A5=768 And A2=12; Given Two Term In A Geometric Equence Find The 8th Term And The Recurive Formula

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Understanding geometric sequences is fundamental in various branches of mathematics, including algebra, calculus, and applied sciences. When dealing with geometric progressions, it is essential to understand how to determine any term in the sequence once the initial terms are known. This article provides a detailed explanation of how to find the 8th term in a geometric sequence when given two specific terms, A2 and A5, along with the recursive formula associated with the sequence.

Introduction to Geometric Sequences

A geometric sequence is a sequence of numbers where each term after the first is obtained by multiplying the previous term by a fixed, non-zero number called the common ratio (r). The general form of a geometric sequence can be written as:

A_n = A_1 r^(n-1)

where:
- A_n is the nth term,
- A_1 is the first term,
- r is the common ratio,
- n is the term number.

In many problems, you are given specific terms of the sequence, and your goal is to find other terms or the recursive formula that generates the sequence.

Understanding the Given Data

In this particular problem, the provided data is:
- A5 = 768
- A2 = 12

From this information, we need to determine:
- The 8th term, A8
- The recursive formula for the sequence

Step-by-Step Solution Process

1. Recall the general formula for the nth term of a geometric sequence:

A_n = A_1 r^(n-1)

2. Express the known terms in terms of A_1 and r:

- For the 2nd term:

$A_2 = A_1 r^{(2-1)} = A_1 r$

- For the 5th term:

$A_5 = A_1 r^{(5-1)} = A_1 r^4$

3. Set up equations based on the given data:

- $A_2 = 12 \rightarrow A_1 r = 12$ ...(Equation 1)
- $A_5 = 768 \rightarrow A_1 r^4 = 768$ ...(Equation 2)

4. Derive the common ratio r:

Divide Equation 2 by Equation 1:

$(A_1 r^4) / (A_1 r) = 768 / 12$

$r^3 = 64$

Taking cube root of both sides:

$r = \sqrt[3]{64} = 4$

5. Find the first term A_1:

Substitute r = 4 into Equation 1:

$A_1 4 = 12$

$A_1 = 12 / 4 = 3$

6. Find the 8th term, A8:

Using the general formula:

$A_8 = A_1 r^{(8-1)} = 3 4^7$

Calculate 4^7:

$4^7 = 4^4 4^3 = (256) (64) = 16{,}384$

Therefore:

A8 = 3 16,384 = 49,152

The Recursive Formula for the Sequence

A recursive formula defines each term based on the previous term. For a geometric sequence, the recursive formula is:

$A_n = r A_{(n-1)}$

Given that $r = 4$ and $A_1 = 3$, the recursive formula is:

- $A_1 = 3$
- For $n \geq 2$: $A_n = 4 A_{(n-1)}$

This recursive formula allows the sequence to be generated step-by-step, starting from the first term.

Summary of Findings

Term	Value
First term (A_1)	3
Common ratio (r)	4
8th term ($A8$)	49,152

Recursive formula:

$A_n = 4 A_{(n-1)}$, with $A_1 = 3$

Additional Insights and Applications

Why is understanding recursive formulas important?

Recursive formulas are essential in computer science, mathematics, and engineering because they provide a straightforward way to generate terms of sequences without explicitly calculating the entire formula each time. They are particularly useful when dealing with large sequences or when implementing algorithms that rely on previous computations.

Real-world applications of geometric sequences

- Finance: Compound interest calculations
- Biology: Population growth modeling
- Physics: Radioactive decay
- Computer Science: Algorithmic complexity analysis

Practice problems for further understanding

1. Given $A3 = 48$ and $A6 = 768$, find the common ratio r and the first term $A1$.
2. Find the 10th term of a geometric sequence if $A2 = 18$ and $A5 = 1458$.
3. Derive the recursive formula for the sequence where $A4 = 81$ and $A7 = 6561$.

