A Straight Line Passes Through The Point (-1,2) And (2,8). Find The Equation Of Thestraight Line.

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Understanding how to find the equation of a straight line passing through two given points is a fundamental concept in coordinate geometry. Whether you're a student preparing for exams or a professional dealing with geometric problems, mastering this process is essential. In this comprehensive guide, we will explore the step-by-step method to determine the equation of the straight line passing through the points (-1, 2) and (2, 8). We will also delve into related concepts such as the slope, point-slope form, slope-intercept form, and how to interpret these equations. By the end of this article, you'll be equipped with the knowledge to solve similar problems confidently and efficiently.

Understanding the Coordinates of the Given Points

Before diving into the calculations, it's important to understand the given points and their significance in the coordinate plane.

Points Provided:

- Point 1: (-1, 2)
- Point 2: (2, 8)

Each point is represented by an ordered pair (x, y), where:
- The first value (x) indicates the position along the horizontal axis.
- The second value (y) indicates the position along the vertical axis.

Knowing these points allows us to determine the slope of the line and subsequently find its equation.

Calculating the Slope of the Line

The slope (m) of a line measures its steepness and direction. It is calculated using the coordinates of two points through the formula:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

where:
- (x_1, y_1) are the coordinates of the first point.
- (x_2, y_2) are the coordinates of the second point.

Applying the Formula

For points (-1, 2) and (2, 8):

$$
x_1 = -1, \quad y_1 = 2
$$
$$
x_2 = 2, \quad y_2 = 8
$$

Calculating the slope:

$$
m = \frac{8 - 2}{2 - (-1)} = \frac{6}{3} = 2
$$

Result: The slope of the line is 2.

Understanding the slope is crucial because it determines the angle of the line and how it progresses across the coordinate plane.

Formulating the Equation of the Line

With the slope known, the next step is to derive the algebraic equation representing the line. Several forms are used in mathematics, including:

- Point-slope form
- Slope-intercept form
- Two-point form

We'll explore each and see how to derive the equation using the point-slope form, which is particularly useful when you know a point and the slope.

Using the Point-Slope Form

The point-slope form of a line's equation is:

$$
y - y_1 = m(x - x_1)
$$

Where:
- (x_1, y_1) is a known point on the line.
- m is the slope.

Choosing a Point: Let's use the point (-1, 2). Substituting the values:

$$
y - 2 = 2(x - (-1))
$$

Simplify the expression:

$$
y - 2 = 2(x + 1)
$$

Now expand the right side:

$$
y - 2 = 2x + 2
$$

Finally, solve for y:

$$
y = 2x + 2 + 2
$$
$$
y = 2x + 4
$$

Result: The equation of the line passing through the points (-1, 2) and (2, 8) is:

$$
\boxed{y = 2x + 4}
$$

Verification with the Second Point

To ensure accuracy, substitute the second point (2, 8) into the equation:

$$
y = 2(2) + 4 = 4 + 4 = 8
$$

Since it satisfies the equation, our derivation is correct.

Understanding Different Forms of the Equation

The line's equation can be expressed in various forms, depending on the context or preference.

Slope-Intercept Form

The slope-intercept form is:

$$
y = mx + c
$$

where:
- m is the slope.
- c is the y-intercept (the point where the line crosses the y-axis).

From the previous calculation:

$$
y = 2x + 4
$$

Thus, the y-intercept $c = 4$.

Standard Form

The standard form of a line's equation is:

$$
Ax + By + C = 0
$$

Convert $y = 2x + 4$ into standard form:

$$
y - 2x = 4
$$

or

$$
-2x + y - 4 = 0

\]

Multiplying throughout by -1 for clarity:

$$
2x - y + 4 = 0
$$

Note: The standard form is often used for geometric proofs and algebraic manipulations.

Graphical Representation of the Line

Plotting the line helps visualize its slope and position in the coordinate plane.

