Change From Rectangular To Cylindrical Coordinates. (let R 0 And 0 2.) A) (-1,1,1) B) (-7,73,7)

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Understanding how to convert between coordinate systems is fundamental in mathematics, physics, and engineering. One of the most common conversions is from rectangular (Cartesian) coordinates to cylindrical coordinates. This transformation is essential, especially when dealing with problems exhibiting symmetry around an axis, such as those involving cylinders, pipes, or rotational systems. In this article, we will explore the step-by-step process of converting rectangular coordinates to cylindrical coordinates, focusing on two specific points: A) (-1, 1, 1) and B) (-7, 73, 7). We will also delve into the significance of parameters R, θ, and z in the cylindrical coordinate system, along with practical applications and tips for accurate conversions.

Understanding Cylindrical Coordinates

What Are Cylindrical Coordinates?

Cylindrical coordinates are a three-dimensional coordinate system that extends polar coordinates into three dimensions by including a height component. The system is particularly useful for describing points located around a central axis, typically the z-axis, with a radius from that axis, an angle, and a height.

The cylindrical coordinates are expressed as:

- R (Radial distance): The distance from the point to the z-axis.
- θ (Angular coordinate): The angle measured in the xy-plane from the positive x-axis.
- z (Height): The same as in the Cartesian system, representing the point's height above the xy-plane.

Mathematically:

$$
\begin{cases}
R = \sqrt{x^2 + y^2} \\
\theta = \arctan\left(\frac{y}{x}\right) \quad \text{(adjusted based on quadrant)} \\
z = z
\end{cases}
$$

Conversion From Rectangular to Cylindrical Coordinates

General Process

Converting a point from rectangular to cylindrical coordinates involves three main steps:

1. Calculate R:

$$
R = \sqrt{x^2 + y^2}
$$

2. Determine θ:

$$
\theta = \arctan\left(\frac{y}{x}\right)
$$

Note: Be cautious of the quadrant in which the point lies, as the arctangent function only returns values between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$. Use the `atan2(y, x)` function or adjust θ based on the signs of x and y.

3. Identify z:

$$
z = z
$$

Special Considerations

- Quadrant Adjustment: Use the `atan2(y, x)` function for accurate angle calculation across all quadrants.
- Angles in Radians or Degrees: Ensure consistency with units when calculating θ.
- Points on the z-axis: When $x = 0$ and $y = 0$, R is zero, and θ is undefined; handle these as special cases.

Applying the Conversion to Specific Points

Let's now apply this process to the given points:

A) Point (-1, 1, 1)

Step 1: Calculate R

\[
R = \sqrt{(-1)^2 + 1^2} = \sqrt{1 + 1} = \sqrt{2} \approx 1.4142
\]

Step 2: Calculate θ

Using `atan2(y, x)`:

\[
θ = \arctan2(1, -1)
\]

Since x is negative and y is positive, the point lies in the second quadrant.

\[
θ = \arctan2(1, -1) \approx 2.3562 \text{ radians} \approx 135^\circ
\]

Step 3: z Coordinate

\[
z = 1
\]

Result:

\[
\boxed{
(R, θ, z) \approx (1.4142, 2.3562 \text{ rad}, 1)
}
\]

B) Point (-7, 73, 7)

Step 1: Calculate R

\[
R = \sqrt{(-7)^2 + 73^2} = \sqrt{49 + 5329} = \sqrt{5378} \approx 73.355
\]

Step 2: Calculate θ

$$
\theta = \arctan2(73, -7)
$$

Here, x is negative, y is positive, so the point is in the second quadrant.

$$
\theta \approx \arctan2(73, -7) \approx 1.6476 \text{ radians} \text{ (or approximately } 94.5^\circ)
$$

Step 3: z Coordinate

$$
z = 7
$$

Result:

$$
\boxed{
(R, \theta, z) \approx (73.355, 1.6476 \text{ rad}, 7)
}
$$

Interpreting the Results and Practical Uses

Converting between coordinate systems allows for more natural problem-solving in various contexts:

- Symmetry Exploitation: Cylindrical coordinates simplify integration and differential equations involving rotational symmetry.
- Physics Applications: Describing electromagnetic fields, fluid flow, and heat transfer around cylinders or wires.
- Engineering Design: Modeling components with cylindrical geometry.

