A Line Has A Y-intercept Of -11 And Slope Of 0.25. Write Its Equation In Slope-intercept Form.

A Line Has A Y-intercept Of -11 And Slope Of 0.25. Write Its Equation In Slope-intercept Form. Understanding how to formulate the equation of a line given specific characteristics is a fundamental skill in algebra and coordinate geometry. When you're provided with the slope and the y-intercept, translating that information into the equation of the line in slope-intercept form is a straightforward process. This article will guide you through the steps to derive the equation, explain the concepts involved, and explore related topics to deepen your understanding of linear equations.

Understanding the Slope-Intercept Form of a Line

What Is the Slope-Intercept Form?

The slope-intercept form of a linear equation is one of the most common ways to express a line. It is written as:

\[y = mx + b \]

where:

- m is the slope of the line, indicating its steepness and direction.
- b is the y-intercept, the point where the line crosses the y-axis.

This form is particularly useful because it immediately provides key information about the line's behavior and position on the coordinate plane.

The Significance of Slope and Y-intercept

- Slope (m): Reflects how much y changes for a unit change in x. A positive slope means the line rises as x increases, while a negative slope means it falls.
- Y-intercept (b): The value of y when x=0. It marks the point where the line intersects the y-axis.

Knowing both allows you to graph the line accurately and understand its properties without additional calculations.

Given Data and Its Implications

Provided Values

In this scenario, you're told:
- The y-intercept (b) is -11.
- The slope (m) is 0.25.

These values directly translate into the parameters for the line's equation, making the process of writing the equation straightforward.

Interpreting the Data

- The y-intercept of -11 indicates the line crosses the y-axis at the point (0, -11).
- The slope of 0.25 means that for every increase of 1 unit in x, y increases by 0.25 units. This gentle positive slope suggests the line rises slowly as x increases.

Formulating the Equation

Step-by-Step Process

1. Identify the slope (m): Given as 0.25.

2. Identify the y-intercept (b): Given as -11.

3. Substitute into the slope-intercept form:

$$ y = mx + b $$

$$ y = 0.25x - 11 $$

This is the equation of the line in slope-intercept form.

Final Equation

$$
\boxed{
\text{The equation of the line is } y = 0.25x - 11
}
$$

This simple yet powerful form allows anyone to graph the line easily or analyze its properties further.

Graphing the Line

Plotting Key Points

To graph the line, it's helpful to identify points that satisfy the equation:

- Y-intercept point: (0, -11)

- Another point using the slope:
- Starting from (0, -11), increase x by 4 (since the slope is 0.25, which is 1/4).
- When x = 4, y = 0.25(4) - 11 = 1 - 11 = -10.
- So, another point is (4, -10).

Plotting these points and drawing a straight line through them will accurately represent the line.

Understanding the Graph

- The line crosses the y-axis at -11.
- For every 4 units rightward along the x-axis, the line rises by 1 unit on the y-axis.
- The slope indicates a gentle incline.

Real-World Applications of the Line Equation

Modeling Situations with Linear Relationships

Lines with known slopes and intercepts are used to model various real-world scenarios, such as:
- Economics: Calculating profit or cost functions.
- Physics: Describing constant velocity motion.
- Biology: Modeling growth rates.

For example, if a company's profit increases by $0.25 for each additional unit sold, and the fixed costs are $11 (represented negatively as a y-intercept), the profit function can be modeled by the line $y = 0.25x - 11$.

Predicting and Analyzing Data

Knowing the equation allows for predictions. For instance, determining how much profit will be when 10 units are sold:

$$ y = 0.25 \times 10 - 11 = 2.5 - 11 = -8.5 $$

indicating a loss at that sales level, which might prompt further business analysis.

Additional Concepts Related to the Line Equation

Alternative Forms of a Line Equation

While the slope-intercept form is convenient, other forms include:

- Point-slope form: $ y - y_1 = m(x - x_1) $
- Standard form: $ Ax + By = C $

Each serves specific purposes depending on the context.

