

What Is The Slope Of A Line Perpendicular To The Line Whose Equation Is $4x + 10y = -140$

What Is The Slope Of A Line Perpendicular To The Line Whose Equation Is $4x + 10y = -140$ is a fundamental question in coordinate geometry that involves understanding the relationship between slopes of lines and how they relate to perpendicularity. Determining the slope of a line perpendicular to a given line requires first identifying the slope of the original line and then applying the concept that the slopes of perpendicular lines are negative reciprocals of each other. This process is essential not only for solving algebraic problems but also for understanding geometric relationships and properties of lines in the coordinate plane.

Understanding the Equation of a Line

Standard Form of a Line Equation

The given line equation, $4x + 10y = -140$, is expressed in standard form, which is generally written as $Ax + By = C$, where A , B , and C are constants. This form is useful because it allows us to easily identify the coefficients associated with x and y , which are essential in calculating the slope.

Converting to Slope-Intercept Form

To find the slope of the given line, it is often most straightforward to convert the equation into slope-intercept form, $y = mx + b$, where m represents the slope and b the y -intercept.

Let's convert $4x + 10y = -140$ to slope-intercept form:

1. Subtract $4x$ from both sides:

$$10y = -4x - 140$$

2. Divide both sides by 10:

$$y = (-4/10)x - 140/10$$

3. Simplify fractions:

$$y = (-2/5)x - 14$$

From this form, the slope (m) of the original line is clearly $-2/5$.

Finding the Slope of a Perpendicular Line

The Concept of Perpendicular Slopes

In the coordinate plane, two lines are perpendicular if they intersect at a right angle (90 degrees). A key property of such lines is the relationship between their slopes:

- The slope of one line is the negative reciprocal of the other.

Mathematically, if the slope of the first line is m , then the slope of a line perpendicular to it, m_{perp} , is:

- $m_{\text{perp}} = -1/m$

Applying this to our problem, since the slope of the original line is $-2/5$, the slope of the perpendicular line is calculated as follows.

Calculating the Perpendicular Slope

Given:

- $m = -2/5$

The negative reciprocal is:

- $m_{\text{perp}} = -1 / (-2/5) = -1 (-5/2) = 5/2$

Therefore, the slope of any line perpendicular to the original line is $5/2$.

Practical Applications of Perpendicular Slopes

Understanding the slope of perpendicular lines is vital in various mathematical and real-world contexts, such as:

- Constructing right angles in geometric figures
- Determining the orientation of roads, ramps, or architectural elements
- Solving systems of equations where perpendicularity constraints exist
- Analyzing slopes in physics problems involving perpendicular forces or motions

Additional Considerations and Related Concepts

Parallel Lines and Their Slopes

It's important to distinguish between perpendicular and parallel lines:

- Parallel lines have the same slope.
- Perpendicular lines have slopes that are negative reciprocals.

Special Cases

- Vertical lines: These have undefined slopes, and any line perpendicular to a vertical line is horizontal, which has a slope of zero.

- Horizontal lines: These have a slope of zero, and any line perpendicular to a horizontal line is vertical, which has an undefined slope.

Summary

To summarize:

- The original line's equation is $4x + 10y = -140$.
- Converting to slope-intercept form yields $y = (-2/5)x - 14$, so the slope is $-2/5$.
- The slope of a line perpendicular to this one is the negative reciprocal: $5/2$.

Understanding how to find the slope of perpendicular lines is a fundamental skill in algebra and geometry that enhances problem-solving capabilities and deepens comprehension of spatial relationships.

Practice Problems

To solidify your understanding, try solving these related problems:

1. Find the slope of the line perpendicular to $y = 3x + 7$.
2. Given a line with slope 4, what is the slope of a line perpendicular to it?
3. Convert the equation $2x - 3y = 6$ into slope-intercept form and find the perpendicular slope.

