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Understanding how to find the slope of the tangent line to a polar curve is a fundamental concept in calculus and analytical geometry. In this article, we will explore the process of calculating the slope of a tangent line to the polar curve given by the equation $r = \sin(30^\circ)$ at a specific angle $\theta = 4$ radians (or degrees, depending on the context). We will break down each step, provide relevant formulas, and interpret the results to enhance your understanding of polar curves and their derivatives.

Introduction to Polar Curves

Before diving into the calculation, it’s essential to understand the basics of polar coordinates and curves.

What Are Polar Coordinates?

- Polar coordinates represent a point in the plane using a radius r and an angle θ measured from the positive x-axis.
- A point's position is given as (r, θ), where:
- r is the distance from the origin to the point.
- θ is the angle between the positive x-axis and the line connecting the origin to the point.

Polar Equations and Curves

- A polar curve is defined by an equation relating r and θ, such as $r = f(\theta)$.
- The shape of the curve depends on the function $f(\theta)$.

Understanding the Given Curve: $r = \sin(30)$

The given polar equation is $r = \sin(30)$. However, this may seem to suggest that r

is a constant, which is the sine of 30 degrees (or radians). Let’s clarify:

- If the equation is $r = \sin(30^\circ)$, then r is a constant since $\sin(30^\circ) = 0.5$.
- If the equation is $r = \sin(30 \theta)$, then r varies with θ.

Given the context, it appears the intended curve is $r = \sin(30^\circ)$, a constant. But since the problem involves finding the slope at $\theta = 4$, it would be more meaningful if the function was $r = \sin(30 \theta)$.

Assumption: The correct equation is likely $r = \sin(30 \theta)$, where 30 is a coefficient, making the curve a classic rose or sinusoidal polar curve.

Calculating the Slope of a Tangent Line in Polar Coordinates

The slope of the tangent line to a polar curve at a point is given by the derivative $\frac{dy}{dx}$. Since polar curves are expressed in terms of r and θ, we need to convert derivatives accordingly.

Formula for the Slope of the Tangent Line

- The slope $\frac{dy}{dx}$ for a polar curve $r = r(\theta)$ is given by:

$$
\frac{dy}{dx} = \frac{r'(\theta) \sin \theta + r(\theta) \cos \theta}{r'(\theta) \cos \theta - r(\theta) \sin \theta}
$$

where:

- $r'(\theta) = \frac{dr}{d\theta}$ is the derivative of r with respect to θ.

This formula is derived from the parametric equations:

$$
x = r \cos \theta,\quad y = r \sin \theta
$$

and their derivatives.

Step-by-Step Solution

Let's proceed step-by-step to find the slope of the tangent line at $\theta = 4$ radians for the curve $r = \sin(30 \theta)$.

Step 1: Identify $r(\theta)$ and compute $r'(\theta)$

- $r(\theta) = \sin(30 \theta)$

- Derivative:

$$
r'(\theta) = \frac{d}{d\theta} \sin(30 \theta) = 30 \cos(30 \theta)
$$

Step 2: Evaluate $r(\theta)$ and $r'(\theta)$ at $\theta = 4$

- $r(4) = \sin(30 \times 4) = \sin(120)$

- $r'(4) = 30 \cos(120)$

Recall:

- $\sin(120^\circ) = \sin(180^\circ - 60^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$

- $\cos(120^\circ) = - \cos 60^\circ = - \frac{1}{2}$

Note: If $\theta = 4$ is in radians, convert to degrees:

$$
4 \text{ radians} \times \frac{180^\circ}{\pi} \approx 4 \times 57.2958^\circ \approx 229.18^\circ
$$

But since the original calculation uses degrees, and the sine and cosine are evaluated at 120°, it suggests θ should be interpreted in degrees. For consistency, assume $\theta = 4^\circ$:

- $r(4^\circ) = \sin(30 \times 4^\circ) = \sin(120^\circ) = \frac{\sqrt{3}}{2}$

- $r'(4^\circ) = 30 \cos(120^\circ) = 30 \times (-\frac{1}{2}) = -15$

Alternatively, if $\theta = 4$ radians, then:

- $r(4) = \sin(30 \times 4) = \sin(120)$ radians, which is approximately:

$$
\sin(120) \text{ radians} \approx \text{Calculate as needed}
$$

Given the context and common practice, we'll proceed assuming degrees for simplicity.

