Beginning With The Equation 3m=7 Write The New Equation Produced By Multiplying Both Sides By 3/8

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Understanding how to manipulate algebraic equations is a fundamental skill in mathematics. Starting from the simple equation 3m = 7, one of the essential operations involves multiplying both sides of the equation by a specific fraction—in this case, 3/8—to produce a new equivalent equation. This process not only demonstrates the principles of maintaining equality but also enhances problem-solving skills. In this comprehensive guide, we will explore the step-by-step method of deriving a new equation by multiplying both sides of the original equation by 3/8, along with practical applications, tips, and common mistakes to avoid.

Understanding the Fundamentals of Algebraic Equations

Before diving into the specific operation, it's important to have a solid grasp of the basic concepts involved in algebraic equations.

What Is an Equation?

An equation is a mathematical statement that asserts the equality of two expressions, often containing variables. The goal in solving equations is to find the value(s) of the variable(s) that satisfy the statement.

Why Is Maintaining Equality Important?

When performing operations on an equation, such as addition, subtraction, multiplication, or division, it's crucial to do the same to both sides to preserve the balance and validity of the equation.

Starting Point: The Equation 3m = 7

The initial equation, 3m = 7, involves a single variable, m, multiplied by 3. To find the value of m, one common method is to isolate the variable by performing operations on both sides of the equation.

Basic Solution: Dividing Both Sides by 3

Dividing both sides by 3 yields:
$$ m = \frac{7}{3} $$
This is a straightforward method to solve for m, but in this article, we focus on an alternative operation: multiplying both sides by 3/8.

Multiplying Both Sides of an Equation by a Fraction

Multiplying both sides of an equation by a fraction is a powerful technique in algebra, often used to manipulate equations into more useful forms or to prepare for substitution.

Key Principles

- Equivalence: Multiplying both sides by the same non-zero fraction preserves the equality.
- Multiplication by a fraction: This involves multiplying each side of the equation by the numerator and denominator accordingly.

Why Multiply by 3/8?

Multiplying both sides by 3/8 can serve various purposes:
- To manipulate the original equation into a form that simplifies solving for the variable.
- To prepare for the substitution or to change the coefficients.
- To illustrate the effect of fractional multiplication on an equation.

Step-by-Step Process of Multiplying Both Sides by 3/8

Let's go through the detailed steps to produce the new equation.

Step 1: Write Down the Original Equation

$$
3m = 7
$$

Step 2: Multiply Both Sides by 3/8

$$
\left(3m\right) \times \frac{3}{8} = 7 \times \frac{3}{8}
$$

Step 3: Simplify Each Side

- Left side:
$$
3m \times \frac{3}{8} = 3m \times \frac{3}{8} = \frac{3 \times 3m}{8} = \frac{9m}{8}
$$
- Right side:
$$
7 \times \frac{3}{8} = \frac{7 \times 3}{8} = \frac{21}{8}
$$

Step 4: Write the New Equation

$$
\boxed{\frac{9m}{8} = \frac{21}{8}}
$$

This is the new equivalent equation produced by multiplying both sides by 3/8.

Interpreting and Solving the New Equation

The new equation, $\frac{9m}{8} = \frac{21}{8}$, maintains the equality established in the original equation but rearranges the coefficients, making it easier in some contexts to solve for m or to analyze the relationship.

Step 5: Solve for the Variable m

Since both sides have the same denominator, 8, you can eliminate the denominator by multiplying both sides by 8:

$$
8 \times \frac{9m}{8} = 8 \times \frac{21}{8}
$$
which simplifies to:
$$
9m = 21
$$

Now, solve for m:

$$
m = \frac{21}{9} = \frac{7}{3}
$$

Note that this matches the original solution, confirming that multiplying both sides by 3/8 did not change the solution's validity.

Key Takeaways and Practical Applications

Understanding how multiplying both sides of an equation by a fraction affects the equation is vital in algebra and beyond.

Key Points

- Multiplying both sides by a non-zero fraction preserves the equality.
- The operation can help manipulate equations into more manageable forms.
- Multiplying by fractions is often used to clear denominators or to set up for other operations.

