What Is The Quotient Of 15a4b3/12a2b ? Assume That The Denominator Does Not Equal Zero.

What Is The Quotient Of 15a4b3/12a2b ? Assume That The Denominator Does Not Equal Zero.

Understanding how to simplify algebraic expressions, especially those involving variables and exponents, is fundamental in mathematics. The given problem asks for the quotient of two algebraic expressions: 15a^4b^3 divided by 12a^2b, with the assumption that the denominator is not zero. This means that the variables a and b do not take on values that make the denominator zero, ensuring the expression is defined.

In this article, we will explore how to approach such simplifications systematically. We will review the basic principles of algebraic division, focus on the rules for dividing coefficients and variables with exponents, and work through the step-by-step process for simplifying the given expression. By the end, you'll have a clear understanding of how to handle similar algebraic division problems efficiently.

Understanding the Components of the Expression

Before diving into the simplification process, it is essential to understand the parts of the given expression:

Numerator: 15a^4b^3

- Coefficient: 15
- Variables: a and b
- Exponents: a^4 (a raised to the 4th power), b^3 (b raised to the 3rd power)

Denominator: 12a^2b

- Coefficient: 12
- Variables: a and b
- Exponents: a^2, b^1

The goal is to divide the numerator by the denominator, simplifying the coefficients and the variable parts separately.

Principles of Algebraic Division

Dividing algebraic expressions involves two main steps:

1. Divide the coefficients (numerical parts)

- Divide 15 by 12, simplifying the fraction as needed.

2. Divide the variables with the same base

- Apply the rule: a^m / a^n = a^{m - n}
- Similarly for b: b^p / b^q = b^{p - q}

The key is to simplify each part systematically to achieve the most reduced form.

Step-by-Step Simplification Process

Let's proceed step-by-step to simplify the given expression:

\[
\frac{15a^{4}b^{3}}{12a^{2}b}
\]

Step 1: Simplify the coefficients

- Write the coefficients as a fraction:

\[
\frac{15}{12}
\]

- Simplify the fraction:

\[
\frac{15 \div 3}{12 \div 3} = \frac{5}{4}
\]

Result after step 1:

\[
\frac{5}{4} \times \frac{a^{4}b^{3}}{a^{2}b}
\]

Step 2: Divide the variable parts

- Divide the powers of 'a':

\[
a^{4} / a^{2} = a^{4 - 2} = a^{2}
\]

- Divide the powers of 'b':

$$
b^{3} / b = b^{3 - 1} = b^{2}
$$

Result after step 2:

$$
\frac{5}{4} \times a^{2} \times b^{2}
$$

Step 3: Write the final simplified expression

Combining all parts:

$$
\boxed{\frac{5a^{2}b^{2}}{4}}
$$

This is the most simplified form of the original expression, assuming the denominator is not zero (i.e., $a \neq 0$ and $b \neq 0$).

Additional Considerations and Notes

Restrictions on Variables

- Since the denominator contains $a^{2}b$, both a and b cannot be zero. This ensures the original expression is defined and division by zero is avoided.

Why Simplify Coefficients and Variables Separately?

- The coefficients are numerical and can be simplified via common factors.
- Variables are handled using the laws of exponents, which provide a straightforward way to combine or reduce powers.

Other Types of Expressions

- Similar techniques apply to more complex expressions involving multiple variables, powers, or coefficients.
- When variables are in different bases or powers, apply the relevant exponent rules carefully for each

part.

Common Mistakes to Avoid

- Forgetting to subtract exponents when dividing variables.
- Not simplifying coefficients fully before combining variables.
- Assuming variables are zero when the problem states they are not, leading to division by zero errors.
- Mixing the rules for multiplication and division of exponents; ensure to follow the correct operation.

Practice Problems for Reinforcement

To solidify understanding, try simplifying the following similar expressions:

1. $\frac{8x^{5}y^{2}}{4x^{3}y}$

2. $\frac{20m^{7}n^{4}}{5m^{3}n^{2}}$

3. $\frac{45p^{6}q^{3}}{15p^{2}q^{1}}$

Solutions involve:

- Simplifying coefficients.
- Applying exponent rules.
- Combining like terms.

