A Quadratic Function F Is Defined By F(x)=(x-7)(x+3). Identify The X-intercepts Of The Graph Of F

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Understanding quadratic functions is fundamental in algebra and helps us analyze the behavior of parabolas, their roots, and their intercepts. In this article, we will explore the quadratic function defined by F(x) = (x - 7)(x + 3), focusing primarily on identifying the x-intercepts of its graph. We will break down the process step-by-step, covering key concepts such as factored forms, x-intercepts, and how to find them.

What Is a Quadratic Function?

A quadratic function is a polynomial function of degree 2, typically expressed in the form:

- Standard form: $ax^2 + bx + c$
- Factored form: $a(x - r_1)(x - r_2)$
- Vertex form: $a(x - h)^2 + k$

In the context of our specific function, F(x) = (x - 7)(x + 3), the function is given in factored form. This form is particularly useful for quickly identifying the roots or x-intercepts of the quadratic function.

Understanding the Given Function: F(x) = (x - 7)(x + 3)

The function F(x) is expressed as the product of two binomials. Each binomial contains a linear expression involving x. To analyze this function, especially to find its x-intercepts, it is essential to understand what the roots of the function are.

Factored Form and Its Significance

The factored form of a quadratic function directly reveals its roots. Roots are the values of x for which the function evaluates to zero, i.e., F(x) = 0.

Given F(x) = (x - 7)(x + 3), for F(x) to be zero:

$(x - 7)(x + 3) = 0$

Applying the Zero Product Property (which states that if a product of two factors equals zero, then at least one of the factors must be zero), we get:

- $x - 7 = 0$
- $x + 3 = 0$

Solving these simple equations yields the roots of the quadratic function.

Finding the X-Intercepts of the Graph of F

The x-intercepts of a graph are the points where the graph crosses or touches the x-axis. These points occur where the output value, F(x), is zero. Therefore, finding the x-intercepts involves solving F(x) = 0.

Step-by-Step Process to Find the X-Intercepts

1. Set the function equal to zero:

$F(x) = (x - 7)(x + 3) = 0$

2. Apply the Zero Product Property:

Either $(x - 7) = 0$ or $(x + 3) = 0$

3. Solve each equation:

- $x - 7 = 0 \rightarrow x = 7$
- $x + 3 = 0 \rightarrow x = -3$

4. Identify the x-intercepts:

The x-intercepts are located at $x = 7$ and $x = -3$.

Therefore, the graph of the quadratic function crosses the x-axis at the points (7, 0) and (-3, 0).

Interpreting the X-Intercepts in Context

The roots of the quadratic function provide vital information about the graph's behavior and the solutions to the related equations.

- At $x = 7$, the graph intersects the x-axis at the point (7, 0). This indicates that when x is 7, F(x) equals zero.
- At $x = -3$, the graph intersects the x-axis at the point (-3, 0). This indicates that when x is -3, F(x)

equals zero.

These points are critical in sketching the graph and understanding the parabola's shape and position.

Additional Insights: Vertex and Axis of Symmetry

While our primary focus is on x-intercepts, understanding other features of the parabola enhances comprehension.

Finding the Vertex

The vertex of a parabola in factored form can be found by calculating the midpoint between the roots (x-intercepts).

- Midpoint formula: $(x_1 + x_2) / 2$

Applying this:

(7 + (-3)) / 2 = (4) / 2 = 2

The x-coordinate of the vertex is at x = 2.

To find the y-coordinate of the vertex, substitute x = 2 into the original function:

F(2) = (2 - 7)(2 + 3) = (-5)(5) = -25

Thus, the vertex of the parabola is at (2, -25).

Axis of Symmetry

The axis of symmetry is the vertical line that passes through the vertex:

x = 2

This line divides the parabola into two symmetric halves.

Converting the Function to Standard Form

While the factored form is excellent for identifying roots, converting to standard form provides additional insights such as the y-intercept and the parabola's shape.

Starting with:

$F(x) = (x - 7)(x + 3)$

Apply the distributive property (FOIL method):

$F(x) = x x + x 3 - 7 x - 7 3$

$F(x) = x^2 + 3x - 7x - 21$

Combine like terms:

$F(x) = x^2 - 4x - 21$

Standard form of the quadratic: $F(x) = x^2 - 4x - 21$

From this form, the y-intercept occurs at $x = 0$:

$F(0) = 0 - 0 - 21 = -21$

So, the graph crosses the y-axis at (0, -21).

Graphing the Quadratic Function

Using the information obtained, one can sketch the graph of F(x):

- The x-intercepts at (-3, 0) and (7, 0)
- The vertex at (2, -25)
- The y-intercept at (0, -21)
- The axis of symmetry at $x = 2$

Plotting these points and drawing a parabola opening upwards (since the coefficient of x^2 is positive) provides a clear visual of the function's behavior.

Applications of Finding X-Intercepts

Identifying x-intercepts is a fundamental skill with numerous applications across various fields:

- Physics: Determining points where an object reaches a certain position or velocity.
- Economics: Finding break-even points in profit or loss analysis.
- Engineering: Analyzing systems where certain variables reach specific thresholds.
- Biology: Modeling population dynamics where populations reach zero.