Steps to graph:

1. Identify the y-intercept: The point (0, 4).
2. Use the slope: Rise over run is 2, so from the y-intercept, move up 2 units and right 1 unit to get another point (1, 6).
3. Plot these points: (-1, 2), (0, 4), and (2, 8).
4. Draw a straight line passing through these points.

This visual representation confirms the correctness of the equation.

Applications of the Equation of a Straight Line

Understanding the equation of a line passing through two points has broad applications:

- Geometry and Trigonometry: Analyzing slopes and angles.
- Physics: Describing motion with linear relationships.
- Economics: Modeling cost or revenue functions.
- Engineering: Designing linear systems and circuits.
- Computer Graphics: Rendering lines and shapes.

Common Mistakes to Avoid

While calculating and deriving the line's equation, be cautious of:

- Incorrect slope calculation: Always subtract the y-values from top to bottom and x-values from left to right.
- Sign errors: Pay attention to negative signs, especially when points have negative coordinates.
- Using the wrong form: Choose the appropriate form based on the problem context.
- Forgetting to verify: Always substitute the second point to verify the derived equation.

Summary and Key Takeaways

- To find the equation of a line passing through two points, first compute the slope using the slope formula:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

- Use the point-slope form with either of the two points:

$$
y - y_1 = m(x - x_1)
$$

- Simplify to get the slope-intercept form ($y = mx + c$) for easier interpretation.
- Confirm the equation by substituting the second point.
- Recognize different forms of the equation and their applications.
- Visualize the line graphically for better understanding.

In conclusion, the equation of the straight line passing through (-1, 2) and (2, 8) is $y = 2x + 4$. This process exemplifies fundamental techniques in coordinate geometry, essential for solving a wide range of mathematical and real-world problems.

Meta Description: Learn how to find the equation of a straight line passing through points (-1, 2) and (2, 8). Step-by-step guide with formulas, verification, and applications in coordinate geometry.

Frequently Asked Questions

How do you find the equation of a straight line passing through two points (-1, 2) and (2, 8)?

First, calculate the slope using m = (8 - 2) / (2 - (-1)) = 6 / 3 = 2. Then, use point-slope form with one of the points, say (-1, 2): y - 2 = 2(x + 1). Simplify to get y = 2x + 4.

What is the slope of the line passing through the points (-1, 2) and (2, 8)?

The slope is calculated as (8 - 2) / (2 - (-1)) = 6 / 3 = 2.

How do I write the equation of a line in slope-intercept

form given two points?

Calculate the slope between the points, then substitute one point into y = mx + c to solve for c. The resulting equation will be y = 2x + 4 for these points.

Can I verify the line passes through both points after finding the equation y = 2x + 4?

Yes. Substitute x = -1: y = 2(-1) + 4 = 2, which matches the point (-1, 2). Substitute x = 2: y = 2(2) + 4 = 8, matching (2, 8).

What is the general method to find the equation of a line passing through two points?

Find the slope using the two points, then use the point-slope form to write the equation, and simplify to slope-intercept form if needed.

Is the line passing through (-1, 2) and (2, 8) unique?

Yes. Given two distinct points, there is exactly one straight line passing through both.

What is the importance of the point-slope form in deriving the line's equation?

The point-slope form allows you to write the equation of a line easily once you know the slope and a point on the line, making it straightforward to find the complete equation.

Additional Resources

Straight Line Equation Through Two Points

Introduction

When exploring the fascinating world of coordinate geometry, one of the fundamental concepts is understanding how to find the equation of a straight line passing through two given points. This is a cornerstone principle that finds applications across various fields—from engineering and physics to computer graphics and navigation systems. Today, we'll delve into a specific problem: finding the equation of a straight line passing through the points (-1, 2) and (2, 8).

This problem may seem straightforward at first glance, but it offers an excellent opportunity to explore core concepts such as the slope of a line, point-slope form, and slope-intercept form. Our comprehensive analysis will guide you through each step, providing clarity and insight into the process.

