Determine The Qualities Of The Given Set. (select All That Apply.) (x, Y)| 9 < X2 + Y2 < 16

Determine The Qualities Of The Given Set. (select All That Apply.) (x, Y)| 9 < X^2 + Y^2 < 16

The set described by the inequality $9 < x^2 + y^2 < 16$ represents a specific region in the Cartesian plane. To analyze this set comprehensively, it is essential to interpret the inequality geometrically, understand its boundary conditions, and identify the qualities that define it. This article aims to explore the various attributes of the set step by step, providing a detailed understanding of its properties, including its shape, boundary, inclusion/exclusion of points, and other relevant features.

Understanding the Inequality: Geometric Interpretation

Basic Concept of the Inequality

The inequality involves $x^2 + y^2$, which is the squared distance from any point (x, y) to the origin $(0, 0)$. Specifically, $x^2 + y^2$ is the radius squared in the polar coordinate system. The inequality $9 < x^2 + y^2 < 16$ establishes a range for this distance squared, defining a particular region in the plane.

Geometric Representation

- Since $x^2 + y^2 = r^2$ in polar coordinates, the inequality translates to:
  - Radius squared r^2 is greater than 9 and less than 16.
- Therefore, the set corresponds to all points whose distance from the origin r satisfies:

- $\sqrt{9} < r < \sqrt{16}$
- $3 < r < 4$

Shape of the Set

In the Cartesian plane, this set is the region between two concentric circles:

1. Inner circle: with radius 3, defined by $x^2 + y^2 = 9$.
2. Outer circle: with radius 4, defined by $x^2 + y^2 = 16$.

The inequalities exclude the boundary circles themselves because they are strict inequalities (greater than and less than, not equal to). Hence, the region is an annulus (ring-shaped region) without the boundary circles.

Qualities of the Set

1. Geometric Shape and Nature

The set is an annulus in the plane, characterized by the following qualities:

- **Type:** Open annulus (ring-shaped region)
- **Shape:** Circular, symmetric about the origin
- **Boundaries:** Neither boundary circle is included because of strict inequalities

2. Boundary Conditions and Inclusion

Understanding whether the boundary points are included is crucial:

- Since the inequalities are strict ($>$ and $<$), the boundary circles $x^2 + y^2 = 9$ and $x^2 + y^2 = 16$ are not part of the set.

- This makes the set an open region in the plane, specifically an open annulus.

3. Location and Extent

The set is centered at the origin $(0,0)$ and extends in all directions equally:

- Radius from 3 to 4 units from the origin
- Infinite in extent tangentially, but confined between the two circles

4. Symmetry

The set possesses symmetry about both axes and the origin:

- **Rotational symmetry:** Invariant under any rotation about the origin
- **Reflective symmetry:** Symmetric about the x-axis and y-axis

5. Openness and Closedness

The set is an open subset of the plane because it does not include its boundary circles:

- Open set in topological terms
- Contains no boundary points, only interior points

6. Connectivity

The region is connected:

- Any two points within the annulus can be connected via a continuous path lying entirely within the set

7. Interior Points

All points within the open annulus are interior points:

- Points satisfy $9 < x^2 + y^2 < 16$

8. Measure and Area

The area of the annulus can be computed:

- Area of the outer circle: $\pi \times 4^2 = 16\pi$
- Area of the inner circle: $\pi \times 3^2 = 9\pi$
- Area of the annulus: $16\pi - 9\pi = 7\pi$

This area is finite and well-defined, indicating the set has a positive measure.