Conclusion

In conclusion, given two terms of a geometric sequence, such as A2 and A5, it is possible to determine the common ratio, the first term, and subsequently find any other term in the sequence, including the 8th term. The process involves setting up equations based on the general formula of geometric sequences, solving for the common ratio and the initial term, and then applying these values to find the desired term.

Understanding how to derive recursive formulas is equally important, as it enables the step-by-step generation of sequence elements, which is useful in practical applications ranging from financial modeling to computer algorithms. Mastering these techniques enhances problem-solving skills and deepens comprehension of mathematical sequences.

By following the systematic approach outlined above, students and enthusiasts can confidently analyze and compute terms in geometric sequences, paving the way for success in advanced mathematics and real-world problem-solving scenarios.

Keywords: geometric sequence, nth term formula, recursive formula, common ratio, sequence calculation, mathematical sequences, algebra, sequence problems, sequence solutions, geometric progression

Frequently Asked Questions

Given A2 = 12 and A5 = 768 in a geometric sequence, how do you find the common ratio (r)?

You can find r by dividing A5 by A2 raised to the power of (5-2), or more directly, r = (A5 / A2)^(1/3). Specifically, r = (768 / 12)^(1/3) = (64)^(1/3) = 4.

What is the formula to find the nth term in a geometric sequence given two terms?

The nth term, An, is given by An = A1 r^(n-1), where A1 is the first term and r is the common ratio. Alternatively, if two terms are known, you can find r and A1 to determine the sequence.

How do you determine the value of the first term A1 in this geometric sequence?

Once you find the common ratio r, you can use one of the known terms, for example, A2 = A1 r, to solve for A1: A1 = A2 / r. Given A2 = 12 and r = 4, then A1 = 12 / 4 = 3.

What is the recursive formula for this geometric sequence?

The recursive formula is An = An-1 r, with the initial term A1 = 3 and r = 4. Therefore, An = An-1 4.

How do you find the 8th term (A8) in this geometric sequence?

Using the formula An = A1 r^(n-1), with A1 = 3 and r = 4: A8 = 3 4^(8-1) = 3 4^7 = 3 16,384 = 49,152.

Can you summarize the steps to find the 8th term given two known terms in a geometric sequence?

Yes. First, find the common ratio r using the known terms. Next, determine the first term A1. Then, apply the nth term formula An = A1 r^(n-1) to find the 8th term.

What is the final recursive formula for the sequence, including initial term and ratio?

The recursive formula is: An = An-1 4, with A1 = 3. This allows you to generate subsequent terms starting from the first.

Additional Resources

Understanding Geometric Sequences: A Comprehensive Guide to Finding the 8th Term and Recursive Formula

When studying sequences in mathematics, one of the fundamental types you encounter is the geometric sequence. These sequences exhibit a consistent ratio between successive terms, which makes them both intriguing and highly applicable across various fields such as finance, physics, and computer science. In this detailed exploration, we will analyze a specific problem involving a geometric sequence where two terms are given, and from those, we aim to find the 8th term and establish the recursive formula governing the sequence.

Introduction to Geometric Sequences

A geometric sequence is a sequence of numbers where each term after the first is found by multiplying the previous term by a constant called the common ratio (denoted as r).

General form:

$$a_n = a_1 \times r^{n-1}$$

where:
- a_n is the nth term,
- a_1 is the first term,
- r is the common ratio,
- n is the position of the term in the sequence.

Key features:
- The ratio r remains constant throughout the sequence.
- The sequence can be increasing, decreasing, or oscillating depending on the value of r.
- The sequence is entirely determined once either the first term or any specific term and the common ratio are known.

Given Data and Problem Statement

In our problem, we're provided with the following:

- A5 = 768
- A2 = 12

Our task is to:
1. Find the 8th term A_8,
2. Derive the recursive formula for the sequence.

Step 1: Understanding the Given Terms

Let's interpret the given data:

- $A_5 = 768$: The fifth term in the sequence is 768.
- $A_2 = 12$: The second term is 12.

