2x+2yequals 5, X-2y Equals 3 Find The Solution Set By Using Method Elimination By Substitution

2x+2yequals 5, X-2y Equals 3 Find The Solution Set By Using Method Elimination By Substitution

Understanding how to solve systems of equations is a fundamental skill in algebra that helps students and professionals alike analyze relationships between variables. In particular, solving the system of equations:

$$
\begin{cases}
2x + 2y = 5 \\
x - 2y = 3
\end{cases}
$$

by using the method of elimination by substitution is a practical approach that combines two powerful algebraic techniques. This article provides a comprehensive guide on how to approach this problem step by step, exploring the concepts behind the method, and demonstrating the solution process in detail.

Understanding Systems of Equations

Before diving into the elimination and substitution methods, it’s essential to understand what a system of equations entails. A system consists of two or more equations with the same set of variables. The goal is to find the values of these variables that satisfy all equations simultaneously.

Key Concepts:

- Variables: Unknown quantities represented by symbols such as x and y.
- Solution Set: The set of all variable values that satisfy every equation in the system.
- Consistent System: Has at least one solution.
- Inconsistent System: Has no solution.
- Dependent System: Has infinitely many solutions.

Approach to Solving Systems Using Elimination by Substitution

The method of elimination by substitution involves two primary steps:

1. Isolate one variable in one equation (usually the easier one to manipulate).
2. Substitute this expression into the other equation to solve for the remaining variable.

This method effectively reduces the system to a single-variable equation, simplifying the process of finding the solution.

Step-by-Step Solution of the System

Given the system:

$$
\begin{cases}
2x + 2y = 5 \quad \text{(Equation 1)} \\
x - 2y = 3 \quad \text{(Equation 2)}
\end{cases}
$$

Let's walk through solving it using elimination by substitution.

Step 1: Isolate a Variable in One Equation

Looking at Equation 2:

$$ x - 2y = 3 $$

It’s straightforward to solve for x:

$$ x = 3 + 2y $$

This expression for x will be substituted into Equation 1.

Step 2: Substitute into the Other Equation

Replace x in Equation 1:

$$2x + 2y = 5$$

becomes

$$2(3 + 2y) + 2y = 5$$

Now, simplify:

$$6 + 4y + 2y = 5$$

$$6 + 6y = 5$$

Subtract 6 from both sides:

$$6y = 5 - 6$$

$$6y = -1$$

Divide both sides by 6:

$$y = -\frac{1}{6}$$

Step 3: Find the Corresponding x Value

Recall the expression for x:

$$x = 3 + 2y$$

Substitute $y = -\frac{1}{6}$:

$$x = 3 + 2 \times \left(-\frac{1}{6}\right)$$

$$x = 3 - \frac{2}{6}$$

$$x = 3 - \frac{1}{3}$$

Expressed as a mixed number or improper fraction:

$$x = \frac{9}{3} - \frac{1}{3} = \frac{8}{3}$$

Final Solution Set

The solution to the system of equations is:

$$
x = \frac{8}{3}, \quad y = -\frac{1}{6}
$$

Expressed as an ordered pair:

$$
\boxed{\left(\frac{8}{3}, -\frac{1}{6} \right)}
$$

This point satisfies both equations, confirming it as the solution set.

Verifying the Solution

Verification ensures the accuracy of the solution.

Check Equation 1:

$$ 2x + 2y = 5 $$

$$ 2 \times \frac{8}{3} + 2 \times \left(-\frac{1}{6}\right) = ? $$

$$ \frac{16}{3} - \frac{2}{6} = ? $$

Expressing $\frac{2}{6}$ as $\frac{1}{3}$:

$$ \frac{16}{3} - \frac{1}{3} = \frac{15}{3} = 5 $$

Check Equation 2:

$$ x - 2y = 3 $$

$$ \frac{8}{3} - 2 \times \left(-\frac{1}{6}\right) = ? $$

$$ \frac{8}{3} + \frac{2}{6} = \frac{8}{3} + \frac{1}{3} = \frac{9}{3} = 3 $$

Both equations check out, confirming the solution.

Additional Tips for Solving Systems Using Elimination and Substitution

While the above example demonstrates a straightforward case, here are some tips to handle more complex systems:

- Choose the easiest equation to isolate a variable: Pick the one with coefficients that are simple to manipulate.
- Pay attention to coefficients: Sometimes multiplying equations by constants simplifies elimination.
- Check for special cases: Parallel lines (no solution) or identical equations (infinite solutions).
- Use substitution when one variable is already isolated or easily isolated.