Conclusion

Simplifying the division of algebraic expressions such as $\frac{15a^{4}b^{3}}{12a^{2}b}$ involves a clear understanding of basic algebraic principles. By systematically dividing coefficients and applying exponent rules, the expression reduces to its simplest form:

$$
\boxed{\frac{5a^{2}b^{2}}{4}}
$$

Always remember to check for restrictions on variables to avoid division by zero. Mastery of these techniques provides a solid foundation for tackling more complex algebraic expressions and equations with confidence.

Frequently Asked Questions

What is the simplified form of the expression $(15a^4b^3)$ divided by $12a^2b$?

The simplified form is $(5/4) a^{2} b^{2}$.

How do you divide algebraic expressions with common bases, like $15a^4b^3 / 12a^2b$?

You divide the coefficients and subtract the exponents of like bases: $(15/12) a^{4-2} b^{3-1} = (5/4) a^{2} b^{2}$.

What assumptions are made about the denominator in the quotient $15a^4b^3 / 12a^2b$?

It is assumed that the denominator $(12a^2b)$ is not equal to zero, so division is valid.

Can the quotient $(15a^4b^3) / (12a^2b)$ be simplified further?

No, the quotient simplifies to $(5/4) a^{2} b^{2}$, which is in simplest form.

What is the importance of assuming the denominator is not zero in algebraic division?

Because division by zero is undefined in mathematics, assuming the denominator is not zero ensures the expression is valid.

How does dividing the coefficients 15 and 12 affect the quotient?

Dividing 15 by 12 simplifies to 5/4, reducing the coefficient to its simplest form.

What is the significance of subtracting exponents in algebraic division, as seen in a^4 / a^2?

Subtracting exponents follows the rule $a^{m} / a^{n} = a^{m - n}$, which simplifies the expression.

If the numerator was $15a^4b^3$ and the denominator was $6a^2b$, what would be the quotient?

The quotient would be $(15/6) a^{4-2} b^{3-1} = (5/2) a^{2} b^{2}$.

Is it necessary to factor out common terms before dividing algebraic expressions like 15a^4b^3 / 12a^2b?

No, direct division and subtraction of exponents are sufficient, but factoring can sometimes make the process clearer.

Additional Resources

What Is The Quotient Of 15a^4b^3 / 12a^2b ?

Understanding algebraic expressions, especially those involving variables and exponents, is vital for students, educators, and professionals who engage with mathematical problem-solving. One common operation in algebra is division of algebraic expressions, which involves simplifying complex fractions to their lowest terms. In this article, we delve deep into the quotient of the algebraic expression 15a^4b^3 divided by 12a^2b, breaking down each component, explaining the principles involved, and illustrating the steps necessary to reach a simplified form. Whether you're reviewing fundamental concepts or seeking to enhance your analytical skills, this comprehensive exploration offers clarity and expertise.

Understanding the Problem: The Expression and Its Components

At the heart of our analysis lies the expression:

(15a^4b^3) / (12a^2b)

This expression involves both numerical coefficients and algebraic variables with exponents. To comprehend it fully, it's essential to understand each element's role.

Numerical Coefficients

- Numerator coefficient: 15
- Denominator coefficient: 12

These are constants that multiply the variables within their respective terms.

Variables and Exponents

- Variable 'a':
- Numerator has a^4
- Denominator has a^2

- Variable 'b':
- Numerator has b^3

- Denominator has b

The exponents indicate the power to which each variable is raised, and they follow the rules of exponents when dividing.

Fundamental Principles for Simplifying Algebraic Expressions

Before diving into the specific problem, understanding some key principles is crucial:

1. Division of Coefficients

Divide the numerical coefficients directly:

$$
\frac{15}{12} = \frac{15 \div 3}{12 \div 3} = \frac{5}{4}
$$

2. Division of Variables with Same Base

When dividing variables with the same base, subtract exponents:

$$
a^m / a^n = a^{m - n}
$$

$$
b^p / b^q = b^{p - q}
$$

Provided that the denominator's exponent is less than or equal to the numerator's; otherwise, the result involves negative exponents.

3. Zero Exponent Rule

Any non-zero variable raised to the zero power equals 1:

$$
a^0 = 1
$$

4. Non-Zero Denominator

The problem explicitly states that the denominator does not equal zero, which assures the division is valid and no undefined expressions occur.