Summary and Key Takeaways

- The quadratic function $F(x) = (x - 7)(x + 3)$ is in factored form, making the identification of roots

straightforward.
- The x-intercepts are at x = 7 and x = -3, corresponding to the points where the graph crosses the x-axis.
- Converting to standard form aids in understanding additional features like the vertex and y-intercept.
- Recognizing the symmetry and shape of the parabola helps in graphing and interpreting the function.
- The process of finding x-intercepts involves setting the function equal to zero and solving for x, leveraging the Zero Product Property.

Conclusion

The quadratic function F(x) = (x - 7)(x + 3) exemplifies how factored form simplifies the process of identifying x-intercepts. By setting the function equal to zero and applying fundamental algebraic principles, we find the roots at x = 7 and x = -3. These intercepts are critical in sketching the graph and understanding the behavior of the quadratic function. Whether used in academic settings or real-world applications, mastering the process of finding x-intercepts enhances problem-solving skills and deepens comprehension of quadratic functions.

Meta Description: Learn how to identify the x-intercepts of the quadratic function F(x) = (x - 7)(x + 3). Step-by-step guide to solving for roots, understanding the graph, and interpreting key features of the parabola.

Frequently Asked Questions

What are the x-intercepts of the quadratic function F(x) = (x - 7)(x + 3)?

The x-intercepts are x = 7 and x = -3.

How do you find the x-intercepts of a quadratic function given in factored form?

Set each factor equal to zero and solve for x; for F(x) = (x - 7)(x + 3), the x-intercepts are x = 7 and x = -3.

What is the significance of the x-intercepts in the graph of a quadratic function?

The x-intercepts are the points where the graph crosses the x-axis, indicating the roots or solutions of the function.

Can the quadratic function F(x) = (x - 7)(x + 3) have any x-intercepts other than 7 and -3?

No, because in factored form, the roots are directly given by the factors set to zero, which are x = 7 and x = -3.

How does the factored form of a quadratic help in quickly identifying x-intercepts?

The factored form explicitly shows the roots as the solutions to each factor set to zero, making it straightforward to find the x-intercepts.

What is the vertex of the quadratic function F(x) = (x - 7)(x + 3)?

The vertex can be found by calculating the axis of symmetry at x = (7 + (-3))/2 = 2, and then plugging x = 2 into the function to find the y-coordinate.

How does the quadratic function F(x) = (x - 7)(x + 3) open (upward or downward)?

Since the quadratic is written in factored form with a positive leading coefficient (implied 1), it opens upward.

What is the importance of identifying x-intercepts in quadratic functions in real-world applications?

X-intercepts help determine solutions to equations modeled by quadratics, such as maximum profit, projectile landing points, or break-even points in business scenarios.

If the quadratic function F(x) = (x - 7)(x + 3) is graphed, where does it cross the x-axis?

It crosses the x-axis at x = 7 and x = -3.

How can you verify the x-intercepts of the quadratic function F(x) = (x - 7)(x + 3)?

Set F(x) = 0 and solve: (x - 7)(x + 3) = 0, which gives x = 7 and x = -3, confirming the x-intercepts.

Additional Resources

A Quadratic Function F Is Defined By F(x) = (x - 7)(x + 3). Identify The X-intercepts Of The Graph Of F

Understanding the behavior of quadratic functions is fundamental in algebra and analytic geometry, serving as a cornerstone for more advanced mathematical concepts. The function F(x) = (x - 7)(x + 3) exemplifies a quadratic expressed in factored form, offering clear insights into its roots, symmetry, and graph. This article aims to thoroughly explore the characteristics of this particular quadratic, with a primary focus on identifying its x-intercepts, while also delving into the broader implications and applications of quadratic functions.

Introduction to Quadratic Functions

Quadratic functions are polynomial functions of degree two, generally expressed in the form:

$$ y = ax^2 + bx + c $$

where $a \neq 0$. These functions are renowned for their parabolic graphs, which open upwards when $a > 0$ and downwards when $a < 0$. The graph's shape, key points, and intercepts reveal much about the function’s properties.

Expressing a quadratic in factored form, like $F(x) = (x - r_1)(x - r_2)$, directly highlights its roots or x-intercepts — the points where the graph crosses the x-axis. This form also makes it straightforward to analyze the function's zeros, vertex, and symmetry.

Understanding the Given Function

The function in question is:

$$ F(x) = (x - 7)(x + 3) $$

This factorized form succinctly indicates that the quadratic is built from two linear factors, each corresponding to a root of the function.

Key features of this function include:

- Factored form: $(x - 7)(x + 3)$
- Degree: 2 (quadratic)
- Leading coefficient: 1 (since the factors are monic)
- Constant term: $(7)(-3) = -21$

By analyzing this form, we can immediately infer several important properties about the graph.