Conclusion

Knowing the slope of a line and how to determine the slope of a perpendicular line is a core concept in algebra. For the specific equation $4x + 10y = -140$, the slope of the line is $-2/5$, and the slope of any line perpendicular to it is $5/2$. Mastery of these relationships allows for accurate geometric constructions, problem-solving, and a deeper understanding of the properties of lines in the coordinate plane.

Remember: Always convert equations into slope-intercept form to easily identify slopes, and use the negative reciprocal rule to find perpendicular slopes.

Frequently Asked Questions

How do you find the slope of a line perpendicular to the line given by $4x + 10y = -140$?

First, rewrite the equation in slope-intercept form to find its slope, then

take the negative reciprocal to find the perpendicular line's slope.

What is the slope of the line $4x + 10y = -140$?

The slope of the line is $-2/5$ after rewriting the equation in slope-intercept form.

What is the slope of a line perpendicular to $4x + 10y = -140$?

The slope of the perpendicular line is $5/2$, which is the negative reciprocal of $-2/5$.

Why do we take the negative reciprocal to find the slope of a perpendicular line?

Because perpendicular lines have slopes that are negative reciprocals of each other, ensuring their slopes multiply to -1 .

Can you provide a step-by-step process to find the perpendicular slope given the equation $4x + 10y = -140$?

Yes. First, rewrite the equation as $y = (-2/5)x - 14$, identify the slope as $-2/5$, then find its negative reciprocal, which is $5/2$, the slope of the perpendicular line.

Additional Resources

Understanding the Slope of a Perpendicular Line to the Equation $4x + 10y = -140$

When delving into the fascinating world of coordinate geometry, one of the foundational concepts is understanding how lines relate to each other through their slopes. Whether you're a student tackling algebra for the first time or a professional mathematician revisiting fundamental principles, grasping the behavior of perpendicular lines is essential. Today, we will explore the question: What is the slope of a line perpendicular to the line whose equation is $4x + 10y = -140$? This exploration will not only answer that specific question but will also serve as a comprehensive guide to understanding slopes, slopes of perpendicular lines, and their significance in geometry.

Deciphering the Equation: From Standard Form to Slope-Intercept Form

Before we can identify the slope of a line perpendicular to the given line, it's crucial to understand the original line's slope. The equation provided is:

$$4x + 10y = -140$$

This is in standard form, which is generally written as:

$$Ax + By = C$$

where A , B , and C are constants. To find the slope, it's often easiest to convert the equation into slope-intercept form:

$$y = mx + b$$

where m is the slope, and b is the y-intercept.

Step-by-step conversion:

1. Subtract $4x$ from both sides:

$$10y = -4x - 140$$

2. Divide every term by 10:

$$y = -\frac{4}{10}x - \frac{140}{10}$$

3. Simplify fractions:

$$y = -\frac{2}{5}x - 14$$

Now, the line's equation in slope-intercept form is:

$$y = -\frac{2}{5}x - 14$$

Key takeaway: The slope of the original line, denoted as m_1 , is:

$$m_1 = -\frac{2}{5}$$

The Concept of Perpendicular Lines in Geometry

Understanding the slope of a perpendicular line requires an appreciation of

what perpendicularity means in the context of lines.

Perpendicular lines are lines that intersect at a right angle (90 degrees). The relationship between their slopes is governed by a simple but powerful rule:

$\text{If two lines are perpendicular, then } m_1 \times m_2 = -1$

where:

- m_1 = slope of the first line
- m_2 = slope of the line perpendicular to the first

This rule stems from the geometric fact that the tangent of the angle between two lines relates to their slopes. When the lines are perpendicular, the angle between them is 90° , and the tangent of 90° is undefined, leading to the relation above.

Implication: To find the slope of a line perpendicular to a given line, you need to find the negative reciprocal of the original line's slope.

