A First-order Reaction Is 58omplete At The End Of 11 Min. What Is The Value Of The Rate Constant?

A First-order Reaction Is 58% Complete At The End Of 11 Min. What Is The Value Of The Rate Constant?

Understanding reaction kinetics is fundamental in chemistry, especially when dealing with reaction rates and their governing parameters. One of the most common types of reactions studied is the first-order reaction, which has a rate proportional to the concentration of a single reactant. In this article, we explore how to determine the rate constant for a first-order reaction given specific data: that the reaction is 58% complete after 11 minutes. We will walk through the principles, formulas, and calculations step-by-step to find the rate constant (k), ensuring clarity and depth for learners and professionals alike.

Introduction to First-Order Reactions

What Is a First-Order Reaction?

A first-order reaction is a chemical process where the rate depends linearly on the concentration of a single reactant. The general form of a first-order reaction is:

$$\text{Rate} = k [A]$$

where:
- $[A]$ is the concentration of the reactant,
- k is the rate constant with units of inverse time (e.g., min^{-1}).

Characteristics of First-Order Reactions

- The half-life ($t_{1/2}$) is constant and independent of concentration.
- The concentration versus time graph is exponential.
- The integrated rate law takes a simple form, allowing easy calculation of reaction progress and rate constants.

Understanding the Data: 58% Completion at 11 Minutes

What Does 58% Completion Mean?

The term "58% complete" indicates that 58% of the reactant has reacted, leaving 42% remaining at the specified time. Mathematically, if:

$[A]_0$ = initial concentration, then
$$[A] = (1 - 0.58) \times [A]_0 = 0.42 \times [A]_0$$

This information allows us to relate concentration changes over time to the rate constant.

Implications for the Reaction Progress

- At $t = 11$ minutes, the concentration of reactant $[A]$ is 42% of the initial concentration.
- The goal is to find the rate constant k based on this data.

Mathematical Framework for First-Order Reactions

Integrated Rate Law for First-Order Reactions

The integrated rate law relates concentration and time:

$$\ln \left(\frac{[A]_0}{[A]} \right) = kt$$

where:
- $[A]_0$ = initial concentration,
- $[A]$ = concentration at time t,
- k = rate constant,
- t = time elapsed.

Rearranged for Rate Constant k

Solving for k:

$$k = \frac{1}{t} \times \ln \left(\frac{[A]_0}{[A]} \right)$$

Since $[A]_0$ and $[A]$ are known or expressed as fractions of the initial concentration, the calculation simplifies.

Step-by-Step Calculation of the Rate Constant

Step 1: Express Known Quantities

- Time $t = 11$ minutes
- Remaining concentration $[A] = 0.42 \times [A]_0$

Step 2: Plug into the Integrated Rate Law

Using the relation:

$$k = \frac{1}{t} \times \ln \left(\frac{[A]_0}{[A]} \right)$$
$$k = \frac{1}{11} \times \ln \left(\frac{[A]_0}{0.42 \times [A]_0} \right)$$

The $[A]_0$ cancels out:

$$k = \frac{1}{11} \times \ln \left(\frac{1}{0.42} \right)$$

Step 3: Calculate the Natural Logarithm

Calculate $\ln (1/0.42)$:

$$\frac{1}{0.42} \approx 2.38095$$
$$\ln(2.38095) \approx 0.8675$$

Step 4: Final Calculation

Now compute k:

$$k = \frac{1}{11} \times 0.8675$$
$$k \approx 0.07886 \, \text{min}^{-1}$$

Thus, the rate constant:

$\boxed{k \approx 0.0789 \, \text{min}^{-1}}$

Interpreting the Result and Additional Insights

Significance of the Rate Constant

- The rate constant k quantifies how quickly the reactant is consumed.
- A higher k indicates a faster reaction.
- In this case, approximately 7.89% of the reactant is converted per minute on average.

Half-Life for the Reaction

For a first-order reaction, the half-life $t_{1/2}$ is:

$$t_{1/2} = \frac{\ln 2}{k}$$

$$ t_{1/2} = \frac{0.693}{0.0789} \approx 8.78 \text{ minutes} $$

This aligns with the data, as roughly 58% completion at 11 minutes suggests a half-life close to 8.78 minutes.

Application in Chemical Engineering and Laboratory Settings

Knowing k allows chemists and engineers to:
- Predict how long a reaction will take to reach a desired conversion level.
- Design reactors and processes for optimal efficiency.
- Understand reaction mechanisms and compare different reaction pathways.