Finding the X-Intercepts of F(x)

X-intercepts, also known as roots or zeros, are the points where the graph intersects the x-axis. At these points, the output of the function is zero:

$$F(x) = 0$$

Given the factored form:

$$(x - 7)(x + 3) = 0$$

we can use the Zero Product Property, which states that if the product of two factors is zero, then at least one of the factors must be zero. Applying this principle:

1. Set each factor equal to zero:

- $x - 7 = 0 \Rightarrow x = 7$
- $x + 3 = 0 \Rightarrow x = -3$

2. These solutions, $x = 7$ and $x = -3$, are the x-intercepts.

Therefore, the graph of $F(x)$ crosses the x-axis at:

- Point 1: $(7, 0)$
- Point 2: $(-3, 0)$

These intercepts serve as critical markers for understanding the shape and position of the parabola.

Graphical Interpretation and Significance of the X-Intercepts

The x-intercepts are more than just solutions; they provide insight into the behavior of the quadratic function.

1. Location on the Cartesian Plane

- The intercept at $(-3, 0)$ lies to the left of the y-axis.
- The intercept at $(7, 0)$ lies to the right of the y-axis.

This symmetric positioning about the parabola's vertex (which we'll analyze later) reflects the function's inherent symmetry, a hallmark of quadratic graphs.

2. Roots and Factorization

- Since the roots are real and distinct, the graph is a parabola that crosses the x-axis at two separate points.
- The roots' values are integral, which simplifies analysis and visualization.

3. Implications for the Graph's Shape

- The parabola opens upward because the leading coefficient (which would be derived from expanded form) is positive.
- The x-intercepts mark the points where the parabola intersects the x-axis, defining the interval over

which the quadratic is negative or positive.

4. Applications

Knowing the x-intercepts is vital in numerous real-world contexts, such as:

- Physics: Determining points where motion or force equations cross a certain threshold.
- Economics: Finding break-even points in profit or cost functions.
- Engineering: Locating points of equilibrium or zero displacement.

Converting to Standard and Vertex Form

While the factored form directly yields the roots, understanding the quadratic's complete behavior benefits from converting it into standard and vertex forms.

1. Expanding to Standard Form

$$ F(x) = (x - 7)(x + 3) $$

Apply the distributive property (FOIL):

$$ F(x) = x \times x + x \times 3 - 7 \times x - 7 \times 3 $$
$$ F(x) = x^2 + 3x - 7x - 21 $$
$$ F(x) = x^2 - 4x - 21 $$

The standard form:

$$ y = x^2 - 4x - 21 $$

2. Finding the Vertex

The vertex of a parabola provides the maximum or minimum point. For a quadratic $ y = ax^2 + bx + c $, the x-coordinate of the vertex is:

$$ x_v = -\frac{b}{2a} $$

Using the standard form:

- $ a = 1 $
- $ b = -4 $

Calculate:

$$ x_v = -\frac{-4}{2 \times 1} = \frac{4}{2} = 2 $$

Next, find the y-coordinate by substituting $ x = 2 $:

$$ y_v = (2)^2 - 4(2) - 21 = 4 - 8 - 21 = -25 $$

Thus, the vertex is at:

$$ (2, -25) $$

The vertex lies below the x-axis, indicating that the parabola opens upward and the minimum point is at $(2, -25)$.

3. Axis of Symmetry

The parabola's axis of symmetry passes through the vertex:

$$ x = 2 $$

This line divides the parabola into two mirror-image halves.

Broader Context and Analytical Significance

The roots and the vertex are fundamental in understanding the behavior of any quadratic function. For the specific function $F(x) = (x - 7)(x + 3)$:

- The roots at $x = -3$ and $x = 7$ define the intervals where the function is positive or negative.

- The vertex at $(2, -25)$ indicates the minimum point, which is crucial in optimization scenarios.

- The parabola's symmetry about the axis $x=2$ reveals the inherent geometric properties of quadratic functions.

Implications for Modeling and Applications:

- Real-world modeling: The roots can correspond to physical points or thresholds, such as the time when an object hits the ground or a profit margin is broken even.
- Design considerations: Engineers and designers can utilize these intercepts to ensure systems operate within desired parameters.
- Mathematical analysis: The roots and vertex are essential in calculus for finding maximum/minimum values and analyzing function behavior.

Conclusion: The Significance of the X-Intercepts

The x-intercepts of the quadratic function $F(x) = (x - 7)(x + 3)$ are critical in understanding its geometric and algebraic properties. They are explicitly given by the solutions to $(x - 7)(x + 3) = 0$, which are $x = 7$ and $x = -3$. These points not only facilitate graphing but also serve as foundational data for applications across diverse fields such as physics, economics, and engineering.

By converting the function into standard and vertex forms, we gain deeper insight into its shape and key features, including the vertex at $(2, -25)$ and the axis of symmetry at $x=2$. The confluence of algebraic solutions and geometric interpretations exemplifies the power of quadratic functions in modeling real-world phenomena and solving practical problems.

In summation, the x-intercepts are more than mere solutions; they are vital landmarks that enable mathematicians, scientists, and engineers to interpret, analyze, and utilize quadratic functions effectively. As demonstrated by this particular function, the roots at \(x = -3 \) and \(x = 7 \) encapsulate the essence of quadratic analysis, bridging algebraic solutions with visual comprehension.

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