Understanding the Problem

Given two points:

- Point 1: (-1, 2)
- Point 2: (2, 8)

Our goal is to find the equation of the straight line that passes through both these points. The line's equation should be expressed in a standard and widely used form, such as the slope-intercept form or point-slope form.

Fundamental Concepts

Before diving into calculations, it’s essential to review some key concepts:

1. Coordinates of a Point

A point in the Cartesian plane is represented as (x, y), where:

- x is the horizontal coordinate.
- y is the vertical coordinate.

2. Slope of a Line

The slope (denoted as m) measures the steepness of the line, calculated as:

$$
m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}
$$

where (x_1, y_1) and (x_2, y_2) are two points on the line.

3. Equation of a Line

Once the slope is known, the line's equation can be written in various forms:

- Point-slope form:

$$
y - y_1 = m (x - x_1)
$$

- Slope-intercept form:

$$
y = mx + c
$$

where c is the y-intercept.

Step-by-Step Solution

Step 1: Calculate the Slope

Using the points (-1, 2) and (2, 8):

$$
m = \frac{8 - 2}{2 - (-1)} = \frac{6}{3} = 2
$$

Interpretation: The slope of the line is 2, meaning the line rises 2 units vertically for every 1 unit horizontally.

Step 2: Write the Equation in Point-Slope Form

Choosing either of the two points, let's pick (-1, 2):

$$
y - y_1 = m(x - x_1)
$$

Substituting the known values:

$$
y - 2 = 2(x - (-1))
$$

Simplify:

$$
y - 2 = 2(x + 1)
$$

Step 3: Convert to Slope-Intercept Form

Expanding the right side:

$$
y - 2 = 2x + 2
$$

Adding 2 to both sides:

$$
y = 2x + 4
$$

Final Equation of the Line

Thus, the equation of the straight line passing through the points (-1, 2) and (2, 8) is:

$$
\boxed{y = 2x + 4}
$$

Validation of the Equation

To ensure accuracy, let's verify the line passes through the second point (2, 8):

Substitute $x = 2$:

$$
y = 2(2) + 4 = 4 + 4 = 8
$$

Since the y-value matches the point's y-coordinate, the equation is confirmed correct.

Additional Insights and Applications

1. Graphical Representation

Plotting the points and the line:

- The point (-1, 2) is located one unit left of the origin and two units above.
- The point (2, 8) is two units right and eight units above.
- The line connecting these points has a slope of 2, indicating a steep incline.

2. Real-World Applications

Understanding how to determine the equation of a line through two points has practical applications, including:

- Navigation and GPS: Calculating straight paths between locations.
- Physics: Describing linear motion with constant velocity.
- Economics: Modeling trends and relationships between variables.
- Computer Graphics: Drawing straight lines efficiently.

3. Variations and Extensions

- Finding the equation of a line parallel or perpendicular to the one we found.
- Calculating the distance between the two points.
- Determining the midpoint of the segment connecting the points.

Summary and Key Takeaways

- The slope is a fundamental property that quantifies the steepness of a line, calculated from two points.
- The point-slope form of a line's equation is convenient for deriving the equation once the slope is known.
- The final equation in slope-intercept form provides a straightforward way to graph and analyze the line.

Conclusion

In conclusion, identifying the equation of a line passing through two points is an essential skill in coordinate geometry, blending algebraic manipulation with geometric intuition. Our detailed walkthrough showcases that:

$$
\boxed{\text{Line passing through } (-1, 2) \text{ and } (2, 8) \text{ is } y = 2x + 4}
$$

This process emphasizes clarity, precision, and understanding—core principles that underpin effective problem-solving in mathematics. Whether you're a student, educator, or professional, mastering these techniques enhances your analytical toolkit and prepares you for more complex geometrical challenges.

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