What Is The Equation Of The Line That Passes Through The Point (3,-7) And Has A Slope Of -2?

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Understanding the equation of a line is fundamental in algebra and analytic geometry. When given a specific point through which the line passes and its slope, the task becomes straightforward: we can derive the equation using the point-slope form of a line. In this case, the point is (3, -7), and the slope (m) is -2. The goal is to find the explicit equation of this line in a form that is easy to interpret and use, such as slope-intercept form or standard form.

Fundamental Concepts in Line Equations

The Slope of a Line

The slope of a line, denoted as m, measures its steepness. It is calculated as the ratio of the change in y to the change in x between two points on the line:

- $m = (y_2 - y_1) / (x_2 - x_1)$

A negative slope indicates the line decreases as x increases, while a positive slope indicates it rises.

The Point-Slope Form of a Line

Given a point (x_1, y_1) and a slope m, the equation of the line can be written as:

- $y - y_1 = m(x - x_1)$

This form is particularly useful when you know a specific point and the slope.

The Slope-Intercept Form

Transforming the point-slope form into slope-intercept form ($y = mx + b$) involves solving for y:

- y = mx + b

where b is the y-intercept, the point where the line crosses the y-axis.

Deriving the Equation for the Given Point and Slope

Using the given data:

- Point: (3, -7)
- Slope: m = -2

Applying the Point-Slope Form

Plugging the point and slope into the point-slope formula:

- y - (-7) = -2(x - 3)

which simplifies to:

- y + 7 = -2(x - 3)

Expanding and Simplifying

Distribute the slope on the right:

- y + 7 = -2x + 6

Subtract 7 from both sides to isolate y:

- y = -2x + 6 - 7

which simplifies further to:

- y = -2x - 1

Final Equation of the Line

The equation of the line passing through (3, -7) with a slope of -2 in slope-intercept form is:

- **y = -2x - 1**

This equation indicates that for every increase of 1 in x, y decreases by 2, and the line crosses the y-axis at y = -1.

Verifying the Equation

Substitute the Point (3, -7)

To confirm, substitute x = 3 into the equation:

- y = -2(3) - 1 = -6 - 1 = -7

which matches the given y-coordinate, confirming the correctness.

Graphical Interpretation

Plotting the point (3, -7):
- The line crosses the y-axis at (0, -1).
- The slope of -2 indicates the line decreases steeply as x increases.
- The point (3, -7) lies exactly on this line, confirming the equation's accuracy.

Other Forms of the Line Equation

Standard Form

The standard form of a line's equation is Ax + By = C. To convert y = -2x - 1 into standard form:

- Bring all terms to one side:
    - 2x + y = -1

This form is useful in certain algebraic contexts and for solving systems of equations.

Graphical Representation

To visualize the line:
- Plot the point (3, -7).
- Use the slope to find another point, for example:
- From (3, -7), move 1 unit right (x = 4), y decreases by 2 (y = -9), giving point (4, -9).
- Draw a straight line passing through these points.

Applications and Importance of the Equation

Understanding the precise equation of a line is useful in various fields:
- Physics: Describing motion with linear relationships.
- Economics: Modeling cost functions.
- Engineering: Designing components with specified linear characteristics.
- Mathematics: Solving systems of equations and analyzing geometric relationships.

Knowing how to derive and manipulate the equation of a line enables problem-solving in academic and real-world contexts.

Summary

In conclusion:
- The line passing through (3, -7) with a slope of -2 has the equation y = -2x - 1.
- The process involves using the point-slope form, simplifying, and optionally converting to other forms.
- Confirming the equation with substitution ensures accuracy.
- Visualizing the line helps in understanding its position relative to the coordinate axes.

Mastering these steps enhances your comprehension of linear equations and strengthens your algebraic skills, laying a foundation for more advanced mathematical topics.

Additional Tips for Working with Line Equations

- Always verify your derived equation by substituting known points.
- Practice converting between different forms of line equations.
- Use graphing tools or graph paper to visualize lines and verify algebraic results.
- Understand the significance of the slope and y-intercept in interpreting the line's behavior.

Frequently Asked Questions

What is the general form of the equation for a line passing through the point (3, -7) with a slope of -2?

The equation can be written in point-slope form as y - (-7) = -2(x - 3), which simplifies to y + 7 = -2(x - 3).

How do I convert the point-slope form to slope-intercept form for this line?

Start with y + 7 = -2(x - 3), expand to y + 7 = -2x + 6, then subtract 7 from both sides to get y = -2x - 1.

What is the slope-intercept form of the line passing through (3, -7) with a slope of -2?

The slope-intercept form is y = -2x - 1.

How can I verify that the point (3, -7) lies on the line y = -2x - 1?

Plug x = 3 into the equation: y = -2(3) - 1 = -6 - 1 = -7. Since y = -7 matches the point's y-coordinate, the point lies on the line.

What is the importance of the point-slope form in finding the equation of a line?

The point-slope form allows you to quickly write the equation of a line when you know a point on the line and its slope, making it useful for deriving the slope-intercept form.

Can I write the equation of the line in standard form based on the given point and slope?

Yes. Starting from y + 7 = -2(x - 3), expand to get y + 7 = -2x + 6, then move all terms to one side: 2x + y + 1 = 0.

