ASAP Write An Equation Of A Quadratic Formula With The Given Properties: F(3)=f(-5)=0;f(-6)=-36

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Introduction

When working with quadratic functions, understanding how to derive the equation based on given properties is a fundamental skill in algebra. In this article, we will analyze the problem of constructing a quadratic function $f(x)$ that satisfies specific conditions: the roots of the function are at $x=3$ and $x=-5$, and the function takes a value of -36 at $x=-6$. By systematically applying algebraic principles, we will develop the explicit quadratic equation that meets these criteria. This process involves identifying roots, formulating the general quadratic form, and solving for any unknown coefficients using the additional point provided.

Understanding the Given Properties

Roots of the Quadratic Function

The problem states that:

- $f(3) = 0$
- $f(-5) = 0$

This indicates that $x=3$ and $x=-5$ are roots of the quadratic function. Since roots correspond to zeroes of the function, the quadratic can be expressed in factored form as:

$$
f(x) = a(x - r_1)(x - r_2)
$$

where r_1 and r_2 are the roots, and a is a leading coefficient that determines the parabola's orientation and width.

Given the roots:

$$
f(x) = a(x - 3)(x + 5)
$$

Note that the second factor is $(x - (-5)) = (x + 5)$.

The Additional Point: $f(-6) = -36$

The problem also provides a specific point on the parabola:

$$
f(-6) = -36
$$

This point allows us to determine the value of a, the leading coefficient, once the quadratic is in factored form.

Formulating the Quadratic Equation

Expressing the Quadratic in Factored Form

As established, the quadratic can be written as:

$$
f(x) = a(x - 3)(x + 5)
$$

Expanding this, we get:

$$
f(x) = a[(x)(x + 5) - 3(x + 5)] = a[x^2 + 5x - 3x - 15] = a[x^2 + 2x - 15]
$$

So the general form, with unknown coefficient a, is:

$$
f(x) = a x^2 + 2a x - 15a
$$

Using the Point $f(-6) = -36$ to Find a

Substitute $x = -6$ and $f(-6) = -36$ into the quadratic:

$$
f(-6) = a(-6)^2 + 2a(-6) - 15a = -36

\]

Calculating step-by-step:

\[
a \times 36 + 2a \times (-6) - 15a = -36
\]

\[
36a - 12a - 15a = -36
\]

Combine like terms:

\[
(36a - 12a - 15a) = -36
\]

\[
(36a - 27a) = -36
\]

\[
9a = -36
\]

Solve for \(a\):

\[
a = \frac{-36}{9} = -4
\]

Final Quadratic Equation

Having determined \(a = -4\), substitute back into the factored form:

\[
f(x) = -4(x - 3)(x + 5)
\]

Expanding the factored form:

\[
f(x) = -4[x^2 + 2x - 15]
\]

Distribute \(-4\):

\[
f(x) = -4x^2 - 8x + 60
\]

Thus, the quadratic function satisfying all the given properties is:

\[
\boxed{
f(x) = -4x^2 - 8x + 60
}
\]

Verification of the Solution

Check Roots

Plugging in $x=3$:

$$
f(3) = -4(3)^2 - 8(3) + 60 = -4 \times 9 - 24 + 60 = -36 - 24 + 60 = 0
$$

Plugging in $x=-5$:

$$
f(-5) = -4(-5)^2 - 8(-5) + 60 = -4 \times 25 + 40 + 60 = -100 + 40 + 60 = 0
$$

Check the Point $x=-6$

$$
f(-6) = -4(-6)^2 - 8(-6) + 60 = -4 \times 36 + 48 + 60 = -144 + 48 + 60 = -36
$$

All conditions are satisfied, confirming the correctness of the quadratic.

Alternative Forms of the Quadratic Equation

While the factored form is often most intuitive when roots are known, it's also useful to express the quadratic in standard form:

$$
f(x) = -4x^2 - 8x + 60
$$

or in vertex form, which makes identifying the vertex straightforward.