Step 3: Calculate $\sin \theta$ and $\cos \theta$ at $\theta = 4^\circ$

- $\sin 4^\circ \approx 0.0698$

- $\cos 4^\circ \approx 0.9976$

Step 4: Plug into the slope formula

$$
\frac{dy}{dx} = \frac{r'(\theta) \sin \theta + r(\theta) \cos \theta}{r'(\theta) \cos \theta - r(\theta) \sin \theta}
$$

Substituting the values:

- $r(4^\circ) = \frac{\sqrt{3}}{2} \approx 0.8660$

- $r'(4^\circ) = -15$

- $\sin 4^\circ \approx 0.0698$

- $\cos 4^\circ \approx 0.9976$

Calculate numerator:

$$
N = (-15)(0.0698) + 0.8660 \times 0.9976 = -1.047 + 0.864 = -0.183
$$

Calculate denominator:

$$
D = (-15)(0.9976) - 0.8660 \times 0.0698 = -14.964 - 0.0605 = -15.0245
$$

Finally, the slope:

$$
\frac{dy}{dx} = \frac{-0.183}{-15.0245} \approx 0.0122
$$

Interpreting the Result

The approximate slope of the tangent line at $\theta = 4^\circ$ (or radians, depending on the initial assumption) is about 0.0122. This indicates that at this point on the polar curve, the tangent line has a very gentle positive slope, meaning it is almost horizontal.

Additional Considerations and Variations

- If the initial r equation was different: For example, if $r = \sin(30 \theta)$, the method remains the same, but the evaluation points differ.

- If the problem involves radians: The calculations should be adapted accordingly, with θ in radians, and sine and cosine evaluated at that value.

- Using software tools: Calculators or software like WolframAlpha, GeoGebra, or graphing calculators can help evaluate complex sine and cosine values precisely.

Summary of Key Steps

1. Identify the polar equation: $r = \sin(30 \theta)$ (assuming the most probable case).

2. Compute derivatives: $r'(\theta) = 30 \cos(30 \theta)$.

Frequently Asked Questions

What is the general method to find the slope of the tangent line to a polar curve at a given point?

The slope of the tangent line to a polar curve $r = f(\theta)$ at a specific angle θ is found using the derivative dy/dx, which can be computed as $(r' \sin \theta + r \cos \theta) / (r' \cos \theta - r \sin \theta)$, where r' is $dr/d\theta$.

How do you evaluate r at θ = 4 for the curve R = Sin(30)?

Since R = sin(30°), r is a constant value of 0.5 regardless of θ, so at θ = 4 radians, r = 0.5.

What is the value of r' (dr/dθ) for R = sin(30) at any θ?

Since R = sin(30°) is a constant, its derivative with respect to θ is zero, so r' = 0.

How do you compute the slope of the tangent line using the formula when r' = 0?

When r' = 0, the slope simplifies to (r' sin θ + r cos θ) / (r' cos θ - r sin θ) = (r cos θ) / (- r sin θ) = - cot θ, since r' = 0.

What is the value of cot θ at θ = 4 radians?

cot θ is the reciprocal of tan θ. To find cot 4, compute tan 4 radians first, then take its reciprocal. tan 4 ≈ 1.1578, so cot 4 ≈ 0.863.

What is the final slope of the tangent line at θ = 4 radians for the given curve?

Using the simplified formula, the slope = - cot 4 ≈ -0.863.

Why does the derivative r' being zero simplify the calculation of the tangent slope?

Because with r' = 0, the formula reduces to a ratio involving only r and θ, making it easier to compute the slope directly as - cot θ.

What is the final answer for the slope of the tangent line at θ = 4 radians to the curve R = sin(30)?

The slope is approximately -0.863.

Additional Resources

Find The Slope Of The Tangent Line To The Polar Curve R = Sin(30) At θ = 4.

Understanding how to find the slope of the tangent line to a polar curve is a fundamental skill in calculus, especially when working with curves expressed in polar coordinates. In this guide, we will explore the process step-by-step to determine the slope of the tangent line to the given polar curve \(r = \sin(30) \) at a specific angle \(\theta = 4 \). This detailed walkthrough aims to clarify the concepts involved and equip you with the techniques necessary to handle similar problems with confidence.

Introduction to Polar Curves and Their Derivatives

Before diving into the specific problem, it’s essential to understand the basics of polar curves and how to differentiate them to find tangents.

What is a Polar Curve?

A polar curve is a graph of a function that relates the distance r from the origin (pole) to an angle θ in the plane. It’s expressed as:

$$
r = f(\theta)
$$

where r is the radius at angle θ. These curves are often used to describe natural shapes like spirals, roses, and other symmetrical patterns.

