Find A Vector Of Magnitude 6 In The Direction Opposite To The Direction Of V= 1/2i +1/2 J +1/2 K

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When working with vectors in physics and mathematics, a common problem involves finding a vector of a specific magnitude that points in a particular direction. In this case, the problem asks us to determine a vector with a magnitude of 6 units that points in the direction opposite to a given vector $\mathbf{V} = \frac{1}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{1}{2}\mathbf{k}$. Understanding how to approach this problem involves several key steps: identifying the direction, finding the unit vector, and then scaling it to the desired magnitude. This article will guide you through the process of solving such problems, providing insights into vector normalization, opposite directions, and scalar multiplication.

Understanding the Given Vector V

Components of the Vector V

The vector $\mathbf{V} = \frac{1}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{1}{2}\mathbf{k}$ has components:

- $\frac{1}{2}$ along the x-axis
- $\frac{1}{2}$ along the y-axis
- $\frac{1}{2}$ along the z-axis

This indicates that $\mathbf{V}$ points equally in the directions of the x, y, and z axes, forming a vector pointing diagonally in the positive octant.

Magnitude of Vector V

Calculating the magnitude of $\mathbf{V}$:

$$
|\mathbf{V}| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}
$$

Knowing the magnitude is crucial for normalizing the vector to a unit vector.

Finding the Unit Vector in the Same Direction as V

Definition of a Unit Vector

A unit vector has a magnitude of 1 and points in the same direction as the original vector. To find the unit vector $\hat{\mathbf{u}}$ in the direction of $\mathbf{V}$:

$$
\hat{\mathbf{u}} = \frac{\mathbf{V}}{|\mathbf{V}|}
$$

Calculating the Unit Vector

Using the components of $\mathbf{V}$ and its magnitude:

$$
\hat{\mathbf{u}} = \frac{\frac{1}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} + \frac{1}{2}\mathbf{k}}{\frac{\sqrt{3}}{2}} = \left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) \mathbf{i} + \left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) \mathbf{j} + \left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) \mathbf{k}
$$

Simplifying each component:

$$
\hat{\mathbf{u}} = \frac{1}{\sqrt{3}} \mathbf{i} + \frac{1}{\sqrt{3}} \mathbf{j} + \frac{1}{\sqrt{3}} \mathbf{k}
$$

This is the unit vector pointing in the same direction as $\mathbf{V}$.

Finding the Opposite Direction

Reversing the Direction

To find a vector pointing in the exact opposite direction, we negate the unit vector:

$$
- \hat{\mathbf{u}} = - \left(\frac{1}{\sqrt{3}} \mathbf{i} + \frac{1}{\sqrt{3}} \mathbf{j} + \frac{1}{\sqrt{3}} \mathbf{k} \right) = - \frac{1}{\sqrt{3}} \mathbf{i} - \frac{1}{\sqrt{3}} \mathbf{j} - \frac{1}{\sqrt{3}} \mathbf{k}
$$

Scaling the Opposite Direction to Magnitude 6

Using Scalar Multiplication

Once the direction is established, scale the unit vector by the desired magnitude:

$$
\mathbf{V}_{\text{desired}} = \text{magnitude} \times (- \hat{\mathbf{u}})
$$

Plugging in the values:

$$
\mathbf{V}_{\text{desired}} = 6 \times \left(- \frac{1}{\sqrt{3}} \mathbf{i} - \frac{1}{\sqrt{3}} \mathbf{j} - \frac{1}{\sqrt{3}} \mathbf{k} \right)
$$

Simplify:

$$
\mathbf{V}_{\text{desired}} = - \frac{6}{\sqrt{3}} \mathbf{i} - \frac{6}{\sqrt{3}} \mathbf{j} - \frac{6}{\sqrt{3}} \mathbf{k}
$$

Since $\frac{6}{\sqrt{3}} = 6 \times \frac{\sqrt{3}}{3} = 2 \sqrt{3}$:

$$
\mathbf{V}_{\text{desired}} = - 2 \sqrt{3} \mathbf{i} - 2 \sqrt{3} \mathbf{j} - 2 \sqrt{3} \mathbf{k}
$$

Final Result

The vector of magnitude 6 pointing in the opposite direction of $\mathbf{V}$ is:

$$
\boxed{
\mathbf{V}_{\text{final}} = - 2 \sqrt{3} \mathbf{i} - 2 \sqrt{3} \mathbf{j} - 2 \sqrt{3} \mathbf{k}
}
$$

This vector has the required magnitude and points directly opposite to the original vector $\mathbf{V}$.

Summary and Key Takeaways

- To find a vector of a specific magnitude in a particular direction, start by determining the unit vector in that direction.