Calculating the Slope of the Perpendicular Line

Given the slope of the original line:

$m_1 = -\frac{2}{5}$

the slope of the line perpendicular to it, denoted as m_2 , is obtained by:

$m_2 = -\frac{1}{m_1}$

Step-by-step calculation:

1. Take the reciprocal of m_1 :

$\frac{1}{-\frac{2}{5}} = -\frac{5}{2}$

2. Change the sign to get the negative reciprocal:

$m_2 = -\left(-\frac{5}{2}\right) = \frac{5}{2}$

Thus,

$$\boxed{\text{Slope of the perpendicular line} = \frac{5}{2}}$$

Summary: The slope of any line perpendicular to the original line $(4x + 10y = -140)$ is $(-\frac{5}{2})$.

Understanding the Significance of the Slope $(-\frac{5}{2})$

The value $(-\frac{5}{2})$ is not just a number; it encapsulates the geometric relationship between the original line and its perpendicular counterpart.

Characteristics of the Slope $(-\frac{5}{2})$:

- Positive Slope: Indicates that the perpendicular line rises as it moves from left to right, contrasting with the original line's negative slope.
- Steepness: The magnitude $(\frac{5}{2})$, or 2.5, signifies a relatively steep incline, emphasizing the angle at which the perpendicular line intersects the original.

Visualizing the Relationship:

Imagine plotting the original line with a gentle downward slope. The perpendicular line, with a steeper upward slope of $(-\frac{5}{2})$, crosses it at a right angle, illustrating the fundamental geometric principle that the slopes are negative reciprocals.

Practical Implications and Applications

Understanding the slope of a perpendicular line has numerous practical applications across various fields, including:

- Engineering and Architecture: Designing structures with precise right-angle intersections.
- Navigation and Mapping: Calculating perpendicular paths or routes.
- Computer Graphics: Rendering objects and calculating perpendicular vectors.
- Physics: Analyzing forces and vectors that are perpendicular in nature.

Example Scenarios:

1. Constructing Perpendicular Lines: Given a line, you can easily determine the slope of any perpendicular line to facilitate construction or design.
2. Finding Equations of Perpendicular Lines: Once the slope is known, the equation of the perpendicular line passing through a specific point can be

formulated using point-slope form.

3. Analyzing Geometric Figures: Recognizing perpendicular slopes aids in identifying right angles within complex figures like polygons.

Formulating the Equation of a Perpendicular Line

Suppose you want to find the equation of the line perpendicular to $(4x + 10y = -140)$, passing through a specific point (x_0, y_0) . The process involves:

1. Using the slope $(m_2 = \frac{5}{2})$.
2. Applying the point-slope form:

$$[y - y_0 = m_2 (x - x_0)]$$

Example:

Let's say the point is $(2, 3)$.

Plugging into the formula:

$$[y - 3 = \frac{5}{2} (x - 2)]$$

Expanding:

$$[y - 3 = \frac{5}{2}x - 5]$$

Adding 3 to both sides:

$$[y = \frac{5}{2}x - 5 + 3]$$

$$[y = \frac{5}{2}x - 2]$$

This is the equation of the line perpendicular to the original, passing through the point $(2, 3)$.

Summary and Final Thoughts

In conclusion, the question of what is the slope of a line perpendicular to the line $(4x + 10y = -140)$ can be answered succinctly: the slope is $(\frac{5}{2})$.

Key points to remember:

- The original line's slope is $-\frac{2}{5}$.
- Perpendicular lines have slopes that are negative reciprocals of each other.
- The negative reciprocal of $-\frac{2}{5}$ is $\frac{5}{2}$.
- Understanding these relationships enables precise calculations and constructions in geometry and related fields.

Having an in-depth grasp of these concepts enhances your ability to analyze geometric figures, solve algebraic problems, and apply mathematical principles to real-world scenarios. Whether you're plotting graphs, designing structures, or exploring theoretical mathematics, recognizing the interplay of slopes and perpendicularity is fundamental.

In essence, the slope of a line perpendicular to $4x + 10y = -140$ is $\frac{5}{2}$, a simple yet powerful reflection of the elegant symmetry inherent in geometric relationships.

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