The Slope of a Tangent Line in Polar Coordinates

In Cartesian coordinates, the slope of the tangent line at a point is straightforward to find. However, in polar coordinates, the process involves converting the polar function into parametric equations:

$$
x = r \cos \theta
$$

$$
y = r \sin \theta
$$

The slope of the tangent line at a specific θ is given by the derivative:

$$
\frac{dy}{dx}
$$

which, via the chain rule, relates to derivatives of r:

$$
\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}
$$

where:

$$
\frac{dy}{d\theta} = \frac{d}{d\theta}(r \sin \theta) = r' \sin \theta + r \cos \theta
$$

$$
\frac{dx}{d\theta} = \frac{d}{d\theta}(r \cos \theta) = r' \cos \theta - r \sin \theta
$$

\]

and

$$
r' = \frac{dr}{d\theta}
$$

This formula allows us to compute the slope directly once r and r' are known.

Step-by-Step Guide to Find the Slope

Now, let’s proceed with the detailed steps to find the slope of the tangent line to the curve $r = \sin(30)$ at $\theta = 4$. Note that $\sin(30)$ is a constant (since 30 degrees is a fixed angle), but based on context, it may be that the curve is $r = \sin \theta$ or similar, as $r = \sin(30)$ (a constant) would be a circle of fixed radius.

Assuming the intended problem is:

Find the slope of the tangent line to the polar curve $r = \sin \theta$ at $\theta = 4$ radians.

---

1. Understanding the Curve $r = \sin \theta$

This is a classic rose curve with petal symmetry. For each θ, the radius is $r = \sin \theta$.

2. Find $r' = \frac{dr}{d\theta}$

Since:

$$
r = \sin \theta
$$

then:

$$
r' = \frac{d}{d\theta} \sin \theta = \cos \theta
$$

3. Calculate r and r' at $\theta = 4$

- $r(4) = \sin 4$
- $r'(4) = \cos 4$

Using approximate values:

$$
\sin 4 \text{ radians} \approx -0.7568
$$

$$
\cos 4 \text{ radians} \approx -0.6536
$$

---

4. Compute $\frac{dy}{d\theta}$ and $\frac{dx}{d\theta}$

Recall:

$$
\frac{dy}{d\theta} = r' \sin \theta + r \cos \theta
$$

$$
\frac{dx}{d\theta} = r' \cos \theta - r \sin \theta
$$

At $\theta = 4$:

- $r = -0.7568$
- $r' = -0.6536$
- $\sin 4 \approx -0.7568$
- $\cos 4 \approx -0.6536$

Calculate:

$$
\frac{dy}{d\theta} = (-0.6536)(-0.7568) + (-0.7568)(-0.6536) = 0.4948 + 0.4948 = 0.9896
$$

$$
\frac{dx}{d\theta} = (-0.6536)(-0.6536) - (-0.7568)(-0.7568) = 0.4272 - 0.5728 = -0.1456
$$

---

5. Find the slope $\frac{dy}{dx}$

$$
\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{0.9896}{-0.1456} \approx -6.79
$$

Therefore, the slope of the tangent line at $\theta=4$ radians is approximately -6.79.

Additional Considerations and Tips

Handling Different Curves

- If the curve is given as $r = \text{constant}$, then the tangent line is a circle, and the slope can be derived differently.
- For more complex $r(\theta)$ functions, always compute r' carefully and verify your calculations.

Converting to Cartesian Coordinates

Alternatively, you can convert the polar curve into x, y:

$$
x = r \cos \theta
$$
$$
y = r \sin \theta
$$

and differentiate directly with respect to θ.

Use of Calculators and Software

Using graphing calculators or software like WolframAlpha, Desmos, or GeoGebra can help verify your derivatives and slopes.

Summary and Final Remarks

In conclusion, finding the slope of the tangent line to a polar curve involves understanding the relationship between the derivatives in polar coordinates and their Cartesian counterparts. The key steps include:

- Computing $r' = \frac{dr}{d\theta}$.
- Using the formulas:

$$
\frac{dy}{d\theta} = r' \sin \theta + r \cos \theta
$$
$$
\frac{dx}{d\theta} = r' \cos \theta - r \sin \theta
$$
- Calculating the ratio $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$.

Applying these steps to the specific curve $r = \sin \theta$ at $\theta = 4$ radians yields an approximate slope of -6.79, indicating a steep negative slope at that point.

By mastering this process, you become better equipped to analyze complex polar curves and understand their tangent lines, which are essential in fields ranging from physics to engineering and beyond.

Remember: Always verify your calculations, especially when dealing with trigonometric functions at specific angles, and consider the context of the curve to ensure your interpretations are accurate.

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