Consider A Circle Whose Equation Is X^2+y^2-2x-8=0. Which Statements Are True? Select Three Options

Consider A Circle Whose Equation Is X^2 + y^2 - 2x - 8 = 0. Which Statements Are True? Select Three Options

Understanding the properties of a circle from its algebraic equation is fundamental in coordinate geometry. When given an equation like X^2 + y^2 - 2x - 8 = 0, the goal is to analyze its components, rewrite it in a more recognizable form, and determine key attributes such as its center, radius, and whether it passes through specific points or satisfies certain conditions. This article provides a comprehensive guide to interpreting the given equation, exploring its geometric meaning, and identifying which statements about the circle are true.

Introduction to the Equation of a Circle

A circle in the coordinate plane is defined as the set of all points equidistant from a fixed point called the center. The standard form of a circle's equation is:

$$ (x - h)^2 + (y - k)^2 = r^2 $$

where:

- (h, k) are the coordinates of the circle's center,
- r is the radius of the circle.

However, many equations are given in a general quadratic form, such as:

$$ x^2 + y^2 + Dx + Ey + F = 0 $$

which can be converted into the standard form by completing the square.

Analyzing the Given Equation

The equation provided is:

$$ x^2 + y^2 - 2x - 8 = 0 $$

To analyze this circle, the first step is to rewrite it in standard form, which makes it easier to identify the center and radius.

Rewriting the Equation in Standard Form

Let's perform the process of completing the square on the x-terms:

1. Group the x and y terms:

$$ (x^2 - 2x) + y^2 = 8 $$

2. Complete the square for the x terms:

- Take half of the coefficient of x, which is -2:

$$ \frac{-2}{2} = -1 $$

- Square it:

$$ (-1)^2 = 1 $$

- Add and subtract this value to complete the square:

$$ (x^2 - 2x + 1) - 1 + y^2 = 8 $$

3. Rewrite as:

$$ (x - 1)^2 + y^2 = 8 + 1 $$

$$ (x - 1)^2 + y^2 = 9 $$

Now, the equation is in standard form:

$$ (x - 1)^2 + y^2 = 9 $$

Identifying the Key Properties of the Circle

From the standard form, we can extract the key properties:

- Center: $(h, k) = (1, 0)$
- Radius: $r = \sqrt{9} = 3$

This information allows us to evaluate various statements about the circle.

Evaluating Statements About the Circle

Suppose we are given a set of statements and asked to identify which three are true based on the analysis above. Let's consider common types of statements that could be posed:

1. The circle has a center at $(1, 0)$.
2. The radius of the circle is 3.
3. The circle passes through the point $(4, 0)$.
4. The circle passes through the point $(1, 3)$.
5. The circle's equation is $(x - 1)^2 + y^2 = 9$.
6. The circle is centered at $(-1, 0)$.
7. The circle includes the point $(1, -3)$.

Let's analyze each statement.

Statement 1: The circle has a center at $(1, 0)$

True.
From the standard form $(x - 1)^2 + y^2 = 9$, the center is clearly at $(1, 0)$.

Statement 2: The radius of the circle is 3

True.
Since $r^2 = 9$, then $r = \sqrt{9} = 3$.

Statement 3: The circle passes through the point $(4, 0)$

Check:
Plug $(x, y) = (4, 0)$ into the standard form:

$$(4 - 1)^2 + 0^2 = 3^2$$
$$3^2 + 0 = 9$$

$$9 = 9$$

This satisfies the equation, so the point $(4, 0)$ lies on the circle, making this statement true.

Statement 4: The circle passes through the point $(1, 3)$

Check:
Plug $(1, 3)$:

$$(1 - 1)^2 + 3^2 = 0 + 9 = 9$$

which equals $r^2 = 9$, so the point $(1, 3)$ lies on the circle. This statement is true.

Statement 5: The circle's equation is $(x - 1)^2 + y^2 = 9$

True.
This is the standard form derived from the original equation.

Statement 6: The circle is centered at $(-1, 0)$

False.
The center is at $(1, 0)$, not $(-1, 0)$.

Statement 7: The circle includes the point $(1, -3)$

Check:
Plug $(1, -3)$:

$$(1 - 1)^2 + (-3)^2 = 0 + 9 = 9$$

which equals r^2. Therefore, the point $(1, -3)$ is on the circle, and this statement is true.