Additional Tips for Accurate Conversion

- Always use the `atan2(y, x)` function when available, as it correctly identifies the angle's quadrant.
- Be mindful of units; convert angles to degrees or radians based on your application.
- When handling points on the axes (where x or y is zero), interpret R and θ carefully to avoid undefined values.

- For visualization, plot the point in the xy-plane to confirm the calculated R and θ.

Summary

Converting from rectangular to cylindrical coordinates involves straightforward calculations of the radius R, the angle θ, and using the existing z-coordinate. This process enhances problem-solving efficiency and provides better insights into the geometry of three-dimensional systems. Applying these conversions to the provided points demonstrates the method's practicality and importance in fields such as physics, engineering, and mathematics.

Conclusion

Mastering the conversion from rectangular to cylindrical coordinates is essential for tackling complex spatial problems involving symmetry and cylindrical geometries. By understanding the underlying principles and practicing with specific points like A) (-1, 1, 1) and B) (-7, 73, 7), students and professionals can develop a robust skill set that enhances analytical capabilities across diverse scientific and engineering disciplines. Remember to always verify your calculations, especially the angles, to ensure accurate and meaningful results in your applications.

Frequently Asked Questions

How do you convert the point (-1, 1, 1) from rectangular to cylindrical coordinates?

To convert from rectangular (x, y, z) to cylindrical (r, θ, z), use $r = \sqrt{x^2 + y^2}$, θ = arctangent(y / x), and z remains the same. For (-1, 1, 1): $r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$, θ = arctangent(1 / -1) = arctangent(-1) = -45° or 135°, depending on the quadrant. Since $x<0$ and $y>0$, θ = 135°. Therefore, the cylindrical coordinates are ($\sqrt{2}$, 135°, 1).

What are the cylindrical coordinates for the point (-7, 73, 7)?

Using $r = \sqrt{x^2 + y^2}$: $r = \sqrt{(-7)^2 + 73^2} = \sqrt{49 + 5329} = \sqrt{5378} \approx 73.34$. θ = arctangent(y / x) = arctangent(73 / -7) ≈ arctangent(-10.43). Since $x<0$ and $y>0$, θ ≈ 180° - arctangent(10.43) ≈ 180° - 84.45° ≈ 95.55°. The z-coordinate remains the same. Thus, the cylindrical coordinates are approximately (73.34, 95.55°, 7).

Why is it necessary to consider the quadrant when calculating

θ in cylindrical coordinates?

Because arctangent alone only gives angles between -90° and 90°, which can lead to ambiguity about the actual quadrant of the point. Determining the correct quadrant ensures θ accurately represents the direction from the positive x-axis, which is essential for precise coordinate conversion.

What is the significance of the range 0 ≤ θ < 2π in cylindrical coordinates?

This range standardizes the angle θ to cover the entire circle from 0 to just less than 2π radians, ensuring each point in space has a unique set of cylindrical coordinates and avoiding ambiguity caused by multiple angles representing the same point.

How does the z-coordinate change when converting from rectangular to cylindrical coordinates?

The z-coordinate remains unchanged during the conversion; it is the same in both coordinate systems. Therefore, z in cylindrical coordinates equals z in rectangular coordinates.

Additional Resources

Change From Rectangular To Cylindrical Coordinates (Let R 0 And 0 2)

Converting from rectangular (Cartesian) coordinates to cylindrical coordinates is a fundamental skill in multivariable calculus, physics, and engineering. This transformation allows us to analyze problems with symmetry around an axis, simplifying calculations involving surfaces, curves, and vector fields. In this guide, we'll explore the process of changing from rectangular to cylindrical coordinates, focusing on the specific cases of points (-1, 1, 1) and (-7, 73, 7), assuming the notation "Let R 0 And 0 2" indicates that the radial coordinate \(R \) ranges from 0 to 2, and the angular coordinate \(\theta \) spans from 0 to 2π radians.

