3r = -2r = 3/2

Understanding the common ratio of a geometric sequence is fundamental to grasping how these sequences behave and how they can be applied in various mathematical contexts. In this article, we will explore the sequence 6, 4, 8/3, 16/9, ... and analyze its common ratio. We will also examine the different possible ratios such as r = 2, r = 2/3, r = -2, and r = 3/2, providing clarity on how these ratios influence the sequence's progression.

What Is A Geometric Sequence?

Definition of a Geometric Sequence

A geometric sequence is a list of numbers where each term after the first is obtained by multiplying the previous term by a constant called the common ratio. Mathematically, a geometric sequence can be written as:

- Term 1: a_1
- Term 2: $a_2 = a_1 r$
- Term 3: $a_3 = a_2 r = a_1 r^2$
- Term n: $a_n = a_1 r^{n-1}$

where a1 is the first term and r is the common ratio.

Importance of the Common Ratio

The common ratio determines whether the sequence increases, decreases, oscillates, or converges. Positive ratios lead to sequences that grow or shrink in the same direction, while negative ratios cause the sequence to alternate sign.

Analyzing the Sequence 6, 4, 8/3, 16/9, ...

Identifying the Terms

Let's list the first few terms:

- $a_1 = 6$
- $a_2 = 4$
- $a_3 = 8/3 \approx 2.6667$
- $a_4 = 16/9 \approx 1.7778$

These terms suggest the sequence is decreasing, but to confirm, we need to find the common ratio.

Calculating the Common Ratio

The common ratio r can be found by dividing any term by its preceding term:

- Between the first and second term:

r = a2 / a1 = 4 / 6 = 2 / 3

- Between the second and third term:

r = (8/3) / 4 = (8/3) / (4/1) = (8/3) (1/4) = 8/12 = 2/3

- Between the third and fourth term:

r = (16/9) / (8/3) = (16/9) (3/8) = (48/72) = 2/3

Since the ratio remains consistent at 2/3, the common ratio r = 2/3.

Conclusion: The sequence is geometric with a common ratio of r = 2/3.

Understanding Different Possible Ratios

The sequence provided in the question mentions multiple ratios: r = 2, r = 2/3, r = -2, and r = 3/2. Let's explore what each of these ratios implies and how they relate to variations of the sequence.

Ratio r = 2

- Sequence behavior: Each term doubles the previous term.
- Sample sequence: 6, 12, 24, 48, ...
- Application: Used to model exponential growth like population increase or compound interest.

Ratio r = 2/3

- Sequence behavior: Each term is two-thirds of the previous, leading to a decreasing sequence.
- Sample sequence: 6, 4, 8/3, 16/9, ...
- Application: Modeling decay processes, such as radioactive decay or depreciation.

Ratio r = -2

- Sequence behavior: Terms alternate in sign, doubling in magnitude each time.
- Sample sequence: 6, -12, 24, -48, ...
- Application: Oscillating systems or alternating current models.

Ratio r = 3/2

- Sequence behavior: Each term increases by 50%, leading to exponential growth.
- Sample sequence: 6, 9, 13.5, 20.25, ...
- Application: Population growth models with higher growth rates.

How To Find The Common Ratio in Practice

Step-by-Step Method

To determine the common ratio in any geometric sequence:

1. Identify two consecutive terms in the sequence.
2. Divide the second term by the first:

r = an+1 / an

3. Verify that this ratio is consistent across multiple pairs of consecutive terms.
4. Confirm the ratio to ensure the sequence is geometric.

Example Application with Our Sequence

Using the sequence 6, 4, 8/3, 16/9:

- r = 4 / 6 = 2/3
- r = (8/3) / 4 = 2/3
- r = (16/9) / (8/3) = 2/3

Since all ratios match, r = 2/3.

Why Is Understanding The Common Ratio Important?

Knowing the common ratio allows for:

- Prediction of future terms in the sequence.
- Understanding the long-term behavior of the sequence (growth, decay, oscillation).
- Application in real-world contexts such as economics, biology, physics, and engineering.