Summary of True Statements

Based on the analysis:

- Statement 1: True
- Statement 2: True
- Statement 3: True
- Statement 4: True
- Statement 5: True
- Statement 6: False
- Statement 7: True

Given the instruction to select three options, the most critical and universally true statements are:

1. The circle has a center at $(1, 0)$.
2. The radius of the circle is 3.
3. The circle passes through points such as $(4, 0)$ and $(1, 3)$.

Additional Insights and Applications

Understanding the properties of a circle from its equation is crucial in many areas, including physics, engineering, and computer graphics. Recognizing the standard form allows for quick identification of a circle's center and radius, which are essential for tasks like collision detection, design, and analysis.

Practical applications include:

- Designing circular components in mechanical engineering.
- Determining distances in navigation and GPS technology.
- Analyzing geometric transformations in computer graphics.

Conclusion

By transforming the given general quadratic equation into the standard form, we can easily extract the circle's key features: center at \((1, 0)\) and radius 3. This understanding helps in verifying various statements about the circle, such as whether specific points lie on it or if particular properties hold true.

In summary, the three most accurate statements about the circle \(x^2 + y^2 - 2x - 8 = 0\) are:

- It has a center at \((1, 0)\).
- Its radius is 3.
- It passes through points like \((4, 0)\), \((1, 3)\), and \((1, -3)\).

Mastering these analytical techniques enhances your ability to interpret and manipulate equations representing circles, a fundamental skill in analytical geometry and related fields.

Frequently Asked Questions

What is the center of the circle given by the equation x^2 + y^2 - 2x - 8 = 0?

The center is at (1, 0).

What is the radius of the circle represented by the equation x^2 + y^2 - 2x - 8 = 0?

The radius is √5.

Does the circle pass through the point (3, 0)?

Yes, because substituting (3, 0) satisfies the equation.

Is the circle's equation in standard form? If not, how can it be rewritten?

No, it can be rewritten as (x - 1)^2 + y^2 = 5 in standard form.

Is the radius of the circle greater than 2?

No, since the radius is √5 ≈ 2.24, it is slightly greater than 2.

Is the circle centered at (1, 0)?

Yes, the standard form reveals the center at (1, 0).

Does the circle intersect the x-axis?

Yes, because at $y=0$, the equation simplifies to $x^2 - 2x - 8=0$, which has real solutions.

Are the statements 'The circle's center is at (1, 0)', 'The radius is √5', and 'The circle passes through (3, 0)' all true?

Yes, all three statements are true.

Additional Resources

Understanding the Equation: A Deep Dive into the Circle

When presented with the equation $X^2 + y^2 - 2x - 8 = 0$, one of the first steps in analyzing it is to recognize its geometric nature. The form hints at a circle, but to confirm this and understand its properties, we need to manipulate the equation into a more familiar form. This process involves completing the square and then interpreting the resulting equation. Let’s explore this in detail.

Transforming the Equation into Standard Form

The original equation:

$$X^2 + y^2 - 2x - 8 = 0$$

is a quadratic in both variables. To analyze it geometrically, the goal is to express it in the standard form of a circle’s equation:

$$(x - h)^2 + (y - k)^2 = r^2$$

where (h, k) is the center of the circle, and r is its radius.

Step-by-step process:

1. Group the x-terms and y-terms:

$$(X^2 - 2x) + y^2 = 8$$

2. Complete the square for the x-terms:

- The coefficient of x is -2. Half of -2 is -1, and squaring it gives 1.

- Add and subtract 1 within the equation to complete the square:

$$ (X^2 - 2x + 1) - 1 + y^2 = 8 $$

which simplifies to:

$$ (X - 1)^2 + y^2 = 8 + 1 $$

3. Simplify the right side:

$$ (X - 1)^2 + y^2 = 9 $$

Now, the equation is in standard form, revealing critical properties of the circle.

Key Properties of the Circle

From the standard form:

$$ (x - 1)^2 + y^2 = 9 $$

we can extract:

- Center: $(h, k) = (1, 0)$

- Radius: $r = \sqrt{9} = 3$

These parameters are fundamental in understanding the geometric placement and size of the circle.

Analyzing the Statements: Which Are True?

Given the equation and its properties, the problem asks to identify which three statements are true about this circle. While the exact options aren’t specified here, typical statements to evaluate might include:

1. The circle is centered at $(1, 0)$.

2. The radius of the circle is 3.

3. The circle passes through the point $(4, 0)$.

4. The circle’s equation can be derived by completing the square.

5. The circle intersects the x-axis at two points.

6. The circle intersects the y-axis at exactly one point.

7. The circle has a diameter of 6 units.

8. The circle’s equation is symmetric with respect to the x-axis and y-axis.

Let’s evaluate common statements based on our findings:

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1. The Center of the Circle Is at $(1, 0)$

Verdict: True

- From the standard form $(x - 1)^2 + y^2 = 9$, the center is explicitly $(1, 0)$.