9. Inclusion and Exclusion of Points

Points lying exactly on the boundary circles are excluded, meaning:

- Points with $x^2 + y^2 = 9$ or $x^2 + y^2 = 16$ are not in the set
- All points with $9 < x^2 + y^2 < 16$ are included

Additional Qualities and Implications

1. Topological Properties

The set is:

- Open and connected, hence an open connected subset of $\mathbb{R}^2$
- Locally Euclidean, meaning it locally resembles $\mathbb{R}^2$

2. Functional and Analytical Considerations

From a mathematical analysis perspective, the set’s properties influence how functions behave within it:

- Potential for defining continuous functions over the set
- Suitable for integration and measure theory applications due to finite area

3. Visual and Practical Understanding

Visualizing the set helps solidify understanding:

- Imagine a ring-shaped region around the origin, with no boundary points included
- Useful in physics and engineering where such regions model zones between two spherical shells

Summary of Selected Qualities

Based on the analysis above, the qualities of the set $\{ (x, y) | 9 < x^2 + y^2 < 16 \}$ are:

- Shape: Open annulus (ring-shaped, bounded by two circles)
- Boundary points: Not included (strict inequalities)
- Location: Centered at the origin, symmetric in all directions
- Connectivity: The set is connected
- Openness: The set is open in $\mathbb{R}^2$
- Area: Finite, equal to 7π
- Symmetry: Symmetric about axes and origin

Conclusion

The set defined by $9 < x^2 + y^2 < 16$ encapsulates a well-understood geometric region—the open annulus between two circles with radii 3 and 4 centered at the origin. Its qualities include being open, connected, symmetric, and having a calculable finite area. Recognizing these properties is fundamental in various mathematical, physical, and engineering contexts where understanding the shape, boundary, and measure of regions in the plane is essential. The strict inequalities exclude the boundaries, emphasizing the importance of boundary conditions when analyzing set properties in topology and geometry.

Frequently Asked Questions

What is the set of all points (x, y) satisfying $9 < x^2 + y^2 < 16$?

It is the region between the circles with radii 3 and 4, excluding the boundaries.

Does the set include points on the circle $x^2 + y^2 = 9$?

No, points on $x^2 + y^2 = 9$ are not included because the inequality is strict ($9 < x^2 + y^2$).

Is the set of points with $x^2 + y^2$ between 9 and 16 bounded?

Yes, the set is bounded because it lies between two circles of finite radii.

Does the set include the origin (0,0)?

No, because at (0,0), $x^2 + y^2 = 0$, which does not satisfy $9 < x^2 + y^2 < 16$.

Is the set open or closed?

The set is open because it does not include the boundary circles where $x^2 + y^2 = 9$ or 16.

What is the shape of the set in the coordinate plane?

It is an annular (ring-shaped) region between two circles.

Can the set be considered a locus of points?

Yes, it can be considered the locus of points whose distance from the origin is between 3 and 4 units.

Which of the following is a property of the set? (select all that apply)

It is bounded, open, consists of all points between two circles, and excludes the boundary circles.

Additional Resources

Determine The Qualities Of The Given Set. (select All That Apply.) (x, Y)| 9 < X^2 + Y^2 < 16

Understanding the properties of sets defined by inequalities is fundamental in the study of geometry and algebra. The given set, which can be expressed as $\{(x, y) \mid 9 < x^2 + y^2 < 16 \}$, describes a specific region in the coordinate plane. The phrase “Determine The Qualities Of The Given Set” prompts us to analyze its geometric, algebraic, and topological features. This article will comprehensively explore the qualities of this set, breaking down its nature, boundaries, and other critical characteristics using structured headings for clarity.

Geometric Interpretation of the Set

Understanding the Inequalities

The set $\{(x, y) \mid 9 < x^2 + y^2 < 16 \}$ involves a double inequality related to the sum of squares of x and y. Since $x^2 + y^2$ is the equation of a circle in the coordinate plane centered at the origin, the inequalities suggest that we are dealing with an annular region, or a ring-shaped area, between two circles.

- Inner Boundary: $x^2 + y^2 = 9$ (a circle with radius 3)
- Outer Boundary: $x^2 + y^2 = 16$ (a circle with radius 4)

The inequalities specify that the set includes all points whose distance from the origin exceeds 3 but is less than 4. This geometric insight is crucial for visualizing the set.

Visual Representation

Graphically, the set looks like a ring or an annulus:

- Inner circle: centered at the origin with radius 3, excluded from the set because of the strict inequality > 9.
- Outer circle: centered at the origin with radius 4, also excluded because of the strict inequality < 16.