What is the significance of knowing both the point and slope when determining the line's equation?

Knowing a point and the slope allows you to uniquely determine the line's equation, as the slope indicates the steepness and the point ensures the line passes through a specific location.

Additional Resources

Equation of the Line Passing Through (3, -7) with a Slope of -2

Understanding the equation of a line that passes through a specific point with a given slope is fundamental in coordinate geometry. In particular, when asked to find the equation of a line passing through the point (3, -7) and having a slope of -2, it involves applying foundational algebraic concepts to derive a precise mathematical expression. This process not only reinforces core algebra skills but also enhances one's ability to model real-world situations where relationships can be represented by linear equations.

Fundamentals of Line Equations in Coordinate Geometry

Before delving into the specific problem, it's essential to grasp the foundational concepts related to lines in coordinate geometry. Lines in a plane can be characterized by their slope and intercepts. The slope indicates the steepness and direction of the line, while the intercepts show where the line crosses the axes.

Understanding the Slope

The slope, often represented by the letter m, quantifies how much y changes for a given change in x. It is calculated as:

$$
m = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x}
$$

A negative slope, such as -2 in our scenario, indicates that as x increases, y decreases. Conversely, a positive slope suggests that both x and y increase together.

The Point-Slope Form of a Line

The point-slope form offers a direct way to write the equation of a line when a point (x_1, y_1) and slope m are known:

$$
y - y_1 = m(x - x_1)
$$

This form is particularly useful because it simplifies the process of deriving the line's equation from a point and slope.

Deriving the Equation of the Line

Given the point $(3, -7)$ and the slope $m = -2$, the goal is to find the explicit equation of the line passing through this point with the specified slope.

Applying the Point-Slope Formula

Using the point-slope form:

$$
y - y_1 = m(x - x_1)
$$

Substitute the known values:

$$
y - (-7) = -2(x - 3)
$$

Simplify:

$$
y + 7 = -2(x - 3)
$$

Next, expand the right side:

$$
y + 7 = -2x + 6
$$

Finally, isolate y to find the slope-intercept form:

$$
y = -2x + 6 - 7
$$
$$
y = -2x - 1
$$

This is the slope-intercept form of the line:

$$
\boxed{y = -2x - 1}
$$

Features and Variations of the Line Equation

Having derived the equation in slope-intercept form, it's important to understand its features and how it relates to different representations.

Standard Form

The line's equation can also be expressed in standard form:

$$
ax + by + c = 0
$$

Rearranging $y = -2x - 1$:

$$
2x + y + 1 = 0
$$

Features:

- Easier to analyze certain properties like intercepts.
- Useful in algebraic operations such as addition or subtraction of equations.

Graphical Interpretation

The graph of $y = -2x - 1$ has the following features:

- Slope: -2, indicating the line falls 2 units vertically for every 1 unit moved horizontally.
- Y-intercept: (-1), where the line crosses the y-axis.
- X-intercept: Set $y=0$:

$$
0 = -2x - 1 \Rightarrow 2x = -1 \Rightarrow x = -\frac{1}{2}
$$

- The line passes through $(-0.5, 0)$ and $((3, -7)$, confirming the initial point.

Applications of the Equation of a Line

Understanding and deriving the equation of a line has multiple applications across various fields:

- Physics: Modeling linear relationships between variables such as speed and time.
- Economics: Representing cost functions or demand curves.
- Engineering: Determining relationships between physical quantities.
- Data Analysis: Fitting linear models to data points.

Pros and Cons of Different Line Equation Forms

When working with line equations, choosing the appropriate form depends on the context.

Slope-Intercept Form ($y = mx + b$)

Pros:

- Straightforward for graphing.
- Clearly shows the slope and y-intercept.
- Easy to interpret and use for quick plotting.

Cons:

- Not ideal for finding x-intercepts directly.
- Less suitable if the line is vertical or nearly vertical.

Standard Form ($ax + by + c = 0$)

Pros:

- Useful for solving systems of equations.
- Facilitates finding intercepts.

Cons:

- Less intuitive for graphing.
- Can be less straightforward to interpret parameters.

Additional Considerations and Tips

- Verifying the Equation: To ensure correctness, substitute the point $(3, -7)$ into the derived equation:

$$
-2(3) - 1 = -6 - 1 = -7
$$

which confirms the point lies on the line.

- Linearity Check: The derivation confirms the relationship is linear, as expected, since the equation is first-degree in x and y.

- Multiple Points and Slopes: If asked to find the line passing through different points or with different slopes, the same process applies, emphasizing the flexibility of the method.

Conclusion

The equation of the line passing through the point (3, -7) with a slope of -2 is $y = -2x - 1$. This derivation demonstrates how foundational algebraic principles—particularly the point-slope form—are employed to translate geometric information into a precise algebraic expression. Whether for graphing, analysis, or application in real-world problems, understanding how to derive and interpret line equations is a vital skill in mathematics. The ability to manipulate these equations into different forms, such as slope-intercept or standard form, further enhances their utility and broadens their applicability across various disciplines. Mastery of these concepts equips learners with the tools to analyze linear relationships effectively, fostering a deeper appreciation for the elegance and power of algebraic methods in geometry.

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