Vertex Form

Complete the square:

$$
f(x) = -4x^2 - 8x + 60
$$

Factor out -4:

$$
f(x) = -4(x^2 + 2x) + 60
$$

Complete the square inside the parentheses:

$$
x^2 + 2x = x^2 + 2x + 1 - 1 = (x + 1)^2 - 1
$$

Substitute back:

$$
f(x) = -4[(x + 1)^2 - 1] + 60 = -4(x + 1)^2 + 4 + 60 = -4(x + 1)^2 + 64
$$

The vertex form:

$$
f(x) = -4(x + 1)^2 + 64
$$

indicates that the parabola opens downward, with vertex at $(-1, 64)$.

Summary and Conclusion

By analyzing the roots and the additional point, we derived the quadratic function:

$$
\boxed{
f(x) = -4x^2 - 8x + 60
}
$$

which satisfies the conditions:

- Roots at $x=3$ and $x=-5$,
- The point $(-6, -36)$ on the parabola.

This process illustrates the importance of understanding the relationship between roots, coefficients, and specific points on a quadratic curve. Whether in factored, standard, or vertex form, the quadratic function can be precisely determined once the roots and a point are known. Mastery of these techniques enables students and mathematicians to efficiently construct quadratic equations based on a variety of properties and constraints.

Frequently Asked Questions

What is the quadratic equation with roots at x=3 and x=-5, given that f(-6) = -36?

First, since the roots are 3 and -5, the quadratic can be written as $f(x) = a(x - 3)(x + 5)$. Using the point (-6, -36):

$-36 = a(-6 - 3)(-6 + 5) = a(-9)(-1) = 9a$

Thus, $a = -36 / 9 = -4$.

The quadratic equation is $f(x) = -4(x - 3)(x + 5)$.

How do you find the quadratic function when the roots are known and a specific point is given?

To find the quadratic function with known roots and a point, write it in factored form using the roots, then substitute the given point to solve for the leading coefficient a. For roots r1 and r2:

$f(x) = a(x - r1)(x - r2)$

Use the given point (x, y) to find a:

$y = a(x - r1)(x - r2) \Rightarrow a = y / [(x - r1)(x - r2)]$

What are the steps to derive the quadratic equation from the given roots and a point?

Step 1: Write the quadratic in factored form using the roots: f(x) = a(x - 3)(x + 5).

Step 2: Plug in the point (-6, -36): -36 = a(-6 - 3)(-6 + 5).

Step 3: Simplify and solve for a: -36 = a(-9)(-1) => -36 = 9a => a = -4.

Step 4: Write the full equation: f(x) = -4(x - 3)(x + 5).

Can you convert the factored quadratic form into standard form with the given properties?

Yes. Starting from f(x) = -4(x - 3)(x + 5), expand:

f(x) = -4[x^2 + 5x - 3x - 15] = -4[x^2 + 2x - 15] = -4x^2 - 8x + 60.

This is the standard form of the quadratic.

Why is the value of 'a' negative in this quadratic function, and what does it imply about the parabola?

The value of 'a' is negative (-4) because the parabola opens downward, indicating that the quadratic has a maximum point. This is consistent with the given points and roots, showing the parabola reaches its vertex at a maximum height.

Additional Resources

ASAP Write An Equation Of A Quadratic Formula With The Given Properties: F(3)=f(-5)=0;f(-6)=-36

When it comes to quadratic functions, understanding how to construct an equation from given properties is a fundamental skill in algebra. Whether you're solving for roots, vertex, or specific point values, being able to "write an equation of a quadratic formula with the given properties" quickly and accurately is essential for students, educators, and professionals alike. In this comprehensive guide, we'll walk through a step-by-step process to derive the quadratic function based on the provided properties: F(3) = 0, f(-5) = 0, and f(-6) = -36. By the end, you'll have a clear understanding of how to approach similar problems efficiently and confidently.