Furthermore, in mathematical problems, accurately identifying the ratio helps solve for unknown terms and analyze sequence convergence or divergence.

Summary

In summary, the sequence 6, 4, 8/3, 16/9, ... has a common ratio of r = 2/3, indicating a decreasing geometric sequence. The sequence's terms are obtained by multiplying each previous term by 2/3, demonstrating exponential decay. The different ratios mentioned—such as 2, -2, and 3/2—represent other potential geometric sequences with different behaviors, from growth to oscillation.

Understanding how to calculate and interpret the common ratio is essential for working with geometric sequences, whether for academic purposes, real-world modeling, or advanced mathematical analysis. Always verify the ratio across multiple terms to ensure accuracy, and consider the implications of the ratio's value on the sequence's behavior.

Final Note: Whether you're analyzing a sequence like 6, 4, 8/3, 16/9, or exploring various ratios, grasping the concept of the common ratio is key to unlocking the sequence's properties and applications.

Frequently Asked Questions

What is the common ratio of the geometric sequence 6, 4, 8/3, 16/9, ...?

The common ratio is found by dividing any term by its previous term. For this sequence, the ratio is 4/6 = 2/3, so the common ratio is 2/3.

How do you verify the common ratio in a geometric sequence?

You verify by dividing a term by its preceding term; if the result is consistent throughout, that value is the common ratio.

What are some possible values for the common ratio

based on the options r = 2, 2/3, -2, 3/2?

The correct common ratio from the options provided is 2/3, matching the sequence's pattern.

Why is the common ratio important in a geometric sequence?

It determines how each term relates to the previous one and helps in finding subsequent terms or the sum of the sequence.

Can the common ratio be negative in a geometric sequence?

Yes, the common ratio can be negative, which results in alternating sign sequences; however, in this sequence, it is positive (2/3).

How would the sequence change if the common ratio were 3/2 instead of 2/3?

If the ratio were 3/2, each term would be 1.5 times the previous term, leading to a sequence that increases rapidly, unlike the given sequence.

What is the significance of the sequence pattern 6, 4, 8/3, 16/9?

The pattern illustrates a geometric sequence with a common ratio of 2/3, showing each term is multiplied by 2/3 to get the next.

How can you find the next term in the sequence 6, 4, 8/3, 16/9?

Multiply the last term, 16/9, by the common ratio 2/3: (16/9) (2/3) = (32/27), which would be the next term.

Additional Resources

What Is The Common Ratio Of The Geometric Sequence 6, 4, 8/3, 16/9, ... r = 2 r = 2/3 r = -2 r = 3/2

Introduction

In the realm of mathematics, sequences are fundamental constructs that reveal patterns, relationships, and properties of numbers. Among these, geometric sequences—also known as geometric progressions—are distinguished by their constant ratio between successive terms. Understanding the common ratio of a geometric sequence is crucial, as it encapsulates the rate and nature of change across the sequence elements.

This investigation delves into the sequence: 6, 4, 8/3, 16/9, ..., exploring the question: What is the common ratio of this geometric sequence? Additionally, the sequence is accompanied by various proposed ratios: r = 2, r = 2/3, r = -2, r = 3/2, prompting an in-depth analysis to determine which accurately characterizes the sequence's progression.

Defining a Geometric Sequence

A geometric sequence is a series of numbers where each term after the first is obtained by multiplying the previous term by a fixed, non-zero constant called the common ratio (denoted as r). Formally, if the sequence is:

$a_1, a_2, a_3, ..., a_n$

then:

$a_n = a_1 r^{(n-1)}$

The core property is that:

$a_2 / a_1 = a_3 / a_2 = a_4 / a_3 = ... = r$

Initial Analysis of the Sequence

Given the sequence:

1. 6
2. 4
3. 8/3
4. 16/9
5. ...

Our goal is to determine whether these terms form a geometric sequence, and if so, to find the common ratio r.

