An Equilateral Triangle Has An Apothem Of 14cm And A Side Length Of 48.5 Cm. What Is Its Area?

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Understanding the area of an equilateral triangle is fundamental in geometry, especially when given specific measurements such as the apothem and side length. In this comprehensive guide, we will explore how to determine the area of an equilateral triangle with an apothem of 14 cm and a side length of 48.5 cm. We will delve into the properties of equilateral triangles, the significance of the apothem, and the step-by-step process to calculate the area using mathematical formulas. Whether you're a student preparing for exams, a teacher preparing lesson plans, or an enthusiast of geometry, this article aims to clarify all aspects related to this problem.

Understanding the Equilateral Triangle

Before calculating the area, it’s important to understand the fundamental properties of an equilateral triangle.

What Is an Equilateral Triangle?

An equilateral triangle is a triangle where all three sides are of equal length. This symmetry results in equal angles and specific relationships between various elements such as the sides, angles, medians, altitudes, and the apothem.

Key properties include:
- All sides are equal in length.
- All interior angles are 60 degrees.
- The centroid, incenter, circumcenter, and orthocenter all coincide at a single point.

Properties Related to the Apothem

The apothem in regular polygons, including equilateral triangles, is the line from the center to the midpoint of a side, perpendicular to that side. It acts as the radius of the inscribed circle.

In an equilateral triangle:
- The apothem, incenter, centroid, and circumcenter all coincide.
- It helps in calculating the area, especially when the apothem is known.

Given Data and Its Significance

The problem provides two vital measurements:
- Apothem (a): 14 cm
- Side length (s): 48.5 cm

Understanding what these measurements represent:
- The side length is straightforward—the length of each side of the triangle.
- The apothem is the distance from the center to the midpoint of a side, perpendicular to it.

Our goal is to find the area of this equilateral triangle based on these measurements.

Mathematical Foundations

To find the area, we need to understand the relationship between the apothem, side length, and the area of an equilateral triangle, and then apply the appropriate formulas.

Formula for the Area of an Equilateral Triangle

The traditional formula for the area (A) of an equilateral triangle with side length s is:

$$A = \frac{\sqrt{3}}{4} s^2$$

However, since the problem provides the apothem, it's often more convenient to use the apothem-based formula:

$$A = \frac{1}{2} \times \text{Perimeter} \times \text{Apothem}$$

This formula is derived from the fact that the area of a regular polygon can be expressed as half of the perimeter times the apothem.

Relationship Between Side Length and Apothem

In an equilateral triangle, the apothem (a) relates to the side length (s) by the formula:

$$a = \frac{\sqrt{3}}{6} s$$

This is because the apothem in an equilateral triangle is also the height of the smaller right triangle formed when you split the triangle down the middle.

If given the side length s, the apothem can be calculated as:

$$a = \frac{\sqrt{3}}{6} s$$

Conversely, if the apothem and side length are known, this relationship can be verified or

used to find missing measurements.

Step-by-Step Calculation of the Area

Now, with the given data, we can proceed step by step to find the area.

Step 1: Verify the Relationship Between Side Length and Apothem

Using the formula:

$$a = \frac{\sqrt{3}}{6} s$$

Substitute s = 48.5 cm:

$$a = \frac{\sqrt{3}}{6} \times 48.5$$

Calculate the numerator:

$\sqrt{3} \approx 1.732$

Then:

$$a \approx \frac{1.732}{6} \times 48.5$$

Calculate:

$\frac{1.732}{6} \approx 0.2887$

Finally:

$$a \approx 0.2887 \times 48.5 \approx 14.03\, \text{cm}$$

This closely matches the given apothem of 14 cm, confirming the consistency of the data.

Conclusion: The given measurements are compatible, and the side length of approximately 48.5 cm aligns with an apothem close to 14 cm.

Step 2: Calculate the Perimeter

Perimeter (P) of an equilateral triangle:

$$P = 3 \times s = 3 \times 48.5 = 145.5\, \text{cm}$$

Step 3: Use the Area Formula

Recall:

$$A = \frac{1}{2} \times P \times a$$

Substitute the known values:

$$A = \frac{1}{2} \times 145.5 \times 14$$

Calculate:

$\frac{1}{2} \times 145.5 = 72.75$

Then:

$$A = 72.75 \times 14 = 1018.5\, \text{cm}^2$$

Result: The area of the triangle is approximately 1018.5 square centimeters.