Understanding the Given Properties

Before diving into the solution, let's interpret what the properties tell us:

- F(3) = 0: The function crosses the x-axis at x = 3.
- f(-5) = 0: The function crosses the x-axis at x = -5.
- f(-6) = -36: The function takes the value -36 at x = -6.

Note that the roots provided are 3 and -5, which are potential solutions to the quadratic equation, assuming it factors nicely. The value at x = -6 gives us additional information to determine the specific quadratic function.

Step 1: Recognize the Roots and Form the Factored Form

Since the roots are given directly, the quadratic function can be initially expressed in factored form as:

$$ f(x) = a \cdot (x - r_1) \cdot (x - r_2) $$

where r_1 and r_2 are the roots.

Given roots:

- $r_1 = 3$
- $r_2 = -5$

Therefore,

$$f(x) = a \cdot (x - 3) \cdot (x + 5)$$

The constant a is unknown and needs to be determined.

Step 2: Use the Additional Point to Find the Leading Coefficient a

Since we know the function's value at $x = -6$, where $f(-6) = -36$, substitute $x = -6$ into the factored form:

$$f(-6) = a \cdot (-6 - 3) \cdot (-6 + 5)$$

Calculate each factor:

- $-6 - 3 = -9$
- $-6 + 5 = -1$

So,

$$-36 = a \cdot (-9) \cdot (-1)$$
$$-36 = a \cdot 9$$

Solve for a:

$$a = \frac{-36}{9} = -4$$

Step 3: Write the Complete Equation

Now that $a = -4$, the quadratic function is:

$$f(x) = -4 \cdot (x - 3)(x + 5)$$

Expanding the factors:

1. Expand $(x - 3)(x + 5)$:

$$(x - 3)(x + 5) = x^2 + 5x - 3x - 15 = x^2 + 2x - 15$$

2. Multiply by -4:

$$f(x) = -4(x^2 + 2x - 15)$$

3. Distribute -4:

$$f(x) = -4x^2 - 8x + 60$$

Final Equation:

$$
\boxed{
f(x) = -4x^2 - 8x + 60
}
$$

This quadratic function satisfies all the given properties:

- Roots at $x=3$ and $x=-5$,
- Function value at $x=-6$ is -36.

Additional Insights and Validation

Verifying the Roots

- At $x=3$:

$$f(3) = -4(3)^2 - 8(3) + 60 = -4(9) - 24 + 60 = -36 - 24 + 60 = 0$$

- At $x=-5$:

$$f(-5) = -4(-5)^2 - 8(-5) + 60 = -4(25) + 40 + 60 = -100 + 40 + 60 = 0$$

Verifying the Point $(-6, -36)$:

$$f(-6) = -4(-6)^2 - 8(-6) + 60 = -4(36) + 48 + 60 = -144 + 48 + 60 = -36$$

All properties check out successfully.

Practical Tips for Writing Quadratic Equations from Properties

1. Identify roots and write in factored form:

- Use the roots directly if provided.

- Express the quadratic as $a \cdot (x - r_1)(x - r_2)$.

2. Use additional point(s) to determine the leading coefficient a:

- Substitute known x and $f(x)$ values into factored form.
- Solve for a.

3. Expand and simplify to get the standard form $ax^2 + bx + c$.

4. Verify all given properties to ensure correctness.

Additional Variations and Considerations

- If roots are not given explicitly but are known to exist, consider factoring or quadratic formula methods.
- When only some points are given, always check for consistency.
- In problems with complex roots or non-integer roots, quadratic formula or completing the square may be necessary.

Conclusion

Writing the equation of a quadratic function given specific properties may seem daunting at first, but following a systematic approach simplifies the process. Start with the roots to form the basic structure, then use known points to solve for the leading coefficient. Always verify your solution against the properties provided. With practice, you'll be able to quickly derive quadratic equations from a variety of given data, enhancing your problem-solving skills and mathematical fluency.

Remember: Practice makes perfect. Try similar problems with different roots and points to build confidence and mastery in constructing quadratic functions from properties!

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