Find An Equation For The Line Perpendicular To The Tangent To The Curvey=x34x+1at The Point(2,1).

Find An Equation For The Line Perpendicular To The Tangent To The Curvey=x34x+1at The Point(2,1) is a common problem in calculus that combines understanding of differentiation, tangent lines, and perpendicular lines. This type of problem is essential for mastering the concepts of derivatives and their geometric interpretations, which are fundamental in calculus and analytical geometry. In this article, we will walk through the steps to find the equation of the line perpendicular to the tangent to the curve y = x^3 + x + 1 at the specific point (2,1). We will explore the necessary concepts, calculations, and reasoning involved in solving this problem in detail.

Understanding the Problem

Before diving into calculations, let's clarify what the problem entails.

Key Components of the Problem

- **The curve:** y = x^3 + x + 1
- **The point on the curve:** (2, 1)
- **The tangent line at that point:** The line tangent to the curve at x = 2.
- **The perpendicular line:** The line perpendicular to the tangent line at (2, 1).

Our goal is to find the equation of this perpendicular line.

Step 1: Confirm the Point Lies on the Curve

Before proceeding, verify that the point (2,1) lies on the given curve.

Calculate y at x=2:

- y = (2)^3 + 2 + 1 = 8 + 2 + 1 = 11

Since y=11 when x=2, the point (2,1) does not lie on the curve y = x^3 + x + 1.

Important: The point given as (2,1) does not satisfy the curve y = x^3 + x + 1.

Possible correction: If the problem states that the point is (2,1), but the curve is $y = x^3 + x + 1$, then the point is not on the curve unless the curve is different.

Alternative: Perhaps there's a typo or misinterpretation.

Assuming the intended curve is $y = x^3 + x + 1$, and the point is (2,11), not (2,1).

Therefore, proceed with the point (2,11).

Clarification:
- If the point is actually (2,1) and the curve is $y = x^3 + x + 1$, then the point does not lie on the curve.
- For the purpose of this problem, assuming the point is (2,11), which lies on $y = x^3 + x + 1$.

Step 2: Find the Derivative of the Curve

The derivative y' gives the slope of the tangent line at any point x on the curve.

Differentiate y = x^3 + x + 1

- $y' = d/dx (x^3 + x + 1) = 3x^2 + 1$

Calculate the slope of the tangent at x=2

- y' (at $x=2$) $= 3(2)^2 + 1 = 3(4) + 1 = 12 + 1 = 13$

Thus, the slope of the tangent line at (2,11) is 13.

Step 3: Find the Equation of the Tangent Line

Using point-slope form:
- $y - y_1 = m (x - x_1)$
- with point (2,11) and slope $m=13$:

$y - 11 = 13(x - 2)$

Simplify:
- $y - 11 = 13x - 26$
- $y = 13x - 15$

Therefore, the equation of the tangent line is $y = 13x - 15$.

Step 4: Find the Equation of the Perpendicular Line

The line perpendicular to the tangent will have a slope that is the negative reciprocal of 13.

Calculate the perpendicular slope

- perpendicular slope $m_p = -1 / 13$

Now, find the equation of this perpendicular line passing through the point (2,11):

$y - 11 = (-1/13)(x - 2)$

Simplify:
- $y - 11 = (-1/13) x + (2/13)$

Add 11 to both sides:
- $y = (-1/13) x + (2/13) + 11$

Express 11 as a fraction with denominator 13:
- $11 = 143/13$

So:
- $y = (-1/13) x + (2/13) + (143/13) = (-1/13) x + (145/13)$

Step 5: Write the Final Equation

The equation of the line perpendicular to the tangent at the point (2,11) is:

$y = (-1/13) x + 145/13$

This line passes through (2,11) and is perpendicular to the tangent line at that point.

Additional Considerations

If the Point Is (2,1) Instead of (2,11)

- The point (2,1) does not lie on $y = x^3 + x + 1$.
- If the problem intended (2,1), then either:
- The curve is different, or
- The point is a different point on the curve with a different x-coordinate.

Alternative Scenario

- Suppose the curve is $y = x^3 + x + 1$, and the point is (2,11). The calculations above hold.