Step-by-Step Simplification of the Expression

Let's analyze the division process systematically:

Step 1: Simplify the Numerical Coefficients

$$
\frac{15}{12} = \frac{5}{4}
$$

This reduces the fraction to its simplest form by dividing numerator and denominator by their greatest common divisor (GCD), which is 3.

Step 2: Simplify the Variable 'a'

Given:

$$
a^4 / a^2
$$

Applying the exponent division rule:

$$
a^{4 - 2} = a^{2}
$$

Step 3: Simplify the Variable 'b'

Given:

$$
b^3 / b
$$

Since:

$$
b = b^1
$$

Applying the exponent division rule:

$$
b^{3 - 1} = b^{2}
$$

Step 4: Combine All Simplified Parts

Putting it all together:

$$
\frac{5}{4} \times a^2 \times b^2
$$

Or, expressed as a single algebraic fraction:

$$
\boxed{\frac{5a^2b^2}{4}}
$$

This is the simplified form of the original expression.

Interpreting the Result: What Does the Quotient Represent?

The simplified expression, $(5a^2b^2)/4$, encapsulates the quotient of the initial algebraic terms in a clear, concise form. It reveals how the coefficients and variables interact when divided, illustrating the power of the laws of exponents and fraction reduction.

Key Takeaways from the Simplification

- Coefficients reduction demonstrates the importance of common factors.
- Exponent subtraction elegantly handles the division of variables with the same base.
- The process underscores the necessity of understanding fundamental algebraic rules to simplify complex expressions efficiently.

Applications and Real-World Relevance

Understanding how to simplify algebraic expressions like this one has numerous applications across various fields:

In Engineering and Physics

- Calculating ratios of quantities with related units often involves similar algebraic manipulations.
- Simplifying expressions aids in deriving formulas for motion, force, or electrical calculations.

In Computer Science and Data Analysis

- Algorithms that involve polynomial or exponential calculations require understanding of exponent rules.
- Data modeling often involves simplifying algebraic expressions for better interpretability.

In Education and Learning

- Mastery of these algebraic operations forms the foundation for advanced mathematics, including calculus and linear algebra.

Common Mistakes and Tips for Accurate Simplification

While the process may seem straightforward, several pitfalls can occur:

1. Forgetting to Subtract Exponents

Ensure you subtract exponents only when dividing like bases. It's a common mistake to add exponents or leave them unchanged.

2. Ignoring the Sign of Exponents

Negative exponents indicate reciprocals; be cautious when exponents involve negative numbers or subtraction results.

3. Not Simplifying Numerical Fractions

Always reduce fractions to their lowest terms to make the final expression cleaner and easier to interpret.

4. Overlooking Zero Exponents

Remember that any variable raised to zero equals 1, which can simplify expressions further.

Tips for Accurate Simplification

- Write each step clearly.
- Use exponent rules systematically.
- Cross-check each part of the simplification.
- Practice with similar problems to develop intuition.

Conclusion: The Significance of Mastering Algebraic Division

The division of algebraic expressions like 15a^4b^3 / 12a^2b exemplifies core mathematical principles that are fundamental to higher-level problem-solving. Through systematic application of numerical reduction, exponent laws, and careful attention to detail, one can arrive at simplified, meaningful expressions that reveal the underlying relationships between variables.

This process is not just an academic exercise but a practical skill essential across science, engineering, technology, and education. Mastery of such algebraic manipulations empowers individuals to approach complex problems with confidence, clarity, and analytical rigor. As algebra forms the backbone of many advanced mathematical concepts, developing proficiency in these basic operations sets the stage for future success in a wide array of disciplines.

Final Answer:

$$
\boxed{\frac{5a^2b^2}{4}}
$$

This is the quotient of the given algebraic expression after thorough simplification, assuming the denominator does not equal zero.

What Is The Quotient Of 15a4b3 12a2b Assume That The Denominator Does Not Equal Zero

What Is The Quotient Of 15a4b3 12a2b Assume That The Denominator Does Not Equal Zero

Related Articles

- Solve 7c - 5c + 4 = 4c - 10c = 5c = 7c = 2c = -7** Giving BRANIEST To FIRST To Answer **
- True Or False: The Transport Layer Provides For Host-to-host Delivery Service? True. False
- How Is An Adrenal Cortex Stress Response Different From An Adrenal Medulla Stress Response?

Back to Home