- Negate the unit vector to reverse the direction.
- Multiply the resulting vector by the desired magnitude to scale it appropriately.
- Always ensure to compute the magnitude of the original vector before normalization.
- Understanding the relationship between vector components, magnitude, and scalar multiplication is essential for solving vector problems efficiently.

Applications of Finding Opposite Vectors

Understanding how to find vectors of specific magnitudes in opposite directions has numerous applications:

- In physics, calculating forces acting in opposite directions.
- In navigation, determining the heading opposite to a current or wind.
- In engineering, analyzing vector quantities such as velocity, acceleration, or force vectors.
- In computer graphics, manipulating object orientations and movements.

Conclusion

Finding a vector of a given magnitude in the opposite direction to a known vector involves a straightforward process of normalization, negation, and scalar multiplication. By understanding the components of the original vector, calculating its magnitude, and then scaling and reversing the direction, you can accurately determine the desired vector. Mastering these fundamental vector operations enhances your problem-solving skills in various scientific and engineering contexts, making complex vector calculations more manageable and intuitive.

Frequently Asked Questions

How do you find a vector of magnitude 6 pointing in the opposite direction of V = 1/2 i + 1/2 j + 1/2 k?

First, find the magnitude of V, then determine its unit vector, and finally multiply by 6 in the opposite direction to get the desired vector.

What is the magnitude of the vector V = 1/2 i + 1/2 j + 1/2 k?

The magnitude is $\sqrt{[(1/2)^2 + (1/2)^2 + (1/2)^2]} = \sqrt{(3/4)} = (\sqrt{3})/2$.

How do you compute the unit vector in the direction of V?

Divide each component of V by its magnitude: $(1/2) / (\sqrt{3}/2) = 1/\sqrt{3}$ for each component, so the unit vector is $(1/\sqrt{3})\, i + (1/\sqrt{3})\, j + (1/\sqrt{3})\, k$.

What is the opposite direction of the vector V?

The opposite direction is obtained by multiplying the unit vector by -1, resulting in $-(1/\sqrt{3})\, i - (1/\sqrt{3})\, j - (1/\sqrt{3})\, k$.

How do you find the vector of magnitude 6 in the opposite direction of V?

Multiply the unit vector in the opposite direction by 6: $6 [-(1/\sqrt{3})\, i - (1/\sqrt{3})\, j - (1/\sqrt{3})\, k] = -(6/\sqrt{3})\, i - (6/\sqrt{3})\, j - (6/\sqrt{3})\, k$.

What is the simplified form of the vector of magnitude 6 opposite to V?

Simplify $6/\sqrt{3}$ to $2\sqrt{3}$, so the vector is $-2\sqrt{3}\, i - 2\sqrt{3}\, j - 2\sqrt{3}\, k$.

Why is it important to normalize the vector before scaling to the desired magnitude?

Normalizing ensures the direction remains the same while scaling correctly to the specified magnitude—in this case, 6 units.

Can you verify the magnitude of the final vector?

Yes, the magnitude of the vector $-2\sqrt{3}\, i - 2\sqrt{3}\, j - 2\sqrt{3}\, k$ is $\sqrt{[(2\sqrt{3})^2 + (2\sqrt{3})^2 + (2\sqrt{3})^2]} = \sqrt{[12 + 12 + 12]} = \sqrt{36} = 6$, confirming correctness.

Additional Resources

Find A Vector Of Magnitude 6 In The Direction Opposite To The Direction Of V= 1/2i + 1/2 j + 1/2 k

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Introduction

In the realm of vector mathematics, understanding how to manipulate and derive vectors with specific properties is fundamental. Whether in physics, engineering, or computer science, vectors serve as essential tools for representing quantities that possess both magnitude and direction. Among the most common tasks encountered by students and professionals alike is finding a vector with a prescribed magnitude that points in a particular direction—especially the opposite direction of a given vector.

This article delves into a detailed, step-by-step process to find a vector of magnitude 6 that points exactly opposite to a given vector V = (1/2) i + (1/2) j + (1/2) k. Through comprehensive explanations, mathematical derivations, and contextual insights, we aim to clarify the underlying concepts and methods involved in this type of vector problem.

Understanding the Problem Statement

Before diving into the solution, it is crucial to interpret what the problem is asking:

- Given vector V = (1/2) i + (1/2) j + (1/2) k, which points in a specific direction in three-dimensional space.
- Find a vector:
- Of magnitude 6.
- In the direction opposite to V.

This involves two main components:

1. Determining the opposite direction of the given vector V.
2. Adjusting the vector's length (magnitude) to exactly 6, while maintaining its direction.

Understanding these components enables us to formulate a systematic approach to find the desired vector.