Since the sequence is geometric, these terms relate through the common ratio r. Our goal is to find the first term a_1 and the common ratio r using these two pieces of data.

Step 2: Establishing Equations from the Given Data

Recall the general term formula:

$$
A_n = a_1 \times r^{n-1}
$$

Applying this to the given terms:

$$
A_2 = a_1 \times r^{2-1} = a_1 \times r = 12 \quad \text{(Equation 1)}
$$

$$
A_5 = a_1 \times r^{5-1} = a_1 \times r^4 = 768 \quad \text{(Equation 2)}
$$

Now, we have two equations:

1. \(a_1 \times r = 12 \)
2. \(a_1 \times r^4 = 768 \)

Step 3: Solving for the Common Ratio \(r \)

To find \(r \), divide Equation 2 by Equation 1:

\[
\frac{a_1 \times r^4}{a_1 \times r} = \frac{768}{12}
\]

Simplifying:

\[
\frac{r^4}{r} = 64
\]

\[
r^{4-1} = r^3 = 64
\]

Since \(r^3 = 64 \), take the cube root of both sides:

\[
r = \sqrt[3]{64} = 4
\]

Thus, the common ratio:

\[
r = 4
\]

Step 4: Finding the First Term \(a_1 \)

Using Equation 1:

\[
a_1 \times r = 12
\]

\[

$$
a_1 \times 4 = 12
$$

$$
a_1 = \frac{12}{4} = 3
$$

Result:

- $a_1 = 3$
- $r = 4$

Step 5: Verifying the Values

Check with A_5:

$$
A_5 = a_1 \times r^{4} = 3 \times 4^4 = 3 \times 256 = 768
$$

which matches the given data, confirming the correctness of our calculations.

Step 6: Finding the 8th Term A_8

Using the general term formula:

$$
A_8 = a_1 \times r^{7} = 3 \times 4^7
$$

Calculate 4^7:

- $4^1 = 4$
- $4^2 = 16$
- $4^3 = 64$
- $4^4 = 256$
- $4^5 = 1024$
- $4^6 = 4096$
- $4^7 = 16384$

Therefore:

$$
A_8 = 3 \times 16384 = 49152
$$

The 8th term of the sequence is 49,152.

Step 7: Deriving the Recursive Formula

A recursive formula relates each term to the previous one. For a geometric sequence, the recursive formula is:

$$
A_{n} = r \times A_{n-1}
$$

Given our calculated common ratio $r = 4$, the recursive formula becomes:

$$
A_{n} = 4 \times A_{n-1}
$$

with an initial term:

$$
A_1 = 3
$$

This recursive relation succinctly captures the geometric pattern: each term is obtained by multiplying the previous term by 4.

Summary of Results

Parameter	Value
First term a_1	3
Common ratio r	4
8th term A_8	49,152
Recursive formula	$A_{n} = 4 \times A_{n-1}$, starting with $A_1 = 3$

Additional Insights and Applications

Understanding the process of deriving the sequence's properties from limited data is a vital skill in mathematics. The approach used here can be extended to more complex sequences, including those with variable ratios or additional constraints.

Practical applications include:

- Financial modeling: Compound interest calculations often follow geometric sequences.
- Population dynamics: Growth or decay models frequently assume geometric progression.
- Computer science: Recursive algorithms and data structures like trees often rely on geometric properties.

Concluding Remarks

This exercise demonstrates the power of algebraic manipulation and sequence formulas to analyze and predict terms within a geometric sequence. Starting from two known terms, we identified the common ratio and first term, verified the sequence, and then projected future terms with confidence. The recursive formula provides a straightforward means to generate subsequent terms iteratively, which is especially useful in programming and algorithm design.

In summary, mastering such problems enhances problem-solving skills, deepens understanding of geometric relations, and illustrates the interconnectedness of algebra and sequences in mathematics.

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