Summary of Steps

1. Verify the point lies on the curve.

2. Differentiate the curve to find the slope of the tangent.

3. Evaluate the derivative at the given point to find the tangent slope.

4. Use the negative reciprocal of the tangent slope to find the perpendicular slope.

5. Apply the point-slope form to find the equation of the perpendicular line.

Conclusion

Finding the equation of a line perpendicular to the tangent at a specific point involves several key steps: confirming the point's position on the curve, differentiating the curve to find the tangent's slope, then calculating the negative reciprocal for the perpendicular slope, and finally writing the equation using point-slope form. This process illustrates the powerful connection between calculus and geometry, allowing us to analyze and interpret the behavior of curves and lines in the coordinate plane.

By mastering these steps, students and professionals can confidently approach a wide range of problems involving tangents, normals, and perpendicular lines in various contexts, from pure mathematics to applied sciences.

Frequently Asked Questions

How do I find the equation of the tangent line to the curve $y = 3x^4 + 1$ at the point (2, 1)?

First, find the derivative $y' = 12x^3$. Then, evaluate at $x=2$: $y' = 12(2)^3 = 12(8) = 96$. The slope of the tangent line is 96. Using point-slope form: y - 1 = 96(x - 2).

What is the slope of the line perpendicular to the tangent line at (2, 1)?

Since the slope of the tangent line is 96, the perpendicular line's slope is the negative reciprocal: -1/96.

How do I write the equation of the line perpendicular to the tangent at the point (2, 1)?

Use point-slope form with the point (2, 1) and slope -1/96: y - 1 = (-1/96)(x - 2).

Can you provide the explicit form of the perpendicular line's equation?

Yes. Simplify: y - 1 = (-1/96)x + (2/96) which simplifies to y = (-1/96)x + 1 + (1/48). So, y = (-1/96)x + (49/48).

Why do we find the negative reciprocal of the tangent slope to get the perpendicular line?

Because in coordinate geometry, perpendicular lines have slopes that are negative reciprocals of each other, ensuring they intersect at right angles.

What is the importance of the point (2, 1) in this problem?

The point (2, 1) is where the tangent line touches the curve, and the perpendicular line we find passes through this same point.

Is the process of finding the perpendicular line similar to finding the tangent line?

Yes, both involve calculating the slope at a point, but for the perpendicular line, you take the negative reciprocal of the tangent's slope.

What are common mistakes to avoid when solving this type of problem?

Avoid mixing up the slopes of the tangent and perpendicular lines, forgetting to evaluate the derivative at the correct x-value, or misapplying the point-slope formula.

Additional Resources

Find An Equation For The Line Perpendicular To The Tangent To The Curve y = x^3 + 1 at the Point (2, 1)

Understanding how to find the equation of a line perpendicular to a tangent line at a specific point on a curve is a fundamental concept in calculus and analytical geometry. This process involves several steps: differentiating the function to find the slope of the tangent, calculating the slope at the given point, determining the perpendicular slope, and finally, writing the equation of the perpendicular line. In this article, we will explore this process in detail, breaking down each step to make it accessible and clear, even for those new to calculus.

Introduction: The Core Problem

Find An Equation For The Line Perpendicular To The Tangent To The Curve y = x^3 + 1 at the Point (2, 1).

At first glance, this problem might seem complex, but it encapsulates several key ideas in calculus: the use of derivatives to find slopes, the concept of perpendicular lines, and the process of deriving equations of lines from given data points and slopes. Our goal is to determine the specific line that is perpendicular to the tangent at the point (2, 1) on the curve y = x^3 + 1.

Understanding the Curve and Its Geometry

The Curve y = x^3 + 1

This is a cubic function, characterized by its smooth, continuous, and differentiable nature across all real numbers. The graph of y = x^3 + 1 is a cubic curve that passes through the point (2, 1). To understand the tangent line at this point, we need to find the slope of the curve at x = 2.

Why Find the Tangent Line?

The tangent line at a particular point on a curve provides the best linear approximation of the curve at that point. It touches the curve exactly at that point and shares the same slope as the curve there. Once we find the tangent line, the line perpendicular to it will be at 90 degrees, which is often relevant in problems involving orthogonality, normals, or related rates.

Step 1: Finding the Derivative of the Curve

Differentiation Basics

The derivative of a function y = f(x), denoted as dy/dx or f'(x), gives the slope of the tangent line to the curve at any point x. For the given function y = x^3 + 1, the differentiation process is straightforward:

- Power rule: $d/dx [x^n] = n x^{(n-1)}$

Calculating the Derivative

Applying the power rule:

$$dy/dx = d/dx [x^3 + 1] = 3x^2 + 0 = 3x^2$$

This derivative function tells us the slope of the tangent line to the curve at any x.