The set includes all points lying strictly between these two circles, forming a hollow ring where the boundary circles themselves are not part of the set.

Topological and Set-Theoretic Properties

Open, Closed, or Neither?

The nature of the set's boundaries determines whether it is open, closed, or neither.

- Since the inequalities are strict ($>$ and $<$), the set does not include the boundary circles where $x^2 + y^2 = 9$ or $x^2 + y^2 = 16$.
- Therefore, the set is open in the Euclidean plane because it does not contain its boundary points.

Features:

- Open set: Yes, because it does not include the boundary points.
- Closed set: No, since the boundary points are not included.
- Boundaries: The boundaries are the two circles $x^2 + y^2 = 9$ and $x^2 + y^2 = 16$, which are not part of the set.

Connectedness and Shape

- The set is connected because any two points within the annulus can be joined by a path lying entirely within the set.
- The shape is annular, a ring-shaped region, symmetric about the origin.

Algebraic and Analytical Qualities

Equation and Inequalities

The set is characterized algebraically by the inequality:

$$
9 < x^2 + y^2 < 16
$$

which describes all points in the plane where the distance from the origin, $\sqrt{x^2 + y^2}$, satisfies:

$$
3 < \sqrt{x^2 + y^2} < 4
$$

This makes it clear that the set is an open annulus between radii 3 and 4.

Symmetry

- The set is radially symmetric about the origin because the defining inequality depends only on $x^2 + y^2$.
- The symmetry implies that for any point (x, y) in the set, the entire circle centered at the origin with the same radius is also symmetric with respect to the set.

Continuity and Differentiability

- The set itself is a geometric region, but the defining functions $x^2 + y^2$ are continuous and differentiable everywhere.
- The boundary circles are level curves of the function $f(x, y) = x^2 + y^2$.

Implications of the Set's Properties

Implications for Calculus and Analysis

- The set is an example of an open domain in $\mathbb{R}^2$, suitable for applying various theorems such as the Open Set Theorem.
- Since it is open, functions defined on this set can be extended or analyzed using calculus techniques without boundary issues, unless boundary behavior is specifically studied.

Applications in Physics and Engineering

- Modeling regions with specific distance constraints, such as zones of influence or safety margins.
- In electromagnetism, the annulus can represent regions where potential or field strength satisfies certain conditions.

Additional Qualities and Selecting All That Apply

Based on the analysis above, the qualities of the set $\{(x, y) \mid 9 < x^2 + y^2 < 16 \}$ include:

- Open set in $\mathbb{R}^2$: Yes
- Bounded: Yes (contained within the circle of radius 4)
- Connected: Yes
- Symmetric about the origin: Yes
- Contains no boundary points: Yes
- Region between two circles: Yes
- Not closed: Yes
- Not compact: No, because it is bounded and open; but in $\mathbb{R}^2$, a set must be closed and bounded to be compact, so this set is not compact.

Pros and Cons of the Set's Qualities

Pros:

- The set's symmetry simplifies analysis and computations.
- Its openness makes it suitable for calculus applications requiring differentiability.
- Its bounded nature ensures it is contained within a finite region, which is useful for numerical methods.

Cons:

- The exclusion of boundary points limits certain boundary-dependent theorems.
- The set's shape might complicate certain types of integrations or boundary value problems where boundary conditions are crucial.
- The "ring" shape may be less intuitive for some applications or visualizations compared to simply connected regions.

Conclusion

The set \(\{(x, y) \mid 9 < x^2 + y^2 < 16 \}\) exemplifies a classic geometric and topological object: an open annular region centered at the origin. Its qualities—such as being open, bounded, symmetric, and connected—are fundamental attributes that influence its behavior and applications across mathematics, physics, and engineering. Recognizing these characteristics helps in understanding the structure of more complex regions and in applying the appropriate theorems and methods for analysis. Whether used in theoretical mathematics or practical modeling, the set's properties make it a versatile and insightful example of geometric and algebraic interplay.

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