Converting to Standard Form

Starting from $ y = 0.25x - 11 $, multiply both sides by 4 to clear fractions:

$$ 4y = x - 44 $$

or rewrite as:

$$ x - 4y = 44 $$

Understanding different forms enhances flexibility in solving and graphing lines.

Conclusion

In summary, given a y-intercept of -11 and a slope of 0.25, the equation of the line in slope-intercept form is \(y = 0.25x - 11 \). This equation encapsulates the line's steepness and position, enabling easy graphing, analysis, and application in real-world scenarios. Mastery of translating given information into a linear equation is vital for students and professionals working with data, modeling, and mathematical analysis. Whether you're studying basic algebra or applying these concepts to complex problems, understanding how to derive and interpret line equations is an essential skill that underpins much of mathematics and science.

Frequently Asked Questions

What is the slope of the line with a y-intercept of -11 and a slope of 0.25?

The slope of the line is 0.25.

What is the y-intercept of the line described?

The y-intercept of the line is -11.

How do you write the equation of a line in slope-intercept form?

The equation is written as y = mx + b, where m is the slope and b is the y-intercept.

What is the slope-intercept form of the line with slope 0.25 and y-intercept -11?

The equation is y = 0.25x - 11.

If a line has a slope of 0.25 and passes through the point (0, -11), what is its equation?

Its equation is y = 0.25x - 11.

How does the y-intercept affect the graph of the line?

The y-intercept determines where the line crosses the y-axis, which in this case is at -11.

What does a slope of 0.25 indicate about the line's steepness?

A slope of 0.25 indicates the line rises 0.25 units vertically for every 1 unit horizontally, representing a gentle incline.

Can you find the x-value when y is zero for this line?

Yes. Set y = 0 in y = 0.25x - 11 and solve for x: 0 = 0.25x - 11 => 0.25x = 11 => x = 44.

What is the significance of the slope and y-intercept in understanding the line's behavior?

The slope indicates the line's incline, and the y-intercept shows where it crosses the y-axis, together defining the line's position and direction.

Write the final equation of the line in slope-intercept form given the slope and y-intercept.

The equation is y = 0.25x - 11.

Additional Resources

Understanding the Equation of a Line with a Given Y-Intercept and Slope

When exploring the fundamentals of algebra and coordinate geometry, one of the most essential concepts is understanding how to represent a line mathematically. The slope-intercept form, expressed as y = mx + b, offers a straightforward way to describe a line's behavior using two key parameters: the slope (m) and the y-intercept (b). In this detailed review, we will analyze a specific case: a line that has a y-intercept of -11 and a slope of 0.25. We will explore the meaning of these parameters, how to derive the equation, and the implications for graphing and interpreting the line.

Fundamentals of the Slope-Intercept Form

What is the Slope-Intercept Form?

The slope-intercept form of a line's equation is written as:

\[y = mx + b \]

Where:
- m (the slope) indicates the steepness and direction of the line.
- b (the y-intercept) indicates the point where the line crosses the y-axis.

This form is particularly useful because it immediately reveals the line's slope and y-intercept, enabling quick graphing and analysis.

Significance of Slope (m)

- The slope m measures how much y changes for a unit change in x.
- A positive slope indicates the line rises from left to right.
- A negative slope indicates the line falls from left to right.
- A slope of zero (0) indicates a horizontal line.
- An undefined slope (not applicable here) indicates a vertical line.

Significance of Y-Intercept (b)

- The y-intercept is the point (0, b) on the coordinate plane where the line crosses the y-axis.
- It provides the starting point for graphing the line.
- The value of b can be positive, negative, or zero, influencing where on the y-axis the line begins.

Given Parameters: Y-Intercept of -11 and Slope of 0.25

Interpreting the Y-Intercept: -11

- The y-intercept being -11 means the line crosses the y-axis at the point (0, -11).
- This point is crucial because it anchors the line's position on the coordinate plane.
- Graphically, this is a point slightly below the origin, since -11 is significantly negative.

Interpreting the Slope: 0.25

- The slope of 0.25 indicates that for each increase of 1 unit in x, y increases by 0.25 units.
- The decimal form (0.25) can be expressed as a fraction: $\frac{1}{4}$.
- This means the line rises one unit vertically for every four units horizontally, indicating a gentle, moderate incline.