Alternative Method: Using the Traditional Side Length Formula

For validation, let’s also calculate the area using the standard formula:

$$A = \frac{\sqrt{3}}{4} s^2$$

Substitute s = 48.5 cm:

$$A = \frac{1.732}{4} \times (48.5)^2$$

Calculate:

$\frac{1.732}{4} \approx 0.433$

Calculate the square of side length:

$(48.5)^2 = 2352.25$

Multiply:

$$A \approx 0.433 \times 2352.25 \approx 1019.9\, \text{cm}^2$$

This aligns closely with the previous result, confirming the accuracy of our calculations.

Conclusion: Both methods yield similar results, with the area approximately 1019 square

centimeters.

Summary of Key Findings

- The side length of the equilateral triangle is approximately 48.5 cm.
- The apothem is approximately 14 cm.
- The area of the triangle is approximately 1018.5 to 1019 square centimeters.

Additional Insights and Tips

- When given the apothem and side length, always verify their relationship to ensure the data is consistent.
- Use the formula involving the perimeter and apothem for straightforward area calculations.
- Remember the standard formula for the area of an equilateral triangle: $A = \frac{\sqrt{3}}{4} s^2$, which is often simpler when side length is known.
- For more complex problems, understanding the link between the apothem, height, and side length can facilitate various calculations.

Conclusion

Calculating the area of an equilateral triangle with known apothem and side length involves understanding the geometric relationships between these elements. Using the provided measurements, we confirmed their consistency, calculated the perimeter, and applied the appropriate formulas to determine the area. The result, approximately 1019 square centimeters, demonstrates the harmony of geometric principles and the power of mathematical formulas in solving real-world problems. Whether you are studying geometry, designing structures, or simply enhancing your mathematical skills, mastering these calculations is essential for analyzing equilateral triangles and other regular polygons.

Meta Description: Discover how to find the area of an equilateral triangle with an apothem of 14 cm and a side length of 48.5 cm. Learn step-by-step calculations and key formulas in this comprehensive guide.

Frequently Asked Questions

How do you find the area of an equilateral triangle when

given its apothem and side length?

The area can be found using the formula: Area = (1/2) × Perimeter × Apothem. For an equilateral triangle, perimeter = 3 × side length. So, substitute the values into the formula to calculate the area.

What is the relationship between the side length and the apothem in an equilateral triangle?

In an equilateral triangle, the apothem is related to the side length by the formula: Apothem = (√3/6) × side length. Given both, you can verify their consistency or use one to find the other.

Given an apothem of 14 cm and side length of 48.5 cm, how do you verify these measurements are correct for an equilateral triangle?

You can verify by checking if the apothem matches the expected value using the formula: Apothem = (√3/6) × side length. Plug in 48.5 cm and see if the result approximates 14 cm.

What is the formula to calculate the area of an equilateral triangle using its side length alone?

The area of an equilateral triangle with side length s is given by: Area = (√3/4) × s^2. Alternatively, if the apothem is known, you can use the perimeter and apothem to find the area.

How do you calculate the area of an equilateral triangle with given side length and apothem in centimeters?

First, find the perimeter: 3 × side length. Then, use the formula: Area = (1/2) × perimeter × apothem. Substitute the given values to compute the area.

What is the approximate area of an equilateral triangle with an apothem of 14 cm and side length of 48.5 cm?

Using the formula: Area = (1/2) × perimeter × apothem = (1/2) × (3 × 48.5) × 14 ≈ 1017.5 cm^2. Therefore, the area is approximately 1017.5 square centimeters.

Additional Resources

An Equilateral Triangle Has An Apothem Of 14cm And A Side Length Of 48.5 Cm. What Is Its Area?
This intriguing question brings together fundamental geometric concepts and calculations, offering a comprehensive exploration of equilateral triangles. Understanding the

relationships between side lengths, apothem, and area not only enhances mathematical skills but also deepens appreciation for the elegance of geometric properties. In this article, we will dissect the problem, analyze relevant formulas, and provide a detailed step-by-step solution, all while highlighting key features, potential pitfalls, and insights related to equilateral triangles.

Understanding the Geometry of Equilateral Triangles

An equilateral triangle is a special type of triangle where all three sides are equal in length, and consequently, all three angles are equal, each measuring 60°. This symmetry makes equilateral triangles particularly interesting in geometry because many of their properties are interrelated in simple, elegant ways.

Key Features of Equilateral Triangles

- Equal sides and angles: All sides are of length 'a'; angles are each 60°.
- Line of symmetry: The triangle has three lines of symmetry, each passing through a vertex and the midpoint of the opposite side.
- Centroid, incenter, circumcenter, and orthocenter coincide: All these centers are at the same point in an equilateral triangle.
- Altitude, median, and angle bisector: These three lines are identical in an equilateral triangle, each dividing the triangle into two 30-60-90 right triangles.

Understanding these features provides a foundation for deriving the area based on known parameters like side length and apothem.

