

What Is The Equation Of The Line That Passes Through The Point (5, 4) And Has A Slope of $\frac{6}{5}$

What Is The Equation Of The Line That Passes Through The Point (5, 4) And Has A Slope of $\frac{6}{5}$ is a fundamental question in coordinate geometry that helps students and enthusiasts understand how to derive the linear equation of a line given specific conditions. Determining the equation of a line involves understanding the relationship between its slope and points through which it passes. In this article, we will explore in detail how to find the equation of such a line, the concepts involved, and practical applications to reinforce your understanding.

Understanding the Basics: What Is a Line Equation?

Before diving into the specific problem, it's essential to grasp the foundational concepts related to line equations.

What Is a Line in Coordinate Geometry?

A line in a two-dimensional plane is a straight one-dimensional figure that extends infinitely in both directions. It can be uniquely identified by its slope and a point through which it passes.

Common Forms of a Line Equation

There are several ways to express the equation of a line, including:

- Slope-Intercept Form: $y = mx + b$
- Point-Slope Form: $y - y_1 = m(x - x_1)$
- Standard Form: $Ax + By = C$

In this context, the slope-intercept form and point-slope form are most relevant because they directly incorporate the slope and specific points.

Key Concepts for Finding the Equation of a Line

To find the equation of a line passing through a point with a known slope, you need to understand and apply the following key concepts:

1. Slope of a Line (m)

The slope measures how steep the line is, calculated as the ratio of the change in y to the change in x between two points on the line: $m = \Delta y / \Delta x$.

2. Coordinates of a Point (x₁, y₁)

The specific point through which the line passes provides a fixed location in the plane.

3. The Point-Slope Formula

This formula combines the slope and a point to define the line:

$$y - y_1 = m(x - x_1)$$

Calculating the Equation of the Line Passing Through (5, 4) With a Slope of 6/5

Applying the key concepts, we can now determine the line's equation step by step.

Step 1: Identify the Known Values

- Point: (x₁, y₁) = (5, 4)

- Slope: m = 6/5

Step 2: Use the Point-Slope Formula

Plugging these into the formula:

$$y - 4 = \frac{6}{5}(x - 5)$$

Step 3: Simplify the Equation

Distribute the slope:

$$y - 4 = \frac{6}{5}x - \frac{6}{5} \times 5$$

$$y - 4 = \frac{6}{5}x - 6$$

Add 4 to both sides to solve for y:

$$y = \frac{6}{5}x - 6 + 4$$

$$y = \frac{6}{5}x - 2$$

Final Equation in Slope-Intercept Form:

$$y = \frac{6}{5}x - 2$$

This is the equation of the line passing through (5, 4) with a slope of 6/5.

Understanding the Slope-Intercept Form of the Line

The final equation:

$$y = \frac{6}{5}x - 2$$

provides valuable insights:

- Slope (m): 6/5, indicating that for every 5 units increase in x, y increases by 6 units.
- Y-intercept (b): -2, meaning the line crosses the y-axis at (0, -2).

Additional Methods to Find the Equation of a Line

While the point-slope form is straightforward when the slope and a point are known, other methods can also be employed:

1. Using Two Points

If two points are known, the process involves:

- Calculating the slope from the two points.
- Using the point-slope form with either point.

2. Standard Form Conversion

Converting the slope-intercept form to standard form:

$$y = \frac{6}{5}x - 2$$

Multiply both sides by 5 to clear the fraction:

$$5y = 6x - 10$$

Rearranged as:

$$6x - 5y = 10$$

Practical Applications of Line Equations

Understanding how to derive the equation of a line has numerous real-world applications:

1. Engineering and Design

Designing structures and mechanical parts often requires precise calculations of lines and slopes.

2. Business and Economics

Linear models are used for trend analysis, forecasting, and cost-profit calculations.

3. Physics

Analyzing motion, velocity, and acceleration often involves linear equations.

4. Navigation and Mapping

Determining routes, distances, and positions relies on line equations.

Common Mistakes and Tips for Correctly Finding the Line Equation

To ensure accuracy, keep in mind these tips:

- Always verify the slope before plugging into formulas.
- When simplifying fractions, reduce to simplest form for clarity.
- Check your work by plugging in the original point to see if it satisfies the equation.
- Be cautious with signs; positive and negative slopes affect the line's orientation.

Summary: How to Find the Equation of a Line Given a Point and Slope

In summary, to find the equation of a line passing through a specific point with a given slope:

1. Use the point-slope formula:

$$y - y_1 = m(x - x_1)$$

2. Substitute (x_1, y_1) and m .

3. Simplify to slope-intercept form or standard form as needed.
4. Verify by substituting the known point.

Applying these steps to the initial problem:

- Point: (5, 4)
- Slope: 6/5

Leads to the final equation:

$$y = \frac{6}{5}x - 2$$

Conclusion

Understanding how to determine the equation of a line passing through a point with a specific slope is fundamental in algebra and geometry. It provides foundational skills for solving more complex mathematical problems and has widespread applications across various fields. By mastering the point-slope formula and practicing with different points and slopes, you can confidently derive line equations, interpret their meaning, and apply them to real-world scenarios.

Whether you are a student preparing for exams, a professional working in engineering, or simply someone interested in mathematical concepts, knowing how to find the line equation through a given point and slope is an essential skill that forms the backbone of many analytical tasks in mathematics.

