Determine The Equation Of The Circle With Center (9,5)(9,5) Containing The Point (10,2)(10,2).

Determine The Equation Of The Circle With Center (9,5) Containing The Point (10,2)

Understanding how to find the equation of a circle when given its center and a point on its circumference is a fundamental skill in geometry and analytic mathematics. In this guide, we will explore the step-by-step process to determine the equation of a circle with the center at point (9, 5) and passing through the point (10, 2). This process involves calculating the radius, understanding the standard form of a circle's equation, and applying these concepts to find the precise equation. Whether you're a student preparing for exams or a math enthusiast seeking clarity, this comprehensive guide will enhance your understanding of circle equations.

Understanding the Equation of a Circle

Standard Form of a Circle's Equation

The general equation of a circle in the coordinate plane is expressed as:

$$(x - h)^2 + (y - k)^2 = r^2$$

where:
- (h, k) is the center of the circle.
- r is the radius of the circle.

This form clearly shows how the center's coordinates influence the position of the circle, and the radius determines its size.

Importance of the Radius in the Equation

The radius r is a crucial component because it defines the distance from the center to any point on the circle. To find the equation completely, we need to determine r accurately, often by using a known point on the circle.

Step-by-Step Process to Determine the Equation

To find the equation of the circle with center $(9, 5)$ that passes through $(10, 2)$, follow these steps:

Step 1: Recall the standard form of the circle

$$
(x - h)^2 + (y - k)^2 = r^2
$$

Substituting the known center $(h, k) = (9, 5)$:

$$
(x - 9)^2 + (y - 5)^2 = r^2
$$

Step 2: Find the radius using the known point

Since the point $(10, 2)$ lies on the circle, the distance from this point to the center is the radius.

Calculate this distance using the distance formula:

$$
r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

where:
- $(x_1, y_1) = (9, 5)$ (center)
- $(x_2, y_2) = (10, 2)$ (point on the circle)

Applying the values:

$$
r = \sqrt{(10 - 9)^2 + (2 - 5)^2}
$$

$$
r = \sqrt{(1)^2 + (-3)^2}
$$

$$
r = \sqrt{1 + 9} = \sqrt{10}
$$

Step 3: Write the complete equation

Now that we have \(r^2 = 10 \), substitute into the standard form:

\[
(x - 9)^2 + (y - 5)^2 = 10
\]

This is the equation of the circle with the given center and passing through the specified point.

Additional Considerations and Variations

Converting to Expanded Form

Sometimes, it is useful to expand the circle's equation for clarity or specific applications:

\[
(x - 9)^2 + (y - 5)^2 = 10
\]

Expanding:

\[
x^2 - 2 \times 9 \times x + 9^2 + y^2 - 2 \times 5 \times y + 5^2 = 10
\]

which simplifies to:

\[
x^2 - 18x + 81 + y^2 - 10y + 25 = 10
\]

Combine like terms:

\[
x^2 - 18x + y^2 - 10y + (81 + 25 - 10) = 0
\]

\[
x^2 - 18x + y^2 - 10y + 96 = 0
\]

This form is often used for graphing or algebraic analysis.

Graphing the Circle

To graph the circle:

- Plot the center $(9, 5)$.
- Use the radius $r = \sqrt{10} \approx 3.16$.
- Draw a circle around the center with this radius.

Applications and Relevance

Understanding how to determine the equation of a circle is vital in various fields:

- Geometry and Trigonometry: Fundamental for solving problems involving shapes, distances, and angles.
- Computer Graphics: Essential in rendering circles and curves accurately.
- Engineering: Used in designing circular components and analyzing mechanical systems.
- Physics: Helps in modeling orbits and circular motion.

Common Mistakes and Tips

- Incorrectly calculating the radius: Always double-check the distance calculation.
- Mixing coordinate signs: Remember the minus signs in the equation correspond to the center's coordinates.
- Forgetting to square the radius: The standard form uses r^2, not r.

Tips:

- Always write down the known points clearly.
- Use the distance formula carefully and verify calculations.
- Confirm the form of the equation before expanding or manipulating.

