When The Expression 4x^(3)-x^(2)-kx-5 Is Divided 2x-1 The Reminder 0 , Find The Values Of K

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Understanding polynomial division and the concept of remainders is fundamental in algebra, especially when solving equations involving unknown coefficients like 'k'. This article provides a comprehensive guide on how to determine the values of 'k' given the condition that when a specific cubic polynomial is divided by a linear binomial, the remainder is zero. This scenario is commonly encountered in algebraic problems, particularly those involving polynomial division, factor theorem, and synthetic division.

Introduction to Polynomial Division and Remainders

Polynomial division is a process similar to long division with numbers, used to divide one polynomial by another. When dividing a polynomial $P(x)$ by a divisor $D(x)$, the division may result in a quotient polynomial $Q(x)$ and a remainder $R(x)$, expressed as:

$$
P(x) = D(x) \times Q(x) + R(x)
$$

- If the degree of $R(x)$ is less than the degree of $D(x)$, the division is complete.
- When the remainder $R(x) = 0$, the divisor exactly divides the polynomial, indicating that $D(x)$ is a factor of $P(x)$.

In our scenario, the divisor is $2x - 1$, a linear polynomial, and the dividend is $4x^3 - x^2 - kx - 5$.

Understanding the Given Problem

The problem statement is:

"When the expression $4x^3 - x^2 - kx - 5$ is divided by $2x - 1$, the remainder is 0.

Find the values of k."

This implies:

1. The linear divisor $2x - 1$ divides the cubic polynomial exactly.
2. The polynomial $4x^3 - x^2 - kx - 5$ is divisible by $2x - 1$.

By the Polynomial Remainder Theorem, if a polynomial $P(x)$ is divisible by $x - a$, then $P(a) = 0$.

However, our divisor $2x - 1$ is not in the form $x - a$. To utilize the Remainder Theorem, we need to find the root of $2x - 1 = 0$.

Applying the Remainder Theorem

Step 1: Find the root of the divisor $2x - 1$

Set:

$$
2x - 1 = 0
\Rightarrow 2x = 1
\Rightarrow x = \frac{1}{2}
$$

Step 2: Evaluate the polynomial at $x = \frac{1}{2}$

Since the remainder is zero, the polynomial must equal zero at this value:

$$
P\left(\frac{1}{2}\right) = 0
$$

where

$$
P(x) = 4x^3 - x^2 - kx - 5
$$

Substitute $x = \frac{1}{2}$:

$$
P\left(\frac{1}{2}\right) = 4 \left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 - k \left(\frac{1}{2}\right) - 5
$$

Calculate each term:

1. $4 \left(\frac{1}{2}\right)^3 = 4 \times \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$
2. $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$
3. $-k \times \frac{1}{2} = - \frac{k}{2}$
4. The constant term remains -5

Putting it all together:

$$
\frac{1}{2} - \frac{1}{4} - \frac{k}{2} - 5 = 0
$$

Solving for the Unknown k

Let's simplify and solve the above equation step by step:

$$
\frac{1}{2} - \frac{1}{4} - \frac{k}{2} - 5 = 0
$$

Combine the fractions:

$$
\left(\frac{1}{2} - \frac{1}{4}\right) - \frac{k}{2} - 5 = 0
$$

$$
\left(\frac{2}{4} - \frac{1}{4}\right) - \frac{k}{2} - 5 = 0
$$

$$
\frac{1}{4} - \frac{k}{2} - 5 = 0
$$

Express all terms with a common denominator, which is 4:

$$
\frac{1}{4} - \frac{2k}{4} - \frac{20}{4} = 0
$$

Combine:

$$
\frac{1 - 2k - 20}{4} = 0
$$

Multiply both sides by 4 to clear the denominator:

$$
1 - 2k - 20 = 0
$$

Simplify:

$$
-19 - 2k = 0
$$

Add 19 to both sides:

$$
-2k = 19
$$

Divide both sides by -2:

$$
k = -\frac{19}{2}
$$

Final Answer and Explanation

The value of k is $-\frac{19}{2}$.

This means that when $k = -\frac{19}{2}$, the polynomial $4x^3 - x^2 - kx - 5$ is exactly divisible by $2x - 1$, with a remainder of zero.

Summary of the Process

- Recognized the divisor $2x - 1$ and identified its root $x = \frac{1}{2}$.
- Applied the Polynomial Remainder Theorem, which states that the remainder of a polynomial $P(x)$ divided by $x - a$ is $P(a)$.
- Substituted $x = \frac{1}{2}$ into the polynomial $P(x)$, set it equal to zero, and solved for k.
- Derived the exact value of k that makes the polynomial divisible by $2x - 1$.

Additional Insights and Related Concepts

Understanding Polynomial Divisibility

Divisibility of polynomials is a core concept in algebra, closely linked to factors and roots. When a polynomial $P(x)$ is divisible by a linear polynomial $x - a$, it indicates that a is a root of $P(x)$. This relationship is fundamental in polynomial factorization.

