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When working with functions involving two variables such as \(r \) and \(\theta \), understanding how to compute second partial derivatives is crucial for analyzing the behavior, curvature, and local extrema of the functions. In this article, we will explore the process of finding all second partial derivatives for the given function \(T(r, \theta) = E9r \cos(\theta) \), including the mixed derivatives \(T_{r\theta} \) and \(T_{\theta r} \), as well as the pure second derivatives \(T_{rr} \) and \(T_{\theta\theta} \). This comprehensive guide aims to clarify each step involved, provide relevant formulas, and discuss the significance of these derivatives in multivariable calculus.

Understanding the Function \(T(r, \theta) = E9r \cos(\theta) \)

Before diving into the derivatives, let's analyze the structure of the given function:

- \(T(r, \theta) = E9r \cos(\theta) \)

Here, \(E9r \) appears to be a constant or notation representing a constant coefficient multiplied by \(r \). For clarity, we assume \(E9 \) is a constant, so:

- \(T(r, \theta) = (E9) \times r \times \cos(\theta) \)

This assumption simplifies calculations and aligns with typical forms in multivariable calculus.

Key observations:

- The function is a product of \(r \) and \(\cos(\theta) \), scaled by the constant \(E9 \).
- The variables are \(r \) and \(\theta \), common in polar coordinate functions.
- The partial derivatives will involve standard derivatives of \(r \) and \(\cos(\theta) \).

First Partial Derivatives of $T(r, \theta)$

Calculating the first derivatives is the foundation for determining second derivatives.

Partial derivative with respect to r, T_r

Given:

$$
T(r, \theta) = E9 \times r \times \cos(\theta)
$$

- $\cos(\theta)$ is treated as a constant when differentiating with respect to r.

Derivative:

$$
T_r(r, \theta) = \frac{\partial}{\partial r} (E9 \times r \times \cos(\theta)) = E9 \times \cos(\theta)
$$

Result:

$$
\boxed{
T_r(r, \theta) = E9 \cos(\theta)
}
$$

Partial derivative with respect to θ, T_θ

- r is treated as a constant when differentiating with respect to θ.

Derivative:

$$
T_\theta(r, \theta) = \frac{\partial}{\partial \theta} (E9 \times r \times \cos(\theta)) = E9 \times r \times (-\sin(\theta))
$$

Simplifies to:

$$
T_\theta(r, \theta) = -E9 r \sin(\theta)
$$

Result:

$$
\boxed{
T_\theta(r, \theta) = -E9 r \sin(\theta)
}
$$

Second Partial Derivatives of $T(r, \theta)$

The second partial derivatives measure the curvature and how the slope of the function changes in the r and θ directions. They are fundamental in determining the nature of critical points and understanding the function’s behavior.

Second partial derivative with respect to r, T_{rr}

Differentiate $T_r(r, \theta)$ with respect to r:

$$
T_r(r, \theta) = E9 \cos(\theta)
$$

Since this expression does not depend on r, its derivative with respect to r is zero:

$$
T_{rr} = \frac{\partial}{\partial r} T_r(r, \theta) = 0
$$

Result:

\[
\boxed{
T_{rr} = 0
}
\]

Second partial derivative with respect to \(\theta \), \(T_{\theta\theta} \)

Differentiate \(T_\theta(r, \theta) \) with respect to \(\theta \):

\[
T_\theta(r, \theta) = -E9 r \sin(\theta)
\]

Derivative:

\[
T_{\theta\theta} = \frac{\partial}{\partial \theta} (-E9 r \sin(\theta)) = -E9 r \cos(\theta)
\]

Result:

\[
\boxed{
T_{\theta\theta} = -E9 r \cos(\theta)
}
\]

Mixed partial derivatives \(T_{r\theta} \) and \(T_{\theta r} \)

In multivariable calculus, under suitable smoothness conditions, mixed partial derivatives are equal, i.e.,

\[
T_{r\theta} = T_{\theta r}
\]

Let's compute each.