Step 2: Finding the Slope at the Point (2, 1)

Substituting $x = 2$

To find the slope of the tangent line at the specific point (2, 1), substitute $x = 2$ into the derivative:

$$m_{tangent} = 3(2)^2 = 3\ 4 = 12$$

So, the slope of the tangent line at (2, 1) is 12.

Confirming the Point Lies on the Curve

Before proceeding, it's good practice to verify the point's coordinates satisfy the original function:

$$y = (2)^3 + 1 = 8 + 1 = 9$$

But the point given is (2, 1), which does not satisfy $y = x^3 + 1$.

Important Note: There appears to be a discrepancy here because the point (2, 1) does not lie on the curve $y = x^3 + 1$, since y should be 9 when $x = 2$.

Possible explanation: The problem might contain a typo or is intentionally focusing on the tangent at $x = 2$, with the point given as (2, 1). Alternatively, perhaps the point is an approximation or a different curve.

For this tutorial, we will proceed under the assumption that the point is (2, 1) and that the tangent line at $x = 2$ has the slope calculated from the derivative, and the goal is to find the perpendicular line passing through that point.

Step 3: Determining the Slope of the Perpendicular Line

The Concept of Perpendicular Slopes

Two lines are perpendicular if the product of their slopes is -1. If the slope of the tangent is $m_{tangent}$, then the slope of the line perpendicular (m_{perp}) is:

$$m_{perp} = -1 / m_{tangent}$$

Calculating the Perpendicular Slope

Using the tangent slope:

m_perp = -1 / 12

This simplifies to:

m_perp = -1/12

So, the line perpendicular to the tangent at the point (2, 1) has a slope of -1/12.

Step 4: Writing the Equation of the Perpendicular Line

Using the Point-Slope Form

The point-slope form of a line's equation is:

$y - y_1 = m (x - x_1)$

Where:

- (x_1, y_1) is the point through which the line passes
- m is the slope of the line

Applying the Data

Given:

- Point: (2, 1)
- Slope: m_perp = -1/12

Plugging into the point-slope formula:

y - 1 = (-1/12)(x - 2)

Simplifying the Equation

Distribute the slope:

y - 1 = (-1/12) x + (1/6)

Now, add 1 to both sides:

y = (-1/12) x + (1/6) + 1

Express 1 as 6/6 to combine:

y = (-1/12) x + (6/6 + 1/6) = (-1/12) x + (7/6)

Final Equation

The equation of the line perpendicular to the tangent line at the point (2, 1) is:

$y = (-1/12) x + 7/6$

Additional Insights and Applications

Visualizing the Geometry

Imagine the cubic curve $y = x^3 + 1$. At $x = 2$, its tangent line slopes upward steeply with a slope of 12. The perpendicular line, which we derived, crosses through the point (2, 1) with a gentle downward slope of -1/12. This line can be viewed as a normal or an orthogonal component relative to the tangent, often useful in physics and engineering when analyzing forces, normals, or reflections.

Extending the Method

This process is not limited to this particular function or point. The general steps involve:

1. Differentiating the function to find the slope of the tangent at any x.
2. Substituting x into the derivative to find the specific slope at the point.
3. Calculating the perpendicular slope as the negative reciprocal.
4. Using point-slope form to write the equation of the perpendicular line.

Conclusion: Summarizing the Process

Finding an equation for a line perpendicular to a tangent involves a sequence of calculus concepts:

- Differentiation to find the slope of the tangent line.
- Evaluation of the derivative at the specific x-coordinate.
- Reciprocal and sign change to find the perpendicular slope.
- Application of the point-slope form to write the equation.

In our example, starting from the curve $y = x^3 + 1$, at the point where $x = 2$, the tangent slope is 12. The perpendicular line passing through (2, 1) has a slope of -1/12, and its equation is:

$y = (-1/12) x + 7/6$

Understanding these steps enhances your grasp of how calculus connects the geometric properties of curves with algebraic equations, providing powerful tools for analysis across mathematics, physics, and engineering disciplines.

Final Note

While the initial point (2, 1) does not lie on the curve $y = x^3 + 1$, the process demonstrated reflects the standard methodology to find a perpendicular line at a specific point and slope. Always ensure that the point in question lies on the curve when performing such calculations for geometric accuracy.

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