Conclusion

Determining the equation of a circle when given a center and a point on the circle is a straightforward process rooted in understanding the standard form of a circle's equation and the distance formula. By calculating the radius from the known point to the center, you can confidently write the circle's equation in either standard or expanded form. This skill is fundamental in geometry, algebra, and many applied sciences, providing a foundation for more complex mathematical concepts involving circles and their properties.

In summary, for the circle with center $(9, 5)$ passing through $(10, 2)$, the equation is:

$$
\boxed{(x - 9)^2 + (y - 5)^2 = 10}
$$

Mastering these steps enhances your ability to analyze and work with circles effectively in various mathematical contexts.

Frequently Asked Questions

What is the general form of the equation of a circle with center (h, k) and radius r?

The general form is $(x - h)^2 + (y - k)^2 = r^2$.

How do you find the radius of a circle given its center and a point on the circle?

Calculate the distance between the center and the point using the distance formula: $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

What are the steps to determine the equation of the circle with center (9,5) passing through (10,2)?

First, find the radius by calculating the distance between (9,5) and (10,2), then substitute the center coordinates and radius into the standard circle equation.

What is the distance between the center (9,5) and the point (10,2)?

The distance is $\sqrt{(10 - 9)^2 + (2 - 5)^2} = \sqrt{1 + 9} = \sqrt{10}$.

What is the numerical value of the radius for this circle?

The radius is $\sqrt{10}$, approximately 3.16 units.

What is the equation of the circle with center (9,5) and radius √10?

$(x - 9)^2 + (y - 5)^2 = 10$.

How can you verify that the point (10,2) lies on the circle $(x - 9)^2 + (y - 5)^2 = 10$?

Substitute (10,2) into the equation: $(10 - 9)^2 + (2 - 5)^2 = 1 + 9 = 10$, which satisfies the equation.

Can you write the equation of the circle in expanded form?

Yes. Expanding: $(x - 9)^2 + (y - 5)^2 = 10$ becomes $x^2 - 18x + 81 + y^2 - 10y + 25 = 10$, which simplifies to $x^2 + y^2 - 18x - 10y + 106 = 0$.

Why is understanding the derivation of the circle's equation important in geometry?

It helps in solving geometric problems involving circles, such as finding intersections, tangents, and understanding properties related to circle equations.

Additional Resources

Determine The Equation Of The Circle With Center (9,5) Containing The Point (10,2)

Understanding how to determine the equation of a circle when given its center and a point on its circumference is a fundamental concept in coordinate geometry. This problem encapsulates key principles of the geometry of circles, providing insight into how algebraic formulas relate to geometric intuition. In this article, we will thoroughly explore the process involved in deriving the equation of a circle with a specific center and a point lying on its perimeter, examine the underlying concepts, and highlight common pitfalls and tips for success.

Introduction to the Equation of a Circle

Before diving into the problem specifics, it’s essential to revisit the general form of the equation of a circle and understand the parameters involved.

Standard Equation of a Circle

The standard form of a circle's equation in the coordinate plane is:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

where:
- (h, k) is the center of the circle.

- r is the radius of the circle.

The goal is often to find the specific equation when the center and a point on the circle are known, which involves determining r.

Given Data and Its Significance

In the problem, the center of the circle is given as $(9, 5)$, and a point on the circle is $(10, 2)$.

Center $(h, k) = (9, 5)$

This point serves as the fixed point around which the circle is symmetric. Knowing the center allows us to formulate the equation with a specific (h, k) and then focus on finding the radius.

Point on the circle: $(10, 2)$

This point is critical because it lies exactly on the circumference. Its coordinates will help us calculate the radius by measuring the distance from the center to this point.

Step-by-Step Solution Process

Let's break down the process into manageable steps to derive the circle’s equation.