Conceptual Foundations

1. Vectors and Their Directions

A vector in three-dimensional space can be represented as:

\[
\vec{V} = a i + b j + c k

\]

where \(a, b, c\) are scalar components corresponding to the x, y, and z axes respectively.

The direction of a vector is determined by its orientation in space, which can be represented using a unit vector—a vector of magnitude 1 pointing in the same direction.

2. Opposite Direction of a Vector

The opposite of a vector \(\vec{V}\) is simply \(\ -\vec{V}\). It points in the same line but in the reverse direction.

Mathematically,

\[
-\vec{V} = -a i - b j - c k
\]

This operation flips the vector's direction without altering its magnitude.

3. Scaling Vectors to a Desired Magnitude

To obtain a vector with a specific magnitude k in a particular direction, we:

- First find the unit vector in that direction.
- Then scale that unit vector by k.

This process ensures the vector has the desired length while maintaining the specified orientation.

Step-by-Step Solution Approach

The problem involves the following steps:

1. Identify the initial vector \(\vec{V} \).
2. Normalize \(\vec{V} \) to find its unit vector.
3. Determine the opposite direction.
4. Scale the unit vector in the opposite direction to have magnitude 6.

Let's explore each step in detail.

Step 1: Identify the Given Vector

The given vector is:

$$
\vec{V} = \frac{1}{2} i + \frac{1}{2} j + \frac{1}{2} k
$$

Expressed in component form:

$$
\vec{V} = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right)
$$

Step 2: Find the Magnitude of $\vec{V}$

The magnitude (also called the length) of a vector $\vec{V} = (a, b, c)$ is:

$$
|\vec{V}| = \sqrt{a^2 + b^2 + c^2}
$$

Applying this to our vector:

$$
|\vec{V}| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{1}{4} + \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}
$$

This is a crucial step because the magnitude determines how to normalize the vector.

Step 3: Find the Unit Vector in the Opposite Direction

The unit vector $\vec{u}$ in the same direction as $\vec{V}$ is:

$$
\vec{u} = \frac{\vec{V}}{|\vec{V}|}
$$

So,

$$
\vec{u} = \frac{\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2} \right)}{\frac{\sqrt{3}}{2}} = \left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}, \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}, \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} \right) = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right)
$$

Note: The division simplifies because:

$$
\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1/2}{\sqrt{3}/2} = \frac{1/2 \times 2}{\sqrt{3}} = \frac{1}{\sqrt{3}}
$$

Since we want the vector in the opposite direction, we negate the unit vector:

$$
\vec{u}_{\text{opp}} = -\vec{u} = - \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right) = \left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right)
$$

Step 4: Scale the Opposite Unit Vector to Magnitude 6

Finally, to find the vector of magnitude 6 pointing in the opposite direction, multiply the unit vector in the opposite direction by 6:

$$
\vec{V}_{\text{desired}} = 6 \times \vec{u}_{\text{opp}} = 6 \times \left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right)
$$

This yields:

$$
\boxed{
\vec{V}_{\text{desired}} = \left(-\frac{6}{\sqrt{3}}, -\frac{6}{\sqrt{3}}, -\frac{6}{\sqrt{3}} \right)
}
$$

which simplifies further:

\[
\frac{6}{\sqrt{3}} = 6 \times \frac{\sqrt{3}}{3} = 2 \sqrt{3}
\]

Thus,

\[
\boxed{
\vec{V}_{\text{desired}} = \left(-2 \sqrt{3}, -2 \sqrt{3}, -2 \sqrt{3} \right)
}
\]

Final Result

The vector of magnitude 6 in the direction opposite to \(\vec{V} = \frac{1}{2} i + \frac{1}{2} j + \frac{1}{2} k\) is:

\[
\boxed{
\mathbf{V} = -2 \sqrt{3} \, i - 2 \sqrt{3} \, j - 2 \sqrt{3} \, k
}
\]

Additional Insights and Context

Significance of the Result

This problem exemplifies the fundamental process of vector normalization and scaling, which is central to vector calculus. The key steps involve:

- Calculating the magnitude of the original vector to understand its scale.
- Normalizing the vector to obtain a pure direction indicator.
- Negating the unit vector to reverse the direction.
- Rescaling to achieve the desired magnitude.

This procedure is widely applicable across physics (e.g., force vectors, velocity vectors), computer graphics (object orientation and scaling), and engineering (structural analysis).

Practical Applications

- Physics: Determining the force vector acting in the opposite direction with a specified strength.
- Navigation: Calculating a displacement vector that points opposite to a known direction with a specific distance.
- Robotics: Programming a robot to move in the exact opposite direction over a fixed distance.

Limitations and Considerations

- The problem assumes the initial vector is non-zero, enabling normalization.
- The method relies on basic algebraic manipulations; in higher dimensions or more complex vector fields, computational tools may be necessary.
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