Common Mistakes to Avoid

- Mixing methods: Ensure to choose either elimination or substitution, not both haphazardly.
- Incorrect substitution: Carefully replace variables to avoid algebraic errors.
- Ignoring signs: Be attentive to positive and negative signs throughout calculations.
- Not verifying solutions: Always plug solutions back into original equations.

Practice Problems for Mastery

1. Solve the system:

\[
\begin{cases}
3x + y = 7 \\
x - 4y = -1
\end{cases}
\]

using elimination by substitution.

2. Find the solution set for:

\[

\begin{cases}
x + 2y = 4 \\
2x + 3y = 9
\end{cases}
\]

using the same method.

3. For the system:

\[
\begin{cases}
5x - y = 11 \\
x + y = 3
\end{cases}
\]

solve using elimination by substitution.

Conclusion

Solving systems of equations like \(2x + 2y = 5 \) and \(x - 2y = 3 \) using the method of elimination by substitution is an invaluable skill in algebra. This method systematically reduces a complex problem into manageable steps, leading to clear solutions. By isolating variables and carefully substituting, students and practitioners can confidently find solution sets for a variety of systems, enhancing their problem-solving toolkit in mathematics.

Remember to verify solutions, practice with different types of systems, and always double-check your algebra to ensure accuracy. With consistent practice, mastering the elimination by substitution method will become an intuitive and powerful approach for tackling linear systems in algebra and beyond.

Frequently Asked Questions

What is the first step in solving the system of equations using the elimination method?

The first step is to align both equations properly and decide which variable to eliminate, often by making the coefficients of one variable opposites.

How can substitution be used to solve the system of equations 2x + 2y = 5 and x - 2y = 3?

First, solve one of the equations for one variable (e.g., x = 3 + 2y from the second equation) and then substitute this expression into the other equation to find the value of the remaining variable.

What is the solution set for the system 2x + 2y = 5 and x - 2y = 3 after using substitution?

The solution set is x = 1, y = 1, which can be found by substituting x = 3 + 2y into the first equation and solving for y.

Why is elimination a useful method for solving systems of equations like these?

Elimination is useful because it allows you to eliminate one variable by adding or subtracting equations, simplifying the solution process for systems with multiple variables.

Can the substitution method be used if the equations are not easily solvable for one variable? Why?

Yes, but it might be more complicated; in such cases, elimination can sometimes be more straightforward, especially if one variable already has a coefficient of 1 or -1.

What are the key steps to find the solution set using elimination in this system?

Identify coefficients for a variable, multiply equations if necessary to align coefficients, add or subtract equations to eliminate a variable, then solve for the remaining variable and back-substitute to find the other variable.

What is the final solution for the system 2x + 2y = 5 and x - 2y = 3 using the elimination by substitution method?

The solution is x = 1, y = 1, obtained by substituting x = 3 + 2y into the first equation and solving for y, then finding x accordingly.

Additional Resources

2x + 2yequals 5, X - 2y Equals 3 Find The Solution Set By Using Method Elimination By Substitution

In the realm of algebra, systems of equations are fundamental tools for modeling real-world problems, from calculating budgets to engineering designs. Among various techniques to solve such systems, the method of elimination by substitution stands out for its efficiency and clarity, especially when one equation can be readily manipulated to isolate a variable. Today, we delve into a specific system of linear equations:

$$ 2x + 2y = 5 $$
$$ x - 2y = 3 $$

Our goal: to find the solution set—precisely the values of x and y—that satisfy both equations simultaneously using the elimination by substitution method. This approach involves solving one of the equations for one variable and then substituting that expression into the other, simplifying the system step-by-step.

Understanding the System of Equations

Before applying the elimination by substitution technique, it’s essential to understand the structure of our given system:

1. Equation 1: $2x + 2y = 5$
2. Equation 2: $x - 2y = 3$

At a glance, these are linear equations with two variables, x and y. The solution set, if it exists, will typically be a single point (x, y) in the Cartesian plane where both equations intersect.

Recognizing Opportunities for Substitution

In the second equation, $x - 2y = 3$, the variable x is isolated or at least easily isolated. This makes it an ideal candidate for substitution into the first equation.