Factors Affecting the Rate Constant

Temperature

- Increasing temperature generally increases k due to the Arrhenius equation.
- Activation energy plays a crucial role in this temperature dependence.

Catalysts

- Catalysts can significantly increase k by providing alternative pathways.

Pressure and Concentration

- For first-order reactions, the rate constant is independent of concentration but sensitive to other factors like temperature.

Conclusion

Determining the rate constant for a first-order reaction based on reaction completion over time is a fundamental skill in chemical kinetics. Given that a reaction is 58% complete after 11 minutes, we utilized the integrated rate law to calculate the rate constant as approximately 0.0789 min^{-1}. This value not only helps in understanding the speed of the reaction but also aids in designing chemical processes, optimizing reaction conditions, and predicting reaction behavior. Mastery of these calculations reinforces a deeper understanding of reaction dynamics and their practical applications in science and industry.

References and Further Reading

- Atkins, P., & de Paula, J. (2014). Physical Chemistry. Oxford University Press.
- Laidler, K. J., Meiser, J. H., & Sanctuary, B. C. (1999). Physical Chemistry. Houghton Mifflin.
- Petrucci, R. H., Herring, F. G., Madura, J. D., & Bissonnette, C. (2017). General Chemistry: Principles & Modern Applications. Pearson Education.
- Online resources for chemical kinetics and rate law calculations.

Note: Always ensure units are consistent during calculations. In this case, minutes were used for time, but the principles hold for any unit of time, provided the units are consistent.

Frequently Asked Questions

What is a first-order reaction?

A first-order reaction is a chemical reaction where the rate depends directly on the concentration of a single reactant, with the rate law expressed as rate = k[A].

How is the rate constant (k) related to the reaction's half-life?

For a first-order reaction, the half-life ($t_1/_2$) is related to the rate constant by the equation $t_1/_2$ = 0.693 / k.

What is the significance of the reaction being 58% complete at 11 minutes?

It indicates that 58% of the reactant has reacted, and 42% remains, which can be used to determine the rate constant.

How can I calculate the rate constant (k) for this reaction?

Use the integrated rate law for first-order reactions: ln([A]_0 / [A]) = kt, where [A]_0 is the initial concentration and [A] is the concentration at time t.

What is the value of the remaining reactant concentration after 11 minutes?

Since 58% is complete, 42% of the reactant remains, so [A] = 0.42 [A]_0.

Can you provide the formula to calculate k given the

percentage completion?

Yes, k = (1 / t) ln([A]_0 / [A]) = (1 / t) ln(1 / remaining fraction).

What is the calculation for the rate constant in this case?

Using the data: remaining fraction = 0.42, time t = 11 minutes = 11 60 = 660 seconds. Then, k = (1 / 660) ln(1 / 0.42).

What is the approximate value of the rate constant (k)?

Calculating: k ≈ (1/660) ln(1 / 0.42) ≈ (0.001515) 0.8675 ≈ 0.001315 s^{-1}.

Why is understanding the rate constant important in kinetics?

The rate constant provides insight into the speed of the reaction and helps predict how quickly reactants are converted to products under certain conditions.

Additional Resources

Understanding the Rate Constant in a First-Order Reaction: A Detailed Analysis

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Introduction

In chemical kinetics, understanding the behavior of reactions over time is crucial for both theoretical insights and practical applications. Among the various types of reactions, first-order reactions are particularly significant due to their simplicity and prevalence in many chemical and biological processes. A key parameter in describing these reactions is the rate constant (k), which quantifies the speed at which a reactant transforms into products.

This discussion centers around a specific scenario: A first-order reaction is 58% complete at the end of 11 minutes. The goal is to determine the rate constant (k) based on this information. To do this thoroughly, we need to delve into the fundamental principles of first-order kinetics, interpret the given data, and apply the appropriate mathematical models.

Fundamental Concepts of First-Order Reactions

Definition and Characteristics

A first-order reaction is characterized by a rate that depends linearly on the concentration of a single reactant:

\[

$$
\text{Rate} = k[A]
$$

where:
- $[A]$ is the concentration of reactant A,
- k is the rate constant with units of min^{-1} (assuming time in minutes).

Key features:
- The rate is proportional to the concentration of the reactant.
- The integrated rate law is exponential in nature.
- The half-life ($t_{1/2}$) is constant and independent of initial concentration.