Partial derivative of $T_r(r, \theta)$ with respect to θ, $T_{r\theta}$

Recall:

$$
T_r(r, \theta) = E9 \cos(\theta)
$$

Differentiate with respect to θ:

$$
T_{r\theta} = \frac{\partial}{\partial \theta} (E9 \cos(\theta)) = -E9 \sin(\theta)
$$

Partial derivative of $T_\theta(r, \theta)$ with respect to r, $T_{\theta r}$

Recall:

$$
T_\theta(r, \theta) = -E9 r \sin(\theta)
$$

Differentiate with respect to r:

$$
T_{\theta r} = \frac{\partial}{\partial r} (-E9 r \sin(\theta)) = -E9 \sin(\theta)
$$

Note: Both mixed derivatives are equal:

$$
T_{r\theta} = T_{\theta r} = -E9 \sin(\theta)
$$

Summary of All Second Partial Derivatives

Derivative	Expression	Description
T_{rr}	0	Second derivative w.r.t. r
$T_{\theta\theta}$	$-E9 r \cos(\theta)$	Second derivative w.r.t. θ
$T_{r\theta}$	$-E9 \sin(\theta)$	Mixed derivative, r then

\(\theta \) |
| \(T_{\theta r} \) | \(-E9 \sin(\theta)\) | Mixed derivative, \(\theta \) then \(r \) |

The symmetry of mixed partial derivatives is confirmed: \(T_{r\theta} = T_{\theta r} \).

Significance of Second Partial Derivatives in Multivariable Calculus

Understanding second derivatives provides insight into the curvature and local extrema of the function:

- Concavity and convexity: The signs of second derivatives indicate whether the function is concave or convex in a particular direction.
- Hessian matrix: The matrix formed by second derivatives:

\[
H = \begin{bmatrix}
T_{rr} & T_{r\theta} \\
T_{\theta r} & T_{\theta\theta}
\end{bmatrix}
\]

is instrumental in classifying critical points via the second derivative test.

- Critical point analysis:

\[
\text{At a critical point, where } T_r = 0 \text{ and } T_\theta = 0,
\]

the Hessian helps determine whether the point is a local maximum, minimum, or saddle point.

Applications of Second Partial Derivatives in Physics and Engineering

Second derivatives are vital in various fields:

- Physics: Analyzing potential energy surfaces and stability.
- Engineering: Structural analysis, stress-strain calculations, and optimization.
- Mathematics: Surface curvature, saddle points, and more complex geometrical analyses.

Conclusion

By systematically calculating each second partial derivative of the function $T(r, \theta) = E9 r \cos(\theta)$, we gain valuable insights into its behavior. The second derivatives reveal the curvature characteristics and help identify critical points essential in optimization and stability analysis. Remember, the symmetry of mixed derivatives under proper smoothness assumptions simplifies analysis, and understanding the Hessian matrix derived from these derivatives is fundamental in multivariable calculus applications.

This step-by-step approach can be applied to more complex functions, reinforcing core calculus concepts and enhancing problem-solving skills in multivariable analysis.

Keywords: Second partial derivatives, multivariable calculus, polar coordinates, Hessian matrix, curvature analysis, mixed derivatives, critical points, surface analysis

Frequently Asked Questions

What is the general approach to finding all second partial derivatives of a function T(r, θ)?

To find all second partial derivatives of T(r, θ), first compute the first derivatives T_r and T_θ, then differentiate these with respect to r and θ again as needed, applying the chain rule and product rule where appropriate.

Given T = E9r Cos(θ), how do you find T_{rr}, the second partial derivative of T with respect to r?

Since T = E9r Cos(θ), differentiate T with respect to r twice: T_r = E9 Cos(θ), then $T_{rr} = d/dr(T_r) = 0$, because E9 is a constant and Cos(θ) does not depend on r.

How do you compute $T_{\theta\theta}$, the second partial derivative of T with respect to θ, for T = E9r Cos(θ)?

First, find T_θ = E9r (-Sin(θ)). Then, differentiate T_θ with respect to θ: $T_{\theta\theta}$ = E9r (-Cos(θ)).

What is $T_{r\theta}$ (or $T_{\theta r}$) for the function T = E9r Cos(θ)?

To find $T_{r\theta}$, differentiate T_r = E9 Cos(θ) with respect to θ: $T_{r\theta}$ = -E9r Sin(θ). Similarly, $T_{\theta r}$ = d/d r of T_θ = E9 (-Sin(θ)), which is zero if E9 is constant; otherwise, account for dependencies accordingly.

Are the mixed partial derivatives $T_{r\theta}$ and $T_{\theta r}$ equal in this context?

Yes, under standard conditions of continuity and differentiability, mixed partial derivatives $T_{r\theta}$ and $T_{\theta r}$ are equal. For T = E9r Cos(θ), both are -E9 Sin(θ) times the relevant derivatives, confirming their equality.