Keywords for SEO optimization:

- Equation of the line passing through (5, 4) with slope 6/5
- How to find the equation of a line given a point and slope
- Point-slope form of a line
- Slope-intercept form of a line
- Standard form of a line equation
- Coordinate geometry line equations
- Linear equations practice problems
- Applications of line equations in real life

Frequently Asked Questions

What is the equation of the line passing through the point (5, 4) with a slope of 6/5?

Using the point-slope form: $y - 4 = (6/5)(x - 5)$. Simplifying, the equation is $y = (6/5)x - 2$.

How do I find the equation of a line given a point and a slope?

Use the point-slope form $y - y_1 = m(x - x_1)$, where (x_1, y_1) is the point and m is the slope. Plug in the values and simplify to get the equation.

What is the slope-intercept form of the line passing through (5, 4) with slope $6/5$?

The slope-intercept form is $y = (6/5)x - 2$.

Can you verify the point (5, 4) lies on the line with slope $6/5$ passing through (5, 4)?

Yes, substituting $x = 5$ into $y = (6/5)x - 2$ gives $y = (6/5)(5) - 2 = 6 - 2 = 4$, matching the point's y-coordinate.

What is the general process to write the equation of a line given a point and slope?

Identify the point and slope, use the point-slope form $y - y_1 = m(x - x_1)$, then simplify to slope-intercept form if needed.

Additional Resources

What Is The Equation Of The Line That Passes Through The Point (5, 4) And Has A Slope Of $6/5$

Understanding how to find the equation of a line given specific conditions is a fundamental skill in algebra and analytical geometry. When you know a point through which the line passes and its slope, you can determine the precise equation that describes it. In this guide, we'll explore what is the equation of the line that passes through the point (5, 4) and has a slope of $6/5$, breaking down the process step-by-step so you can confidently approach similar problems.

Introduction to Line Equations

Before diving into the specifics of this problem, it's essential to understand the basic concepts behind line equations, slope, and point-slope form.

What Is a Line Equation?

A line in a two-dimensional plane can be described mathematically using various forms, with the most common being:

- Slope-Intercept Form: $y = mx + b$
- Point-Slope Form: $y - y_1 = m(x - x_1)$
- Standard Form: $Ax + By = C$

Each form can be useful depending on the given data. Our focus will primarily be on the point-slope form because it directly incorporates a specific point and the slope.

The Significance of Slope and a Point

The slope (m) indicates the steepness or incline of the line. It measures the rate at which y changes for a unit change in x .

The point (x_1, y_1) through which the line passes provides a specific location on the line.

Given these two pieces of information, we can derive the equation of the line uniquely.

Step-by-Step Guide to Find the Equation

Let's now focus on the specific problem: Find the equation of the line passing through $(5, 4)$ with a slope of $6/5$.

1. Recall the Point-Slope Formula

The point-slope form is:

$$y - y_1 = m(x - x_1)$$

Where:

- (x_1, y_1) is the given point
- m is the slope of the line

2. Substitute the Known Values

Plugging in the point $(5, 4)$ and the slope $m = 6/5$, we get:

$$y - 4 = (6/5)(x - 5)$$

This is the core equation representing the line.

Simplifying the Equation

While the point-slope form is useful, sometimes it's preferable to express the line in slope-intercept form ($y = mx + b$) for clarity and easier graphing.

3. Expand and Simplify

Starting with:

$$y - 4 = (6/5)(x - 5)$$

Distribute the slope:

$$y - 4 = (6/5)x - (6/5)5$$

Calculate:

$$(6/5)5 = (6/5) 5 = 6$$

So,

$$y - 4 = (6/5)x - 6$$

Next, solve for y:

$$y = (6/5)x - 6 + 4$$

Simplify:

$$y = (6/5)x - 2$$

This is the slope-intercept form of the line.

Final Equation of the Line

$$y = (6/5)x - 2$$

This equation fully describes the line passing through the point (5, 4) with a slope of 6/5.

Additional Insights and Applications

Understanding this line's equation opens doors to various applications, from graphing to solving systems of equations.

Graphing the Line

- Intercepts:

- Y-intercept: When $x = 0$, $y = -2$

- X-intercept: When $y = 0$, solve for x :

$$0 = (6/5)x - 2$$

$$(6/5)x = 2$$

$$x = (2) (5/6) = 10/6 = 5/3$$

- Graph Points:
- (0, -2)
- (5/3, 0)

Plotting these points and drawing a straight line through them visualizes the line.

Real-World Contexts

Lines with specific slopes and points are used in various fields:

- Physics: Describing motion with constant velocity
- Economics: Modeling cost or revenue functions
- Engineering: Analyzing relationships between variables

Summary of Key Steps

To summarize, here's a concise checklist for solving similar problems:

1. Identify the known point (x_1, y_1) and the slope m .
2. Use the point-slope form: $y - y_1 = m(x - x_1)$.
3. Substitute known values into the formula.
4. Simplify to slope-intercept form (if desired).
5. Interpret the results for graphing or analysis.

Practice Problems

To reinforce understanding, try solving these:

- Find the equation of a line passing through (2, -3) with a slope of 4/7.
- Write the equation of the line passing through (-1, 2) and has a slope of -3/2.
- Determine the equation of a line with slope 0 (horizontal line) passing through (7, -5).

Conclusion

Knowing what is the equation of the line that passes through the point (5, 4) and has a slope of 6/5 is foundational in algebraic problem-solving. By applying the point-slope formula, simplifying, and understanding the geometric implications, you gain a comprehensive grasp of how lines are represented algebraically. This not only enhances your mathematical skills but also prepares you for more complex topics in analytic geometry and real-world applications.

Remember, mastering these concepts involves practice—so keep working through different problems, and soon, deriving line equations will become second nature!

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