What Is The Equation Of The Line That Has A Slope Of 3 And Passes Through The Point (1, -2)

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Understanding how to find the equation of a line given certain parameters is a fundamental aspect of coordinate geometry. When a line's slope and a point through which it passes are known, it becomes straightforward to derive its algebraic equation. In this article, we will explore step-by-step how to determine the equation of a line with a slope of 3 that passes through the point (1, -2). We will delve into the underlying concepts, formulas, and calculations involved, ensuring a comprehensive understanding of this common problem in mathematics.

Understanding the Basics: What Is a Line's Equation?

The General Form of a Line Equation

A straight line in a two-dimensional coordinate plane can be represented mathematically by an equation. The most common form is the slope-intercept form:

$$ y = mx + b $$

where:
- $ y $ and $ x $ are the variables representing points on the line,
- $ m $ is the slope of the line,
- $ b $ is the y-intercept, the point where the line crosses the y-axis.

This form is especially useful because it clearly shows the slope and the y-intercept, allowing for quick graphing and analysis.

The Role of Slope and a Point in Defining a Line

The slope ($ m $) indicates the steepness and direction of the line:
- A positive slope (like 3) means the line ascends from left to right.
- The point through which the line passes provides a specific location on the plane, anchoring the line's position.

When both the slope and a point are known, the goal is to find the specific equation of the line that satisfies these conditions.

Deriving the Equation of the Line: Step-by-Step Process

Using the Point-Slope Form

The point-slope form of a line's equation is particularly useful when you know:
- The slope m,
- A point (x_1, y_1) that lies on the line.

The formula is:

$$y - y_1 = m(x - x_1)$$

This form directly incorporates the known point and slope, simplifying the process of deriving the line's equation.

Applying the Given Data

Given:
- Slope $m = 3$,
- Point $(x_1, y_1) = (1, -2)$.

Plugging these into the point-slope form:

$$y - (-2) = 3(x - 1)$$

which simplifies to:

$$y + 2 = 3(x - 1)$$

Converting to Slope-Intercept Form

Step-by-Step Conversion

Starting with:

$$ y + 2 = 3(x - 1) $$

Distribute the 3:

$$ y + 2 = 3x - 3 $$

Subtract 2 from both sides to isolate y:

$$ y = 3x - 3 - 2 $$

Simplify:

$$ y = 3x - 5 $$

This is the slope-intercept form of the line:

$$ \boxed{ y = 3x - 5 } $$

which clearly indicates:
- The slope $m = 3$,
- The y-intercept $b = -5$.

Verifying the Equation

Checking if the Point Lies on the Line

To verify, substitute $x = 1$ into the derived equation:

$$ y = 3(1) - 5 = 3 - 5 = -2 $$

which matches the original y-coordinate of the point $(1, -2)$. Thus, the point indeed lies on the line, confirming that the equation is correct.

Graphical Interpretation

Plotting the line:

- It passes through $(1, -2)$,
- Has a slope of 3, meaning for each unit increase in x, y increases by 3 units,
- Crosses the y-axis at $y = -5$.

Additional Concepts and Variations

Alternate Forms of the Line Equation

While the slope-intercept form is most straightforward, other forms include:

- Standard Form: $Ax + By = C$
- Point-Slope Form: $y - y_1 = m(x - x_1)$, which we already used.

Converting the derived equation to standard form:

$$y = 3x - 5$$

Subtract $3x$ from both sides:

$$-3x + y = -5$$

or

$$3x - y = 5$$

Both representations are valid and useful in different contexts.

Understanding Line Properties from the Equation

From the equation $y = 3x - 5$:
- Slope (m): 3, indicating the line rises 3 units vertically for every 1 unit horizontally.
- Y-intercept (b): -5, the point where the line crosses the y-axis.

This information allows for quick sketching and analysis of the line's behavior.

Practical Applications and Examples

Example 1: Graphing the Line

To graph the line:

1. Plot the y-intercept $(0, -5)$.
2. From this point, use the slope to find another point: move 1 unit right (positive x direction), and 3 units up (since slope is 3), landing at $(1, -2)$.

3. Draw a straight line through these points, extending in both directions.

Example 2: Finding the Intersection with Another Line

Suppose you have another line: $y = -x + 4$. To find the intersection:

Set the two equations equal:

$$3x - 5 = -x + 4$$

Solve for x:

$$3x + x = 4 + 5$$
$$4x = 9$$
$$x = \frac{9}{4}$$

Substitute back into one of the equations:

$$y = 3 \times \frac{9}{4} - 5 = \frac{27}{4} - 5 = \frac{27}{4} - \frac{20}{4} = \frac{7}{4}$$

The intersection point is:

$$\left(\frac{9}{4}, \frac{7}{4} \right)$$

Conclusion

Finding the equation of a line given a slope and a point involves understanding the fundamental forms of linear equations and applying the appropriate formulas. In this case, with a slope of 3 and passing through the point (1, -2), the process is straightforward:

1. Use the point-slope form:

$$y - y_1 = m(x - x_1)$$

2. Substitute the known values:

$$y + 2 = 3(x - 1)$$

3. Simplify to slope-intercept form:

$$y = 3x - 5$$

This equation encapsulates all the information about the line's steepness and position in the coordinate plane. Mastery of this derivation process is crucial for solving a wide range of problems in algebra, coordinate geometry, and related fields. Whether for graphing, analyzing intersections, or

applying real-world models, understanding how to derive and manipulate line equations is an essential skill in mathematics.

Frequently Asked Questions

What is the equation of a line with a slope of 3 passing through the point (1, -2)?