Step 1: Recall the formula for the radius

The radius r can be calculated using the distance formula between the center (h, k) and the point (x, y):

$$
r = \sqrt{(x - h)^2 + (y - k)^2}
$$

Step 2: Plug in the known values

Using the given data:

- Center: $(9, 5)$
- Point: $(10, 2)$

Calculate:

$$
r = \sqrt{(10 - 9)^2 + (2 - 5)^2} = \sqrt{(1)^2 + (-3)^2} = \sqrt{1 + 9} = \sqrt{10}
$$

Thus, the radius $r = \sqrt{10}$.

Step 3: Write the equation of the circle

Substitute the center $(h, k) = (9, 5)$ and $r^2 = 10$ into the standard form:

$$
(x - 9)^2 + (y - 5)^2 = 10
$$

This is the equation of the circle.

Features and Analysis of the Derived Equation

Having obtained the equation, it's beneficial to analyze its features and understand what it reveals about the circle.

Properties of the circle

- Center: $(9, 5)$
- Radius: $\sqrt{10} \approx 3.16$
- Equation: $(x - 9)^2 + (y - 5)^2 = 10$

Graphical interpretation

On the Cartesian plane, the circle is centered at $(9, 5)$ with a radius slightly over 3 units. It passes through the point $(10, 2)$, confirming the correctness of the calculation.

Implications for geometry

- The circle is symmetric about the line $x = 9$ and $y = 5$.
- The point $(10, 2)$ is approximately 3.16 units from the center, confirming it lies on the circle.

Common Challenges and How to Address Them

While the steps seem straightforward, several common pitfalls can occur when solving such problems.

1. Confusing the Radius and Diameter

- Pitfall: Using the diameter instead of the radius.
- Solution: Remember that the radius is the distance from the center to any point on the circle, and the equation uses r^2.

2. Incorrectly Calculating Distance

- Pitfall: Mixing up subtraction signs or squaring terms.
- Solution: Carefully apply the distance formula and double-check calculations.

3. Forgetting to Square the Radius

- Pitfall: Using r instead of r^2 in the equation.
- Solution: Always square the radius when substituting into the standard form.

4. Misidentifying Coordinates

- Pitfall: Confusing the order of points or misreading given coordinates.
- Solution: Clearly label the center and point on the circle before calculations.

Extensions and Additional Considerations

Understanding this fundamental problem opens the door to more advanced topics and applications.

1. Equation of a Circle in General Form

Transforming the standard form into the general form:

$$
x^2 + y^2 + Dx + Ey + F = 0
$$

can be useful for certain algebraic manipulations or intersection problems. This involves expanding and rearranging the standard form.

2. Finding the Equation When the Center is Unknown

If only a point on the circle and the radius are given, one would reverse the process to find the center.

3. Use in Coordinate Geometry Problems

The derived equation can help solve problems involving tangency, intersections with other shapes, or locus problems.

4. Application in Real-World Contexts

Circles are prevalent in engineering, physics, and computer graphics; understanding how to derive their equations is crucial in these fields.

Summary and Final Remarks

Determining the equation of a circle with a given center and a point on its circumference involves a clear understanding of the relationship between the center, a point on the circle, and the radius. The process hinges on calculating the distance between the center and the point to find the radius, then substituting into the standard form. This fundamental skill is not only vital in pure mathematics but also in applied sciences and engineering disciplines.

Key steps recap:
- Use the distance formula to find the radius.
- Square the radius for substitution.
- Write the circle's equation in standard form.
- Analyze the geometric properties from the equation.

Pros of this approach:

- Straightforward application of algebraic formulas.
- Clear geometric interpretation.
- Easily adaptable to more complex problems.

Cons or limitations:
- Requires careful calculation to avoid errors.
- Assumes familiarity with coordinate geometry concepts.
- Less straightforward if the circle's center or radius is not directly given.

Mastering this process enhances one's ability to handle a broad array of geometry problems and deepens understanding of the relationship between algebra and geometry. It lays a solid foundation for tackling more advanced topics, such as conic sections and their various equations.

In conclusion, the question of determining the equation of a circle with a given center and a point on its circumference is an excellent illustration of fundamental geometric principles. Through systematic application of the distance formula and algebraic substitution, one can accurately derive the equation, gaining both computational skills and geometric intuition in the process.

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