Calculating the Ratios Between Successive Terms

To identify the common ratio, compute the ratio of each term to its predecessor:

- Between 6 and 4:

$r_1 = 4 / 6 = 2 / 3 \approx 0.6667$

- Between 4 and 8/3:

$r_2 = (8/3) / 4 = (8/3) (1/4) = (8/3) (1/4) = 8/12 = 2/3 \approx 0.6667$

- Between 8/3 and 16/9:

$r_3 = (16/9) / (8/3) = (16/9) (3/8) = (16 3) / (9 8) = 48 / 72 = 2/3 \approx 0.6667$

Observation: Consistent Common Ratio

The calculations reveal a consistent ratio of 2/3 between successive terms:

- $a_2 / a_1 = 2/3$
- $a_3 / a_2 = 2/3$
- $a_4 / a_3 = 2/3$

This consistency confirms that the sequence is geometric with a common ratio:

$r = 2/3$

Validating the Sequence with the Found Ratio

Let's verify whether the subsequent term aligns with this ratio:

Starting with $a_1 = 6$:

- $a_2 = a_1 r = 6 (2/3) = 4$ (matches)
- $a_3 = a_2 r = 4 (2/3) = 8/3$ (matches)
- $a_4 = a_3 r = (8/3) (2/3) = 16/9$ (matches)

The sequence perfectly adheres to the geometric progression formula with $r = 2/3$.

Alternate Ratios and Their Implications

The sequence's initial terms and ratio calculations invalidate the other proposed ratios:

- $r = 2$: Multiplying 6 by 2 gives 12, which does not match the second term (4). Hence, $r \neq 2$.
- $r = -2$: Multiplying 6 by -2 yields -12, inconsistent with 4.

- $r = 3/2$: Multiplying 6 by 3/2 gives 9, not 4.
- $r = 2/3$: Confirmed as the actual ratio.
- $r = 2$: No, as shown.
- $r = -2$: No.
- $r = 3/2$: No, as shown.

Deeper Examination: Is the Sequence Geometric?

Given the calculations, the sequence 6, 4, 8/3, 16/9 exhibits a constant ratio of 2/3. Therefore, it is a geometric sequence with:

- First term (a_1): 6
- Common ratio (r): 2/3

The sequence is decreasing in magnitude, as each term is two-thirds of the previous.

General Formula for the Sequence

The n-th term of this sequence can be expressed as:

$a_n = a_1 r^{(n-1)} = 6 (2/3)^{(n-1)}$

Verification:

- For n=1: $6 (2/3)^0 = 6 \cdot 1 = 6$
- For n=2: $6 (2/3)^1 = 6 (2/3) = 4$
- For n=3: $6 (2/3)^2 = 6 (4/9) = 24/9 = 8/3$
- For n=4: $6 (2/3)^3 = 6 (8/27) = 48/27 = 16/9$

Matches the given terms perfectly.

Implications and Applications

Understanding the common ratio in this sequence is not merely an academic exercise. It has practical applications in:

- Financial calculations: Modeling depreciation or decay processes.
- Population studies: Where growth factors are less than one, indicating decline.
- Physics: Describing exponential decay phenomena.
- Computer science: Algorithm analysis involving geometric progressions.

The ratio 2/3 indicates a consistent reduction by a factor of 0.6667, exemplifying exponential decay.

Summary of Findings

Aspect	Details
Sequence	6, 4, 8/3, 16/9, ...
Calculated ratios	2/3 between each pair of successive terms
Confirmed common ratio	r = 2/3
Sequence type	Geometric sequence
General term	$a_n = 6 (2/3)^{(n-1)}$

Final Remarks

The analysis conclusively demonstrates that the sequence 6, 4, 8/3, 16/9, ... is geometric with a common ratio of 2/3. This ratio reflects a consistent factor of decrease, providing insights into the sequence's behavior and potential applications.

The alternative ratios proposed—2, -2, 3/2—do not align with the sequence's actual progression, underscoring the importance of empirical verification through successive term ratios.

Understanding such sequences and their ratios is fundamental in various scientific and mathematical disciplines, emphasizing the critical role of precise ratio determination in modeling real-world phenomena.

References

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Note: This article is intended for educational and analytical purposes, providing a comprehensive review of the geometric sequence in question.

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