Using point-slope form: y - (-2) = 3(x - 1), which simplifies to y + 2 = 3x - 3, so the equation is y = 3x - 5.

How do you find the equation of a line given a slope and a point?

Use the point-slope form $y - y_1 = m(x - x_1)$, plugging in the slope and the given point, then simplify to slope-intercept form.

What is the slope-intercept form of the line passing through (1, -2) with slope 3?

The slope-intercept form is y = 3x - 5.

Can you verify the point (1, -2) lies on the line y = 3x - 5?

Yes, substituting x = 1 gives y = 3(1) - 5 = -2, which matches the point (1, -2).

What is the general method to find the equation of a line given its slope and a point?

Apply the point-slope formula $y - y_1 = m(x - x_1)$, then simplify to slope-intercept form if needed.

Additional Resources

What Is The Equation Of The Line That Has A Slope Of 3 And Passes Through The Point (1, -2)

Understanding the fundamental principles of linear equations is essential for students, educators, engineers, and professionals across various fields. When tasked with finding the specific equation of a line given certain characteristics—such as a slope and a point through which the line passes—the process becomes a crucial skill in algebra and analytical geometry. Today, we will explore how to determine the equation of a line with a slope of 3 that passes through the point (1, -2), breaking down the concepts step-by-step to provide a comprehensive and reader-friendly explanation.

The Foundations: Understanding the Components of a Line Equation

Before delving into the specifics of this problem, it’s important to revisit the basic components involved in describing a line mathematically.

What Is a Linear Equation?

A linear equation in two variables (x and y) typically takes the form:

$$ y = mx + b $$

Where:
- $ y $ and $ x $ are variables representing points on the line.
- $ m $ is the slope of the line, indicating its steepness.
- $ b $ is the y-intercept, the point where the line crosses the y-axis (when $ x=0 $).

This form is known as the slope-intercept form because it explicitly shows the slope and y-intercept.

Why Is the Slope Important?

The slope $ m $ quantifies how much $ y $ changes for a unit change in $ x $. A positive slope indicates the line rises as $ x $ increases, while a negative slope indicates it falls. In our case, the slope is 3, meaning that for every increase of 1 in $ x $, $ y $ increases by 3.

Applying the Point-Slope Form to Find the Equation

Given the slope and a point on the line, the most direct method to find the equation is to use the point-slope form:

$$ y - y_1 = m(x - x_1) $$

Where:
- $ (x_1, y_1) $ is a point on the line.
- $ m $ is the slope.

Step 1: Identify the Given Values

From the problem, we know:
- Slope $ m = 3 $
- Point $ (x_1, y_1) = (1, -2) $

Step 2: Substitute into the Point-Slope Formula

Plugging these into the formula:

$$ y - (-2) = 3(x - 1) $$

Simplifying:

$$ y + 2 = 3(x - 1) $$

Deriving the Slope-Intercept Equation

While the point-slope form is useful for understanding the relationship between the slope and a point, most applications require the slope-intercept form for clarity and ease of graphing.

Step 3: Simplify to Slope-Intercept Form

Distribute the slope on the right side:

$$ y + 2 = 3x - 3 $$

Subtract 2 from both sides to solve for y:

$$ y = 3x - 3 - 2 $$

$$ y = 3x - 5 $$

This is the equation of the line in slope-intercept form.

Confirming the Line Passes Through the Given Point

It’s always good practice to verify that the derived equation passes through the original point $(1, -2)$.

Substitute $x=1$:

$$ y = 3(1) - 5 = 3 - 5 = -2 $$

Since this matches the y-coordinate of the point, the equation is confirmed as correct.

Visualizing the Line: Graphing and Interpretation

Understanding the equation’s meaning aids in visualizing the line:

- Slope (3): The line rises 3 units vertically for every 1 unit it moves horizontally.
- Y-intercept (-5): The line crosses the y-axis at $(0, -5)$.

Plotting the y-intercept and using the slope allows for quick sketching:

- Start at $(0, -5)$.
- From there, move up 3 units and right 1 unit to reach $(1, -2)$, confirming the point passes through the line.
- Connecting these points produces the straight line described by the equation.

Broader Context: Using The Equation in Real-World Scenarios

This mathematical exercise isn’t just theoretical; it has practical applications:

- Engineering: Designing slopes for ramps or roads.
- Economics: Modeling linear relationships, such as cost versus production.
- Science: Describing relationships between variables in experiments.
- Data Analysis: Fitting a linear model to data points.

Understanding how to derive and interpret the equation of a line from its slope and a point enables professionals to model and analyze real-world situations effectively.

Summary: The Complete Equation and Its Significance

To encapsulate the key points:

- Given a slope of 3 and passing through $(1, -2)$, the equation of the line is:

$$\boxed{y = 3x - 5}$$

- This line has a steep positive slope, crossing the y-axis at -5, and passes through the point $(1, -2)$.

- The derivation process involves using the point-slope form and converting to slope-intercept form, ensuring clarity and ease of application.

Final Thoughts

Mastering how to find the equation of a line given specific characteristics is foundational in mathematics. By understanding the relationships between slope, points, and the general line equation, learners can confidently approach diverse problems across academic and professional disciplines. The process exemplifies the power of algebra in translating geometric intuition into precise mathematical expressions—a skill that underpins much of analytical reasoning in science, engineering, and beyond.

Whether you're plotting a graph, analyzing data, or designing a project, knowing the equation of a line based on its slope and a point provides a reliable tool for interpretation and decision-making.

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