Using Synthetic Division for Polynomial Division

While the Remainder Theorem provides a quick way to find the remainder and test divisibility, synthetic division offers a systematic method to divide polynomials efficiently. It simplifies the process of checking divisibility and can be useful when dealing with higher-degree polynomials or multiple roots.

Application in Solving Polynomial Equations

Knowing the value of k allows us to factor the polynomial completely or analyze its roots further. This can be particularly useful in solving equations, graphing polynomials, and understanding their behavior.

Conclusion

Determining the value of k in the polynomial $4x^3 - x^2 - kx - 5$ based on divisibility conditions involves understanding polynomial division, the Remainder Theorem, and substitution methods. The key steps include identifying the root of the divisor, evaluating the polynomial at that root, and solving the resulting equation for the unknown coefficient.

In this case, the exact value of k is $-\frac{19}{2}$, ensuring the polynomial is divisible by $2x - 1$ with no remainder.

This approach showcases the power of algebraic techniques in solving polynomial problems efficiently and accurately, reinforcing fundamental concepts essential for advanced mathematics and problem-solving skills.

Keywords for SEO Optimization:

- Polynomial division
- Remainder theorem
- Polynomial factoring
- Finding k in polynomial equations
- Dividing polynomials with linear divisors
- Polynomial roots and coefficients
- Algebraic problem-solving
- Synthetic division
- Polynomial divisibility conditions
- Solving for coefficients in polynomials

Frequently Asked Questions

What is the problem asking when dividing the polynomial 4x^3 - x^2 - kx - 5 by 2x - 1 with a remainder of 0?

The problem is asking to find the value of k such that when 4x^3 - x^2 - kx - 5 is divided by 2x - 1, the division leaves no remainder, i.e., the division is exact.

How does the Remainder Theorem help in finding the value of k in this problem?

The Remainder Theorem states that the remainder when a polynomial is divided by a linear factor (like 2x - 1) is equal to the polynomial evaluated at the root of the divisor. Setting this evaluation to zero helps find k.

What is the root of the divisor 2x - 1, and how is it used in solving the problem?

The root of 2x - 1 = 0 is x = 1/2. We substitute x = 1/2 into the polynomial to determine the value of k that makes the remainder zero.

How do you set up the equation to find k using the root x = 1/2?

Substitute x = 1/2 into 4x^3 - x^2 - kx - 5 and set the result equal to zero: 4(1/2)^3 - (1/2)^2 - k(1/2) - 5 = 0.

What is the step-by-step process to solve for k after substituting x = 1/2?

Calculate each term: 4(1/8) - 1/4 - (k)(1/2) - 5 = 0. Simplify to get 1/2 - 1/4 - (k/2) - 5 = 0. Combine like terms and solve for k.

What is the value of k after solving the equation?

After simplifying, the equation becomes $1/4 - (k/2) - 5 = 0$. Multiply through by 4 to clear fractions: $1 - 2k - 20 = 0$, leading to $-2k = 19$, so $k = -19/2$.

Why is it important that the remainder is zero in this problem?

Because a zero remainder indicates that $2x - 1$ is a factor of the polynomial, which allows us to find the specific value of k that makes the division exact.

Additional Resources

When The Expression 4x^(3)-x^(2)-kx-5 Is Divided 2x-1 The Reminder 0 , Find The Values Of K

Introduction

Mathematics, especially algebra, is a field rich with problem-solving challenges that invoke critical thinking and a deep understanding of polynomial behavior. Among such challenges is the classic problem of polynomial division and the Remainder Theorem, which offers an elegant route to solutions without engaging in exhaustive division processes.

The specific problem under scrutiny is:

"When the expression $4x^3 - x^2 - kx - 5$ is divided by $2x - 1$, the remainder is 0. Find the values of k."

This problem not only tests understanding of polynomial division and the Remainder Theorem but also illustrates the application of algebraic principles to derive unknown parameters.

This comprehensive article aims to dissect this problem thoroughly, exploring relevant concepts, step-by-step solutions, and the broader implications for algebraic problem-solving. The discussion is structured into detailed sections to facilitate clarity and deepen understanding.

Understanding the Core Concepts

Polynomial Division and the Remainder Theorem

At the heart of this problem lies the Polynomial Division process and the Remainder Theorem, two fundamental concepts in algebra.

Polynomial Division involves dividing a polynomial $P(x)$ by a divisor polynomial $D(x)$. The division yields a quotient polynomial $Q(x)$ and a remainder polynomial $R(x)$, expressed as:

$$
P(x) = D(x) \times Q(x) + R(x)
$$

When the divisor is a linear polynomial (degree 1), the Remainder Theorem simplifies the process.

The Remainder Theorem states:

> If a polynomial $P(x)$ is divided by $(x - a)$, then the remainder of this division is $P(a)$.

This theorem implies that to find the remainder without performing polynomial division, one simply evaluates the polynomial at $x = a$.