Integrated Rate Law for First-Order Reactions

The fundamental equation governing a first-order reaction is:

$$
\ln\left(\frac{[A]_0}{[A]}\right) = kt
$$

or equivalently,

$$
[A] = [A]_0 e^{-kt}
$$

where:
- $[A]_0$ is the initial concentration,
- $[A]$ is the concentration at time t,
- k is the rate constant,
- t is the elapsed time.

This exponential relation allows us to calculate k if we know the initial concentration, the concentration at a given time, or the fraction reacted.

Interpreting the Given Data: 58% Completion at 11 Minutes

Clarifying "Complete" and "58% Complete"

In the context of reaction progress:
- 58% completion indicates 58% of the reactant has reacted, or, equivalently,
- 42% of the reactant remains at $t = 11$ minutes.

Expressed mathematically:

$$
\frac{[A]}{[A]_0} = 1 - 0.58 = 0.42
$$

Thus, at $t = 11$ min:

$$
\frac{[A]}{[A]_0} = 0.42
$$

Calculating the Rate Constant k

Step 1: Apply the Integrated Rate Law

Using the relation:

$$
\ln\left(\frac{[A]_0}{[A]}\right) = kt
$$

Substitute the known ratio:

$$
\ln\left(\frac{1}{0.42}\right) = k \times 11
$$

Calculating the numerator:

$$
\frac{1}{0.42} \approx 2.38095
$$

then,

$$
\ln(2.38095) \approx 0.8675
$$

Step 2: Solve for k:

$$
k = \frac{0.8675}{11} \approx 0.07886\, \text{min}^{-1}
$$

Answer: The rate constant $k \approx 0.0789\, \text{min}^{-1}$.

Validating the Calculation: Additional Considerations

Checking Half-Life for Consistency

The half-life $t_{1/2}$ of a first-order reaction is:

$$
t_{1/2} = \frac{\ln 2}{k} \approx \frac{0.693}{0.0789} \approx 8.78\, \text{min}
$$

Given that after 11 minutes, 58% of the reactant has reacted, this aligns with the expected reaction progress:

- Approximately one half-life (~8.78 min) would have passed, reducing concentration from 100% to about 50%.
- The observed 42% remaining at 11 minutes indicates the reaction is slightly beyond one half-life, which is consistent with the calculated k.

Reaction Completion and Extent of Reaction

The data supports the understanding that the reaction proceeds exponentially, with a predictable decay of reactant concentration over time. The rate constant directly influences how quickly the reaction reaches a certain extent of completion.

Practical Applications and Implications

Understanding and calculating the rate constant is crucial in various fields:

- Pharmaceuticals: Predicting drug degradation rates.
- Environmental Chemistry: Modeling pollutant breakdown.
- Chemical Manufacturing: Designing reactors for optimal reaction times.
- Biochemistry: Understanding enzyme kinetics.

Knowing k allows scientists and engineers to estimate how long a reaction will take to reach a desired conversion level, optimize process conditions, and evaluate reaction mechanisms.

Broader Context and Related Concepts

Effect of Temperature on k

The rate constant is temperature-dependent, often described by the Arrhenius equation:

$$
k = A e^{-\frac{E_a}{RT}}
$$

where:
- A is the pre-exponential factor,
- E_a is the activation energy,
- R is the gas constant,
- T is the temperature in Kelvin.

This highlights that even a small change in temperature can significantly alter k, affecting reaction rates.

Reaction Order and Its Identification

While this analysis assumes a first-order process, it's important to verify reaction order experimentally:

- Plotting $\ln [A]$ versus t yields a straight line for first-order reactions.
- Deviations from linearity suggest alternative kinetic orders.

Summary of Key Points

- First-order reactions follow the integrated rate law: $\ln([A]_0 / [A]) = kt$.

- Given that 58% of the reaction is complete at 11 minutes, the remaining reactant concentration is 42%.
- The rate constant \(k\) is calculated as approximately 0.0789 min\(^{-1}\).
- This value aligns with the expected reaction progress based on half-life calculations.
- Understanding \(k\) aids in process design, reaction control, and mechanistic insights.

Final Remarks

The process of determining the rate constant from experimental data exemplifies the power of chemical kinetics in predicting and controlling reactions. The ability to relate observable quantities like reaction completion percentage and time to fundamental parameters like \(k\) underpins much of chemical research and industrial application. Mastery of these concepts enables chemists and engineers to design efficient, safe, and predictable chemical processes.

In conclusion, the calculation of the rate constant from the given data demonstrates the practical application of first-order kinetics principles and highlights their importance across scientific disciplines.

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