What is the significance of calculating all second partial derivatives in understanding the behavior of T(r, θ)?

Calculating all second partial derivatives helps analyze the curvature, concavity, and stability of the function T(r, θ), which is essential in fields like differential equations, physics, and engineering for understanding how T changes locally.

Additional Resources

Find All The Second Partial Derivatives. T = E9r Cos(theta) Trr = Trtheta = Tthetar = Tthetatheta =

In the realm of mathematical analysis, understanding the behavior of functions of multiple variables is fundamental, especially in fields such as physics, engineering, and applied mathematics. One crucial aspect of this understanding involves the computation of second-order partial derivatives, which reveal information about the curvature, concavity, and local extrema of functions. Today, we delve into a specific function expressed in cylindrical coordinates and explore how to systematically find all second partial derivatives associated with it.

Our focus centers on a function given by:

$T = E9r \cos(\theta)$

where the derivatives are denoted as:

- T_{rr} (second partial derivative with respect to r twice),
- $T_{r\theta}$ (mixed second partial derivative with respect to r and θ),
- $T_{\theta r}$ (mixed second partial derivative with respect to θ and r),
- $T_{\theta\theta}$ (second partial derivative with respect to θ twice).

This exploration not only involves the direct calculation of these derivatives but also emphasizes the underlying principles, techniques, and the importance of the symmetry of mixed derivatives.

Understanding the Function and Its Context

Before embarking on the calculations, it's vital to interpret the given function and its derivatives correctly.

The Function T(r, θ)

The function is expressed as:

$T(r, \theta) = E9r \cos(\theta)$

This suggests that T depends on the variables r and θ, typical in cylindrical or polar coordinate systems. Here:

- E is likely the base of the natural logarithm (Euler's number), approximately 2.71828.
- 9r is a function of r, possibly indicating a multiplicative factor involving r.
- Cos(θ) is the cosine of the angle θ, reflecting angular dependence.

If we interpret E9r as a function of r, possibly meaning E raised to the power of 9r, then:

$T(r, \theta) = e^{9r} \cos(\theta)$

This is a common form in physics for functions involving exponential decay or growth coupled with oscillatory behavior.

Clarifying the Derivative Notation

The derivatives are denoted as:

- T_{rr}: second derivative of T with respect to r,
- $T_{r\theta}$: mixed partial derivative first with respect to r, then θ,
- $T_{\theta r}$: mixed partial derivative first with respect to θ, then r,
- $T_{\theta\theta}$: second derivative with respect to θ.

According to Clairaut's theorem, if T is sufficiently smooth, then:

$$T_{r\theta} = T_{\theta r}$$

which simplifies calculations and confirms the symmetry of mixed derivatives.

Step 1: Computing the First-Order Partial Derivatives

To find second derivatives, we begin with first derivatives.

Derivative with respect to r: T_r

Given:

$$T(r, \theta) = e^{9r} \cos(\theta)$$

Since $\cos(\theta)$ is treated as a constant when differentiating with respect to r:

$$T_r(r, \theta) = \frac{d}{dr} [e^{9r} \cos(\theta)] = \cos(\theta) \frac{d}{dr} [e^{9r}] = \cos(\theta)\, 9\, e^{9r}$$

Result:

$$T_r(r, \theta) = 9 e^{9r} \cos(\theta)$$

Derivative with respect to θ: T_θ

Similarly, when differentiating with respect to θ, e^{9r} is treated as a constant:

$$T_\theta(r, \theta) = \frac{d}{d\theta} [e^{9r} \cos(\theta)] = e^{9r} \frac{d}{d\theta} [\cos(\theta)] = -e^{9r} \sin(\theta)$$

Result:

$$T_\theta(r, \theta) = - e^{9r} \sin(\theta)$$

Step 2: Computing the Second-Order Partial Derivatives

Now, we proceed to differentiate the first derivatives to obtain the second derivatives.