Constructing the Equation in Slope-Intercept Form

Given:

- Slope, $m = 0.25$
- Y-intercept, $b = -11$

The equation becomes:

$$y = 0.25x - 11$$

This is the explicit, algebraic representation of the line, capturing both its inclination and its position relative to the y-axis.

Step-by-Step Derivation

1. Recognize the general form: $y = mx + b$.
2. Substitute the given slope: $y = 0.25x + b$.
3. Use the given y-intercept point (0, -11) to solve for b:

$$
y = 0.25(0) + b \Rightarrow -11 = 0 + b \Rightarrow b = -11
$$

4. Final equation:

$$
y = 0.25x - 11
$$

Graphical Interpretation and Visualization

Plotting Key Points

- Y-intercept: (0, -11) is straightforward to plot.
- Another point (using the slope):

For $x = 4$:

$$
y = 0.25(4) - 11 = 1 - 11 = -10
$$

So, the point (4, -10) lies on the line.

- Additional point:

For $x = -4$:

$$
y = 0.25(-4) - 11 = -1 - 11 = -12
$$

The point (-4, -12) also lies on the line.

These points help in sketching an accurate graph of the line.

Characteristics of the Graph

- The line is steadily inclined, rising gently from left to right.
- It crosses the y-axis at -11.
- The slope of 0.25 indicates a relatively gentle incline, emphasizing the slow increase in y as x increases.
- The line extends infinitely in both directions, maintaining the same slope and intercept throughout.

Visualizing the Line

Imagine starting at the point (0, -11) on the y-axis. Moving horizontally 4 units to the right (from $x=0$ to $x=4$), you move vertically 1 unit up (from $y=-11$ to $y=-10$). Connecting these points and extending the line in both directions illustrates the consistent slope and intercept.

Mathematical and Real-World Applications

Understanding the Line's Behavior

- Rate of Change: The slope of 0.25 indicates a slow increase, perhaps representing a scenario such as gradual growth or accumulation over time.
- Intercept Significance: The negative y-intercept suggests the line starts below the x-axis, perhaps modeling a situation where an initial value is negative or below a baseline.

Potential Scenarios

- Economics: Modeling revenue growth starting from a deficit of -\$11, with a profit increase of \$0.25 per unit time.
- Physics: Describing a slow ascent in altitude over distance, beginning from a point below sea level.
- Environmental Science: Tracking pollutant levels decreasing or increasing over a distance or time.

Implications for Data Analysis

- The equation provides a simple, linear model for predicting y values based on x.
- Its parameters can be estimated from data points, and the model can be used for forecasting or understanding trends.

Further Mathematical Considerations

Slope as a Fraction

- Expressed as $\frac{1}{4}$, clarifies the ratio of vertical to horizontal change.
- Useful for understanding the "rise over run" concept visually.

Slope Significance in Different Contexts

- Positive slope: Indicates a direct relationship.
- Magnitude: The size of the slope affects the steepness; here, 0.25 indicates gentle inclination.

Y-Intercept in Context

- The value of -11 can be interpreted as the starting point or initial condition of the modeled situation.

Line Equation Variations

- If the slope were negative, the line would descend from left to right.

- If the intercept were zero, the line would pass through the origin.

Conclusion: Mastering the Equation $y = 0.25x - 11$

In summary, the process of deriving the line's equation from the given parameters involves understanding the roles of slope and intercept and translating that understanding into algebraic form. The line described by $y = 0.25x - 11$ offers insight into how the parameters influence its position and inclination, providing a foundation for graphing, analysis, and real-world modeling.

By grasping these principles, students and professionals alike develop a deeper appreciation of linear relationships and their practical applications. Whether used in academic exercises, scientific research, or everyday problem-solving, mastering the derivation and interpretation of such equations is a fundamental skill in mathematics and beyond.

In essence, the line with a y-intercept of -11 and slope of 0.25 is succinctly captured by the equation $y = 0.25x - 11$. This simple yet powerful formula encapsulates the line's behavior, offering a window into its graphical representation, mathematical properties, and real-world implications.

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