Understanding Cylindrical Coordinates

Cylindrical coordinates are a three-dimensional coordinate system characterized by three parameters:

- \(R \): the radial distance from the z-axis (\(R \geq 0 \))
- \(\theta \): the angle in the xy-plane measured from the positive x-axis (\(0 \leq \theta < 2\pi \))
- \(z \): the height above the xy-plane (same as in Cartesian coordinates)

The relationships between Cartesian coordinates \((x, y, z) \) and cylindrical coordinates \((R, \theta, z) \) are:

\[
x = R \cos \theta

$$
y = R \sin \theta
$$

$$
z = z
$$

Conversely,

$$
R = \sqrt{x^2 + y^2}
$$

$$
\theta = \arctan \left(\frac{y}{x} \right)
$$

$$
z = z
$$

Step-by-Step Guide to Converting Points

Let's analyze how to convert specific points from rectangular to cylindrical coordinates, focusing on points:

- A) $(-1, 1, 1)$
- B) $(-7, 73, 7)$

1. Converting Point A: (-1, 1, 1)

Step 1: Calculate R

$$
R = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + 1^2} = \sqrt{1 + 1} = \sqrt{2}
$$

Step 2: Calculate θ

$$
\theta = \arctan \left(\frac{y}{x} \right) = \arctan \left(\frac{1}{-1} \right) = \arctan(-1)
$$

Since $x = -1$ and $y = 1$, the point lies in the second quadrant (because x is negative and y is positive). The principal value:

$$
\arctan(-1) = -\frac{\pi}{4}
$$

But in the second quadrant, θ should be:

$$
\theta = \pi - \left| \arctan \left(\frac{y}{x} \right) \right| = \pi - \frac{\pi}{4} = \frac{3\pi}{4}
$$

Step 3: Retain $z = 1$.

Result:

$$
\boxed{
(R, \theta, z) = \left(\sqrt{2}, \frac{3\pi}{4}, 1 \right)
}
$$

---

2. Converting Point B: (-7, 73, 7)

Step 1: Calculate R

$$
R = \sqrt{(-7)^2 + 73^2} = \sqrt{49 + 5329} = \sqrt{5378}
$$

Simplify if needed:

$$
R \approx \sqrt{5378} \approx 73.36
$$

Step 2: Calculate θ

$$
\theta = \arctan \left(\frac{73}{-7} \right) = \arctan(-10.43)
$$

Since $x = -7$ (negative) and $y = 73$ (positive), the point lies in the second quadrant. The principal value:

$$
\arctan(-10.43) \approx -1.471 \text{ radians}
$$

Adjust for the second quadrant:

$$
\theta = \pi + \arctan \left(\frac{y}{x} \right) = \pi + (-1.471) = 3.1416 - 1.471 \approx 1.6706 \text{ radians}
$$

Step 3: Retain $z = 7$.

Result:

$$
\boxed{
(R, \theta, z) \approx \left(73.36, 1.6706, 7 \right)
}
$$

Visualizing the Transformation

Understanding the geometric interpretation of converting points can deepen comprehension:

- The radius R measures the distance from the z-axis.
- The angle θ indicates the position around the z-axis, measured from the positive x-axis.
- The height z remains unchanged in the transformation.

In practice, this conversion simplifies analyzing problems with cylindrical symmetry, such as fluid flow in pipes or electromagnetic fields around wires.

Additional Considerations

Range of R and θ:

- R is always non-negative.
- θ is typically taken in $[0, 2\pi)$, but can be extended beyond for certain applications.

Special Cases:

- When $x = 0$ and $y > 0$, $\theta = \frac{\pi}{2}$.
- When $x = 0$ and $y < 0$, $\theta = \frac{3\pi}{2}$.

Practical Applications and Limitations

Transitioning between rectangular and cylindrical coordinates is critical in:

- Calculating integrals over cylindrical regions.
- Solving differential equations with symmetry.
- Modeling physical phenomena like heat conduction in cylinders or wave propagation.

However, care must be taken with the inverse tangent function, as it only returns principal values. Always consider the signs of x and y to determine the correct quadrant.

Summary

- To change from rectangular to cylindrical coordinates:
- Compute $R = \sqrt{x^2 + y^2}$.
- Determine $\theta = \arctan \left(\frac{y}{x} \right)$, adjusting for the correct quadrant.
- Keep z the same.
- For the specific points:
- $(-1, 1, 1) \rightarrow (\sqrt{2}, 3\pi/4, 1)$
- $(-7, 73, 7) \rightarrow (73.36, 1.6706, 7)$

Mastering this transformation enhances your ability to analyze complex three-dimensional problems with cylindrical symmetry efficiently.

Final Notes

Remember, the key to successful coordinate transformation lies in understanding the geometric implications and being mindful of the principal values and quadrants. Practice with various points and functions to develop intuition and confidence in switching between coordinate systems seamlessly.

Change From Rectangular To Cylindrical Coordinates Let R 0 And 0 2 A 1 1 1 B 7 73 7

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