The Apothem and Its Role in Area Calculation

The apothem of a regular polygon, including an equilateral triangle, is the line segment from the center of the polygon to the midpoint of a side, perpendicular to that side. It essentially acts as the radius of the inscribed circle (incircle).

Apothem in Equilateral Triangles

- In an equilateral triangle, the apothem is also the inradius—the radius of the inscribed circle.
- It divides the triangle into two 30-60-90 right triangles, which makes calculations straightforward.

Given the apothem (14 cm) and the side length (48.5 cm), we can use these measurements to find the area of the triangle.

Deriving the Area of the Equilateral Triangle

The area of a triangle can be calculated through various methods, but for an equilateral triangle, the most straightforward formulas involve side length or inradius (apothem).

Method 1: Using Side Length

The standard formula for the area of an equilateral triangle with side length 'a' is:
$$
\text{Area} = \frac{\sqrt{3}}{4}a^2
$$

Method 2: Using Apothem (Inradius)

The area can also be expressed as:
$$
\text{Area} = \frac{1}{2} \times \text{Perimeter} \times \text{Inradius}
$$
where:
- Perimeter $P = 3a$,
- Inradius $r = 14\,cm$ (given).

Thus:
$$
\text{Area} = \frac{1}{2} \times 3a \times r = \frac{3a\, r}{2}
$$

Since the side length $a = 48.5\,cm$, substituting values yields:

$$
\text{Area} = \frac{3 \times 48.5 \times 14}{2}
$$

Calculating step by step:

$$
3 \times 48.5 = 145.5
$$
$$
145.5 \times 14 = 2037
$$
$$
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$$\frac{2037}{2} = 1018.5\, \text{cm}^2$$

So, the area based on the apothem is 1018.5 square centimeters.

Verifying the Consistency of the Given Data

It's prudent to verify whether the given side length and apothem are consistent with each other, as inconsistencies could suggest errors or misinterpretations.

Calculating the Apothem from the Given Side Length

In an equilateral triangle, the apothem (inradius) 'r' can be derived from the side length:

$$r = \frac{\sqrt{3}}{6}a$$

Substituting $a = 48.5\,cm$:

$$r = \frac{\sqrt{3}}{6} \times 48.5$$

Calculating:

$$\sqrt{3} \approx 1.732$$
$$r \approx \frac{1.732}{6} \times 48.5 \approx 0.2887 \times 48.5 \approx 14.02\,cm$$

This is very close to the given apothem of 14 cm, confirming the data's consistency within acceptable measurement or rounding errors.

Final Calculation of the Area

Given the confirmation that the apothem and side length are consistent, the most accurate and straightforward calculation for the area uses the formula involving the side length:

$$

$$
\boxed{
\text{Area} = \frac{\sqrt{3}}{4}a^2
}
$$

Substituting $a = 48.5\,cm$:

$$
\text{Area} = \frac{\sqrt{3}}{4} \times (48.5)^2
$$

Calculations:

$$
(48.5)^2 = 2352.25
$$
$$
\frac{\sqrt{3}}{4} \approx \frac{1.732}{4} = 0.433
$$
$$
\text{Area} \approx 0.433 \times 2352.25 \approx 1019\, \text{cm}^2
$$

This aligns closely with the previous calculation based on the apothem, reaffirming that the area of the equilateral triangle is approximately 1019 square centimeters.

Summary and Insights

This problem showcases how understanding the relationships between side length, apothem, and area can lead to efficient calculations. The key takeaways include:

- The geometric properties of equilateral triangles simplify many calculations; knowing one element (like side length or apothem) allows the derivation of others.
- The apothem (or inradius) is directly related to the side length via the formula $r = \frac{\sqrt{3}}{6}a$.
- The area can be accurately calculated using either the side length or the apothem, with both approaches converging on a similar result.

Pros and Cons of the Approaches

Using Side Length:
- Pros:

- Straightforward application of a well-known formula.
- Less computational complexity.
- Cons:
- Requires precise knowledge of side length.

Using Apothem (Inradius):
- Pros:
- Useful when the inradius is given, especially in complex polygons.
- Reinforces understanding of the relationship between inscribed circles and the polygon.
- Cons:
- Slightly more involved if the inradius needs to be derived.

Overall, knowing both formulas and their derivations enhances problem-solving flexibility.

Conclusion

The problem of determining the area of an equilateral triangle with a given apothem and side length offers a rich exploration of geometric principles. By verifying the consistency of the data and applying fundamental formulas, we find that the area is approximately 1019 square centimeters. This exercise underscores the importance of understanding the relationships between various elements of regular polygons, and how different approaches can lead to the same result, deepening mathematical insight and appreciation for geometric harmony.

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