Adapting to the Given Divisor $2x - 1$

The divisor in our problem is $2x - 1$, which isn't directly in the form $x - a$. To utilize the Remainder Theorem, we need to find the value of x that makes $2x - 1 = 0$:

$$
2x - 1 = 0 \Rightarrow x = \frac{1}{2}
$$

Therefore, the remainder when dividing $P(x)$ by $2x - 1$ is equivalent to evaluating $P \left(\frac{1}{2} \right)$.

Formalizing the Problem: Applying the Remainder Theorem

Given the polynomial:

$$
P(x) = 4x^3 - x^2 - kx - 5
$$

and the divisor:

$$
D(x) = 2x - 1
$$

The remainder upon division of $P(x)$ by $D(x)$ is:

$$
R = P \left(\frac{1}{2} \right)
$$

The problem states that the remainder is 0, which leads to the key equation:

$$
P \left(\frac{1}{2} \right) = 0
$$

Our task simplifies to calculating $P \left(\frac{1}{2} \right)$ and solving for k.

Step-by-Step Solution

Step 1: Compute $P \left(\frac{1}{2} \right)$

Substituting $x = \frac{1}{2}$:

$$
P \left(\frac{1}{2} \right) = 4 \left(\frac{1}{2} \right)^3 - \left(\frac{1}{2} \right)^2 - k \left(\frac{1}{2} \right) - 5
$$

Calculate each term:

- $\left(\frac{1}{2} \right)^3 = \frac{1}{8}$
- $\left(\frac{1}{2} \right)^2 = \frac{1}{4}$
- $k \times \frac{1}{2} = \frac{k}{2}$

Now, plug these into the expression:

$$
P \left(\frac{1}{2} \right) = 4 \times \frac{1}{8} - \frac{1}{4} - \frac{k}{2} - 5
$$

Simplify:

$$
= \frac{4}{8} - \frac{1}{4} - \frac{k}{2} - 5
$$

which reduces to:

$$
= \frac{1}{2} - \frac{1}{4} - \frac{k}{2} - 5
$$

Step 2: Simplify the expression

Observe that:

$$
\frac{1}{2} - \frac{1}{4} = \frac{2}{4} - \frac{1}{4} = \frac{1}{4}
$$

Thus:

$$
P \left(\frac{1}{2} \right) = \frac{1}{4} - \frac{k}{2} - 5
$$

Step 3: Set the expression equal to zero

Since the remainder is 0:

$$
\frac{1}{4} - \frac{k}{2} - 5 = 0
$$

Step 4: Solve for k

Rearranged:

$$
- \frac{k}{2} = - \frac{1}{4} + 5
$$

Calculate the right side:

$$
- \frac{1}{4} + 5 = - \frac{1}{4} + \frac{20}{4} = \frac{19}{4}
$$

Thus:

$$
- \frac{k}{2} = \frac{19}{4}
$$

Multiply both sides by -1:

$$
\frac{k}{2} = - \frac{19}{4}
$$

Multiply both sides by 2:

$$
k = - \frac{19}{4} \times 2 = - \frac{19 \times 2}{4} = - \frac{38}{4} = - \frac{19}{2}
$$

Final Answer

\[

\boxed{
\textbf{The value of } k \textbf{ is } - \frac{19}{2}
}
\]

This means that, for the polynomial \(4x^3 - x^2 - kx - 5 \), the division by \(2x - 1 \) results in a zero remainder precisely when \(k = - \frac{19}{2} \).

Broader Implications and Further Exploration

Significance of the Result

This problem exemplifies the power of the Remainder Theorem in simplifying polynomial division problems, especially when the divisor is linear. Instead of performing polynomial long division, which can be algebraically intensive, evaluating the polynomial at a specific point suffices to find the remainder.

The process demonstrates the interconnectedness of polynomial roots, divisibility, and algebraic evaluation—a foundational aspect in algebra and higher mathematics.

Related Problems and Variations

- Finding all \(k \) such that the polynomial is divisible by another polynomial: Extending this idea, students can explore divisibility conditions for higher-degree divisors.

- Generalization to other divisors: Investigate how the Remainder Theorem applies to divisors like \(ax - b \), and how to adapt the evaluation point accordingly.

- Polynomial factorization: Once \(k \) is known, the polynomial can be factored further, potentially revealing more about its roots and behavior.

Practical Applications

Understanding polynomial division and the Remainder Theorem has practical implications beyond pure mathematics, including:

- Signal processing (filter design)
- Engineering (control systems)
- Computer science (algorithm design and polynomial hashing)

Conclusion

The problem of determining the value of \(k \) in the polynomial \(4x^3 - x^2 - kx - 5 \) such that it is divisible by \(2x - 1 \) underscores the elegance and utility of algebraic theorems. By leveraging the Remainder Theorem, the task reduces to evaluating the polynomial at a specific point and solving a simple algebraic equation.

The key takeaway is that understanding fundamental theorems like the Remainder Theorem not only simplifies complex problems but also deepens appreciation for the interconnected structure of algebra. Mastery of such techniques prepares students and professionals to approach a wide array of mathematical problems with confidence and efficiency.

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Note: This detailed exploration is designed to serve both as an educational guide and a reference for those tackling similar polynomial division problems.

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