Practical Applications of Fractional Multiplication in Algebra

- Simplifying complex equations involving fractions.
- Solving equations with fractional coefficients.
- Re-arranging equations to isolate variables.
- Preparing equations for graphing or substitution.

Common Mistakes to Avoid

While multiplying both sides by a fraction is straightforward, some frequent errors include:

- **Multiplying by zero**: This would invalidate the equation since multiplying both sides by zero results in 0=0, losing the original information.
- **Neglecting to multiply both sides**: Only multiplying one side breaks the equality and produces incorrect results.
- **Incorrect simplification**: Failing to simplify the resulting expressions correctly can lead to errors in the solution.

Additional Tips for Working with Fractions in Equations

- Always verify that the fraction multiplied is not zero.
- When dealing with multiple fractions, consider finding a common denominator to simplify calculations.
- Use cross-multiplication to compare or solve equations involving fractions efficiently.

- Remember that the inverse operation of multiplication by a fraction is division by the same fraction.

Summary

Beginning with the equation 3m = 7, multiplying both sides by 3/8 produces the new equation $\frac{9m}{8} = \frac{21}{8}$. This process exemplifies the core principle of maintaining equality by performing the same operation on both sides of an equation. The resulting equation can be further simplified to solve for the variable m, ultimately confirming that the solution remains consistent with the original equation.

In the broader context of algebra, mastering such operations enhances problem-solving flexibility and deepens understanding of equation manipulation. Whether you're solving for variables, simplifying expressions, or preparing equations for graphing, knowing how to correctly multiply both sides by fractions is an essential skill.

Additional Resources:
- Algebraic Equation Solving Techniques
- Simplifying Fractions and Rational Expressions
- Common Mistakes in Algebra and How to Avoid Them
- Step-by-Step Guide to Solving Equations with Fractions

By practicing these methods, students and learners can strengthen their algebraic intuition and become proficient in manipulating equations of varying complexity.

Frequently Asked Questions

What is the original equation given in the problem?

The original equation is 3m = 7.

What mathematical operation should be performed on both sides of the equation?

Multiply both sides of the equation by 3/8.

How do you write the new equation after multiplying both sides by 3/8?

The new equation is (3/8) 3m = (3/8) 7.

What is the simplified form of the new equation after

multiplication?

The simplified form is (9/8)m = 21/8.

Why is multiplying both sides by 3/8 important in solving for m?

Multiplying both sides by 3/8 helps to isolate the variable m by adjusting its coefficient, facilitating further solving steps.

Additional Resources

Beginning With The Equation 3m=7 Write The New Equation Produced By Multiplying Both Sides By 3/8

Introduction

In the realm of algebra, the manipulation of equations is fundamental to solving for unknown variables. One common operation involves multiplying both sides of an equation by a specific value to simplify or transform the equation into a more workable form. The process not only demonstrates the core principle of maintaining equality but also illustrates broader concepts such as proportional reasoning and algebraic integrity.

This article delves into a specific example: starting with the equation 3m = 7, then multiplying both sides by 3/8 to produce a new, equivalent equation. Through a detailed exploration, we aim to elucidate the underlying principles, step-by-step procedures, and the significance of such transformations within mathematical problem-solving and real-world applications.

The Foundational Equation: 3m = 7

At the core of our investigation lies the linear equation:

3m = 7

This simple algebraic statement asserts that three times some unknown value m equals seven. The goal often involves solving for m, which requires isolating the variable on one side of the equation.

Key points:

- The coefficient of m is 3.
- The constant term on the right side is 7.

- The solution involves manipulating the equation to express m explicitly.

Multiplication of Both Sides by a Fraction: Conceptual Framework

Before proceeding to the specific operation, it's essential to understand the principle behind multiplying both sides of an equation by a fraction:

Principle:
Multiplying both sides of an equation by the same non-zero value preserves the equality.

When the multiplier is a fraction, such as 3/8, the operation involves scaling both sides proportionally. This can be used to:

- Simplify coefficients.
- Isolate variables.
- Transform the equation into a more convenient form.

Important considerations:

- The multiplier must not be zero.
- The operation must be applied uniformly to both sides.
- The algebraic integrity of the equation remains intact.

Step-by-Step Transformation: From 3m = 7 to the New Equation

Let's examine the precise steps involved in multiplying both sides of the initial equation 3m = 7 by 3/8.