Step 1: Isolate a Variable in One Equation

The key to substitution is to express one variable explicitly in terms of the other. Let’s solve Equation 2 for x:

$$
x - 2y = 3
$$

Adding $2y$ to both sides gives:

$$
x = 3 + 2y
$$

This expression allows us to replace x in the first equation with an equivalent expression involving y.

Step 2: Substitute into the Other Equation

Now that we have x expressed as $3 + 2y$, substitute this into Equation 1:

$$
2x + 2y = 5
$$

Replacing x:

$$
2(3 + 2y) + 2y = 5
$$

Distribute the 2:

$$
6 + 4y + 2y = 5
$$

Combine like terms:

$$
6 + 6y = 5
$$

Subtract 6 from both sides:

$$
6y = 5 - 6
$$
$$
6y = -1
$$

Divide both sides by 6:

$$
y = -\frac{1}{6}

$$
$$

At this stage, we have found the value of y.

Step 3: Find the Corresponding Value of x

Recall our earlier expression:

$$
x = 3 + 2y
$$

Substitute $y = -\frac{1}{6}$:

$$
x = 3 + 2 \times \left(-\frac{1}{6}\right)
$$

Calculate the multiplication:

$$
x = 3 - \frac{2}{6}
$$

Simplify $\frac{2}{6}$ to $\frac{1}{3}$:

$$
x = 3 - \frac{1}{3}
$$

Express 3 as a fraction with denominator 3:

$$
x = \frac{9}{3} - \frac{1}{3} = \frac{8}{3}
$$

Thus, the solution for the system is:

$$
x = \frac{8}{3}, \quad y = -\frac{1}{6}
$$

Step 4: Verify the Solution

Verification ensures that our solution satisfies both original equations.

Check Equation 1:

\[
2x + 2y = 5
\]
\[
2 \times \frac{8}{3} + 2 \times \left(-\frac{1}{6}\right) = 5
\]

Calculate:

\[
\frac{16}{3} - \frac{2}{6} = 5
\]

Simplify \(\frac{2}{6}\) to \(\frac{1}{3}\):

\[
\frac{16}{3} - \frac{1}{3} = 5
\]

Subtract numerator:

\[
\frac{15}{3} = 5
\]
\[
5 = 5
\]

Confirmed.

Check Equation 2:

\[
x - 2y = 3
\]

Substitute:

\[
\frac{8}{3} - 2 \times \left(-\frac{1}{6}\right) = 3
\]

Calculate:

\[
\frac{8}{3} + \frac{2}{6} = 3
\]

Simplify \(\frac{2}{6}\) to \(\frac{1}{3}\):

\[

$$
\frac{8}{3} + \frac{1}{3} = 3
$$

Add numerators:

$$
\frac{9}{3} = 3
$$

$$
3 = 3
$$

Again, the solution satisfies both equations.

The Final Solution Set

The systematic application of the substitution method yields the unique solution:

$$
\boxed{
\left(\frac{8}{3}, -\frac{1}{6} \right)
}
$$

This point in the coordinate plane is where both lines intersect, representing the solution to the system.

Broader Implications of the Method

The elimination by substitution method is particularly powerful when one of the equations can be easily solved for a variable. It simplifies solving systems manually and provides a clear pathway to the solution. This method is also foundational for understanding more complex algebraic systems, including those involving inequalities or systems in higher dimensions.

Advantages of the Method:

- Straightforward when one variable is isolated.
- Reduces a system of two equations to a single-variable problem.
- Facilitates verification of solutions.

Limitations:

- Less efficient if equations are complicated or not easily manipulated.
- Not as suitable when equations are nonlinear or involve higher powers.

Practical Applications

Understanding how to solve systems of equations using substitution extends beyond classroom exercises:

- Engineering: Calculating forces and stresses where multiple variables are involved.
- Economics: Determining equilibrium prices and quantities.
- Computer Science: Solving for variables in algorithms and data modeling.
- Physics: Computing intersection points of trajectories or wave functions.

Conclusion

The process of solving the system:

$$
2x + 2y = 5
$$
$$
x - 2y = 3
$$

via the method of elimination by substitution exemplifies a systematic approach to algebraic problem-solving. By isolating a variable, substituting, and simplifying, we found the unique solution $\left(\frac{8}{3}, -\frac{1}{6}\right)$. This method underscores the importance of strategic manipulation of equations and reinforces core algebraic principles that are crucial for both academic pursuits and real-world applications.

Mastering such techniques equips students and professionals alike with the tools to tackle complex systems efficiently, fostering logical thinking and analytical skills that transcend mathematics.

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