Which Conic Section Is Represented By The Following Equation?5x^2 - X + 5y^2 - 3y - 12 =0

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Understanding conic sections is fundamental in algebra and geometry, as they appear frequently in various mathematical contexts and real-world applications. The given equation, $5x^2 - X + 5y^2 - 3y - 12 = 0$, is a quadratic equation involving both x and y variables. To identify which conic section it represents, we need to analyze and manipulate the equation into a standard or recognizable form. This process involves several steps, including rewriting, completing the square, and examining the coefficients.

In this comprehensive guide, we will explore how to determine the conic section represented by the given equation, including a detailed step-by-step solution, explanations of key concepts, and tips for identifying conics from equations.

Understanding Conic Sections

Before delving into the specific equation, it is essential to understand what conic sections are and their standard forms.

What Are Conic Sections?

Conic sections are the curves obtained by intersecting a plane with a double-napped cone. Depending on the angle of intersection, the resulting curves can be:

- Circle
- Ellipse
- Parabola
- Hyperbola

Each conic has a standard equation form, which helps in identifying and graphing the conic.

Standard Forms of Conic Sections

| Conic Type | Standard Equation Form | Key Characteristics |

Circle	$(x - h)^2 + (y - k)^2 = r^2$	Equal radii, centered at (h, k)
Ellipse	$((x - h)^2)/a^2 + ((y - k)^2)/b^2 = 1$	Sum of distances constant
Parabola	$y - k = a(x - h)^2$ or $x - h = a(y - k)^2$	Quadratic in one variable
Hyperbola	$((x - h)^2)/a^2 - ((y - k)^2)/b^2 = 1$ or vice versa	Difference of distances constant

The general second-degree equation for conic sections is:

$$A x^2 + B xy + C y^2 + D x + E y + F = 0$$

Analyzing the coefficients helps us classify the conic.

Analyzing the Given Equation

The given equation is:

$$5x^2 - X + 5y^2 - 3y - 12 = 0$$

However, it appears there might be a typographical error with the variable 'X' capitalized; assuming the variable is 'x' (lowercase), the equation is:

$$5x^2 - x + 5y^2 - 3y - 12 = 0$$

If that's the case, our goal is to identify the conic section this equation represents.

Step 1: Write the Equation Clearly

Equation:

$$5x^2 - x + 5y^2 - 3y - 12 = 0$$

Note that this is a quadratic in both x and y, indicating the conic is likely an ellipse, hyperbola, or circle.

Step 2: Group Like Terms

Group x and y terms:

$$(5x^2 - x) + (5y^2 - 3y) = 12$$

Our goal is to complete the square for both x and y terms.

Step 3: Complete the Square

For the x-terms:

$5x^2 - x$

Factor out 5:

$5(x^2 - (1/5) x)$

Complete the square inside parentheses:

- Take half of the coefficient of x: $(1/5) / 2 = (1/10)$
- Square it: $(1/10)^2 = 1/100$

Add and subtract this inside the parentheses, considering the factor 5:

$5[x^2 - (1/5) x + (1/100) - (1/100)] = 5[(x - 1/10)^2 - 1/100]$

Similarly for the y-terms:

$5y^2 - 3y$

Factor out 5:

$5(y^2 - (3/5) y)$

Complete the square:

- Half of $(3/5)$: $(3/5)/2 = 3/10$
- Square: $(3/10)^2 = 9/100$

Add and subtract inside:

$5[(y - 3/10)^2 - 9/100] = 5(y - 3/10)^2 - 45/100 = 5(y - 3/10)^2 - 9/20$

Now, rewrite the entire equation:

$5(x - 1/10)^2 - 5/100 + 5(y - 3/10)^2 - 9/20 = 12$

Simplify the constants:

- $5/100 = 1/20$

Combine constants on the left:

- $(1/20) + (9/20) = (10/20) = 1/2$

Bring all constants to the right:

5(x - 1/10)^2 + 5(y - 3/10)^2 = 12 + 1/2

Simplify right side:

12 + 1/2 = (24/2) + (1/2) = 25/2

Step 4: Write in Standard Form

Divide both sides by 25/2 to normalize:

[5(x - 1/10)^2] / (25/2) + [5(y - 3/10)^2] / (25/2) = 1

Simplify:

Multiply numerator and denominator:

5 / (25/2) = 5 (2/25) = (10/25) = (2/5)

Thus,

(2/5)(x - 1/10)^2 + (2/5)(y - 3/10)^2 = 1

Alternatively, write as:

(x - 1/10)^2 / (25/4) + (y - 3/10)^2 / (25/4) = 1

Because:

(2/5) = 1 / (5/2), so the denominators are 25/4

Final standard form:

$$
\frac{(x - 0.1)^2}{\frac{25}{4}} + \frac{(y - 0.3)^2}{\frac{25}{4}} = 1
$$

Since both denominators are equal, this is the equation of an ellipse centered at (0.1, 0.3).

Conclusion: The Equation Represents an Ellipse

Given the positive coefficients for both squared terms and the equal denominators, the original equation describes an ellipse.

Key Points to Remember

- The original quadratic equation, once simplified and completed the square, results in a standard form indicative of an ellipse.
- Equal denominators in the standard form confirm the shape is an ellipse with equal axes.

Additional Insights and Tips

How to Identify Conic Sections from General Equations

When given a general quadratic equation:

A x^2 + B xy + C y^2 + D x + E y + F = 0

follow these steps:

1. Check the coefficients:

- If $B \neq 0$, the conic is rotated; analyze discriminant.
- If $B = 0$, the conic is aligned with axes.

2. Calculate the discriminant:

$$
\Delta = B^2 - 4AC
$$

- If $\Delta < 0$ and $A = C$, circle.
- If $\Delta < 0$ and $A \neq C$, ellipse.
- If $\Delta = 0$, parabola.
- If $\Delta > 0$, hyperbola.

3. Complete the square:

Rewrite the equation to standard form to identify the specific conic.

4. Graph and verify:

Plot the conic to confirm its shape.

Common Mistakes to Avoid

- Not simplifying the equation before analysis.
- Forgetting to complete the square.
- Confusing the signs or coefficients.
- Overlooking rotation effects when $B \neq 0$.

Real-World Applications of Conic Sections

Understanding the nature of conic sections is crucial in various fields:

- Astronomy: Orbits of planets are elliptical.
- Engineering: Design of reflective telescopes and satellite dishes.
- Physics: Projectile motion often involves conic sections.
- Architecture: Arches and bridges utilize elliptical and parabolic shapes.
- Navigation and GPS: Satellite orbits and signal paths are modeled using conics.

Summary

- The given quadratic equation, when properly analyzed, simplifies to an equation of an ellipse.
- Completing the square and rewriting in standard form are essential steps.
- Recognizing the form of the equation helps in classifying the conic.
- Conic sections have broad applications across science and engineering.

Final Thoughts