Step 1: Write the initial equation

$$
3m = 7
$$

Step 2: Multiply both sides by 3/8

$$
\left(3m \right) \times \frac{3}{8} = 7 \times \frac{3}{8}
$$

Step 3: Simplify each side

Left side:

$$
3m \times \frac{3}{8} = m \times 3 \times \frac{3}{8} = m \times \frac{3 \times 3}{8} = m \times \frac{9}{8}
$$

\]

Right side:

\[
7 \times \frac{3}{8} = \frac{7 \times 3}{8} = \frac{21}{8}
\]

Resulting equation:

\[
m \times \frac{9}{8} = \frac{21}{8}
\]

The New Equation: Interpretation and Significance

The transformed equation is:

\[
\frac{9}{8} m = \frac{21}{8}
\]

This form is especially useful because it expresses a direct proportional relationship between m and a known fraction. To solve for m, the next logical step involves isolating m.

Solving for m: Further Algebraic Manipulation

Given:

\[
\frac{9}{8} m = \frac{21}{8}
\]

We observe that both sides share the denominator 8, which can be exploited to simplify the process.

Step 1: Multiply both sides by 8 to clear denominators

\[
8 \times \left(\frac{9}{8} m \right) = 8 \times \frac{21}{8}
\]

\[
9m = 21
\]

Note: This step simplifies the equation back to a form similar to the original, but with a

different coefficient.

Step 2: Solve for m

$$
m = \frac{21}{9} = \frac{7}{3}
$$

Result:

$$
m = \frac{7}{3}
$$

Broader Implications and Analytical Insights

The process of multiplying both sides by 3/8 illustrates several fundamental algebraic principles:

- Preservation of Equality: Applying the same operation to both sides ensures the equation remains valid.
- Fractional Scaling: Multiplying by a fraction scales the entire equation, affecting coefficients and constants proportionally.
- Equation Transformation: Such operations can simplify or reframe equations, making variable isolation more straightforward or exposing intrinsic relationships.

Furthermore, this example demonstrates how algebraic manipulation is not just a mechanical process but a strategic tool for problem-solving, especially in contexts involving proportions, ratios, and scaling.

Practical Applications and Real-World Relevance

Understanding how to manipulate equations through multiplication by fractions has numerous applications beyond pure mathematics:

- Engineering: Adjusting formulas involving ratios or scaling models.
- Economics: Modifying equations representing proportional relationships, such as cost per unit.
- Science: Rescaling experimental data or converting units.
- Data Analysis: Normalizing data or adjusting variables through proportional transformations.

In each case, maintaining the integrity of the initial relationships is critical, and multiplying both sides by a fraction is a powerful method to achieve these goals.

Common Pitfalls and Misconceptions

While the process appears straightforward, students and practitioners should be aware of potential pitfalls:

- Multiplying by zero: This would destroy the equation's information, leading to an invalid statement.
- Incorrect application: Failing to multiply both sides equally violates the fundamental principle of equation manipulation.
- Misinterpretation of fractions: Understanding that fractions act as ratios or scaling factors, not just as division.

A thorough grasp of these concepts ensures accurate and effective algebraic transformations.

Summary and Conclusions

Starting with the equation $3m = 7$, multiplying both sides by $3/8$ transforms the original equation into:

$$
\frac{9}{8} m = \frac{21}{8}
$$

This operation exemplifies core algebraic principles:

- Preserving equality through uniform operations.
- Utilizing fractional multipliers for equation manipulation.
- Facilitating variable isolation and solution simplification.

The process underscores the importance of understanding proportional relationships and the power of algebraic transformations in both theoretical and applied contexts.

By mastering these techniques, students and professionals can confidently manipulate complex equations, ensuring accuracy and efficiency in diverse problem-solving scenarios.

Final Thoughts

Algebraic operations such as multiplying both sides of an equation by a fraction serve as fundamental tools in mathematics. They exemplify the elegance and rigor of algebra, allowing us to transform, analyze, and solve equations systematically. Whether in academic settings or real-world applications, these techniques form the backbone of quantitative reasoning, highlighting the importance of conceptual understanding alongside procedural mastery.

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