2.1 T_{rr}: Second derivative with respect to r

Differentiate $T_r(r, \theta) = 9 e^{9r} \cos(\theta)$ again with respect to r:

$$T_{rr}(r, \theta) = \frac{d}{dr} [9 e^{9r} \cos(\theta)] = 9 \frac{d}{dr} [e^{9r}] \cos(\theta) = 9\, 9\, e^{9r}$$

$\cos(\theta) = 81 e^{9r} \cos(\theta)$

Result:

$$T_{rr}(r, \theta) = 81 e^{9r} \cos(\theta)$$

2.2 $T_{\theta\theta}$: Second derivative with respect to θ

Differentiate $T_\theta(r, \theta) = - e^{9r} \sin(\theta)$ with respect to θ:

$$T_{\theta\theta}(r, \theta) = d/d\theta [- e^{9r} \sin(\theta)] = - e^{9r} d/d\theta [\sin(\theta)] = - e^{9r} \cos(\theta)$$

Result:

$$T_{\theta\theta}(r, \theta) = - e^{9r} \cos(\theta)$$

2.3 $T_{r\theta}$ and $T_{\theta r}$: Mixed derivatives

Since the function is smooth, Clairaut's theorem states $T_{r\theta} = T_{\theta r}$.

2.4 $T_{r\theta}$: Derivative of T_r with respect to θ

Recall:

$$T_r(r, \theta) = 9 e^{9r} \cos(\theta)$$

Differentiating with respect to θ:

$$T_{r\theta}(r, \theta) = d/d\theta [9 e^{9r} \cos(\theta)] = 9 e^{9r} d/d\theta [\cos(\theta)] = -9 e^{9r} \sin(\theta)$$

Result:

$$T_{r\theta}(r, \theta) = -9 e^{9r} \sin(\theta)$$

2.5 $T_{\theta r}$: Derivative of T_θ with respect to r

Recall:

$$T_\theta(r, \theta) = - e^{9r} \sin(\theta)$$

Differentiating with respect to r:

$T_{\theta r}(r, \theta) = d/dr [- e^{(9r)} \sin(\theta)] = - d/dr [e^{(9r)}] \sin(\theta) = - 9 e^{(9r)} \sin(\theta)$

Result:

$T_{\theta r}(r, \theta) = -9 e^{(9r)} \sin(\theta)$

Note: Both mixed derivatives are equal, confirming Clairaut’s theorem.

Summary of All Second Partial Derivatives

Derivative	Expression	Comments
T_{rr}	$81 e^{(9r)} \cos(\theta)$	Second derivative w.r.t. r
$T_{\theta\theta}$	$- e^{(9r)} \cos(\theta)$	Second derivative w.r.t. θ
$T_{r\theta} = T_{\theta r}$	$-9 e^{(9r)} \sin(\theta)$	Mixed derivatives

---

Significance and Applications

Calculating second derivatives is more than an academic exercise; it provides vital insights into the function's behavior. For example:

- Concavity and Convexity: The signs and magnitudes of second derivatives indicate whether the function is concave or convex in a particular region.
- Curvature Analysis: In physics, the second derivatives can relate to curvature, stability, or oscillations.
- Optimization: Finding critical points involves setting first derivatives to zero and examining second derivatives for local maxima, minima, or saddle points.

In our case, the exponential dependence on r combined with sinusoidal angular variation suggests complex behavior, such as oscillations with exponentially increasing amplitude as r increases.

Practical Considerations in Derivative Computation

When approaching such derivatives, keep in mind:

- Chain Rule: Essential for derivatives involving exponential functions and trigonometric functions.
- Product Rule: Used when differentiating products like $e^{(9r)} \cos(\theta)$.
- Symmetry of Mixed Derivatives: Under suitable smoothness conditions, $T_{r\theta} = T_{\theta r}$ simplifies calculations and provides consistency checks.

- Interpretation: Understanding the physical or geometric context can guide expectations about the signs and magnitudes of derivatives.

Final Remarks

The process of finding all second partial derivatives for a multivariable function involves systematic application of fundamental calculus rules, a clear understanding of the function's structure, and recognition of mathematical theorems like Clairaut's. For the function T = e^(9r) cos(θ), we've demonstrated how to derive each second derivative explicitly, revealing the intricate relationship between exponential growth and oscillatory behavior.

This exercise underscores the importance of mastery over derivative rules and the significance of second derivatives in analyzing the local and global properties of functions. Whether in theoretical physics, engineering, or advanced mathematics, such calculations form the backbone of deeper insights into complex systems modeled by multivariable functions.

In conclusion, understanding how to find all second partial derivatives not only enhances mathematical proficiency but also equips scientists and engineers with essential tools for analyzing the behavior of complex functions across various disciplines.

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