- The process of completing the square confirms this point, and the algebra is straightforward.

Implication: Knowing the center allows us to visualize the circle’s position relative to the coordinate axes.

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2. The Radius of the Circle Is 3

Verdict: True

- Since the right side of the standard form is 9, the radius:

$$ r = \sqrt{9} = 3 $$

- This is a fundamental property directly read from the equation.

Implication: The circle extends 3 units in all directions from its center.

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3. The Circle Passes Through the Point $(4, 0)$

Verdict: True

- To verify, substitute $(x, y) = (4, 0)$ into the original equation:

$$ (4)^2 + (0)^2 - 2(4) - 8 = 0 $$

$$ 16 + 0 - 8 - 8 = 0 $$

$$ 0 = 0 $$

- Since the equation holds true, the point $(4, 0)$ lies on the circle.

Implication: The circle intersects the x-axis at $(4, 0)$ and, due to symmetry, also at $(-2, 0)$.

Additional Statements to Consider

While the three statements above are confirmed as true, let’s analyze other potential statements for completeness:

a) The circle intersects the x-axis at two points

- The x-axis corresponds to $y=0$.

- Substituting $y=0$ into the standard form:

$$ (x - 1)^2 + 0^2 = 9 \Rightarrow (x - 1)^2 = 9 $$

- Solving:

$$ x - 1 = \pm 3 \Rightarrow x = 1 \pm 3 $$

$$ x = 4 \quad \text{or} \quad x = -2 $$

- Points: $(4, 0)$ and $(-2, 0)$

- Both are on the circle, confirming that the circle intersects the x-axis at two points.

b) The circle intersects the y-axis at exactly one point

- The y-axis corresponds to $x=0$.

- Substituting $x=0$:

$$ (0 - 1)^2 + y^2 = 9 \Rightarrow 1 + y^2 = 9 \Rightarrow y^2 = 8 $$

$$ y = \pm \sqrt{8} = \pm 2 \sqrt{2} $$

- Points: $(0, 2\sqrt{2})$ and $(0, -2\sqrt{2})$

- Both points are on the circle, so the circle intersects the y-axis at two points, not just one.

c) The diameter of the circle is 6 units

- Since the radius is 3, diameter:

$$ d = 2r = 2 \times 3 = 6 $$

- This statement is true.

d) The equation can be derived by completing the square

- As demonstrated earlier, the standard form was obtained via completing the square, confirming this is true.

Implications of the Circle’s Properties in the Coordinate Plane

Understanding the circle's position and size helps in various applications, from geometry to real-world modeling.

Geometric insights include:

- Symmetry: The circle is symmetric about both the line $x=1$ and the x-axis, owing to its center at $(1, 0)$.

- Quadrant location: Since the center is at $(1, 0)$ and radius is 3, the circle extends into quadrants I, II, and possibly III depending on the exact position.

- Intersections with axes: As established, it intersects the x-axis at $(4, 0)$ and $(-2, 0)$, and the y-axis at $(0, 2\sqrt{2})$ and $(0, -2\sqrt{2})$.

Conclusion: Summarizing the Truths

Based on the detailed analysis, the three most accurate and relevant statements about the circle with equation $X^2 + y^2 - 2x - 8=0$ are:

1. The circle is centered at $(1, 0)$.

2. The radius of the circle is 3 units.

3. The circle passes through the point $(4, 0)$.

These conclusions are directly supported by algebraic manipulation and geometric interpretation.

Final Remarks and Broader Context

Understanding the properties of circles from their equations is a foundational skill in coordinate geometry. The process demonstrated here—completing the square to derive the standard form—is crucial for analyzing conic sections.

Additional considerations include:

- The importance of standard form in visualizing the circle's position.

- The role of symmetry and intercepts in understanding the circle’s intersection points.

- How the algebraic process translates into geometric insights.

In real-world applications, such as engineering, physics, and computer graphics, being able to interpret and manipulate such equations allows for precise modeling of circular objects and phenomena.

In summary, the equation $X^2 + y^2 - 2x - 8=0$ describes a circle with a clear center and radius, passing through specific points, and exhibiting expected geometric properties. Recognizing these features enables a deep understanding of the circle’s behavior in the coordinate plane, illustrating the power of algebraic transformations combined with geometric reasoning.

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