Find An Equation Of The Tangent Line To The Curve At The Given Point. Y = Ln(x2 4x + 1), (4, 0)

Find An Equation Of The Tangent Line To The Curve At The Given Point. Y = Ln(x^2 - 4x + 1), (4, 0)

Understanding how to find the equation of a tangent line to a curve at a specific point is a fundamental concept in calculus. It involves determining the slope of the curve at that point and then using the point-slope form of a line to find the equation. In this article, we will explore this process step-by-step for the function $y = \ln(x^2 - 4x + 1)$ at the point $(4, 0)$. We will discuss the necessary derivatives, verify the point lies on the curve, and then derive the tangent line's equation, providing a comprehensive guide suitable for learners at various levels.

Understanding the Problem: The Curve and the Point

Function Definition

The curve is defined by the function:

$$
y = \ln(x^2 - 4x + 1)
$$

This is a logarithmic function where the argument inside the natural log is a quadratic expression.

Given Point

The point provided is:

$$
(4, 0)
$$

This indicates that when $x = 4$, $y = 0$. Our goal is to find the equation of the tangent line to the curve at this specific point.

Verifying the Point Lies on the Curve

Before proceeding, it is essential to confirm that the point $(4, 0)$ indeed lies on the curve $y = \ln(x^2 - 4x + 1)$.

Substitute $x = 4$ into the function:

$$
y = \ln(4^2 - 4 \times 4 + 1) = \ln(16 - 16 + 1) = \ln(1) = 0
$$

Since the computed y matches the given point's y-coordinate, the point $(4, 0)$ is on the curve, confirming our starting point.

Finding the Derivative: Slope of the Tangent Line

The slope of the tangent line at a point on a curve is given by the derivative $\frac{dy}{dx}$.

Applying the Chain Rule

Given:
$$
y = \ln(u) \quad \text{where} \quad u = x^2 - 4x + 1
$$
The derivative of y with respect to x using the chain rule is:
$$
\frac{dy}{dx} = \frac{1}{u} \times \frac{du}{dx}
$$

Calculating $\frac{du}{dx}$

Differentiate $u = x^2 - 4x + 1$:
$$
\frac{du}{dx} = 2x - 4
$$

Expressing $\frac{dy}{dx}$

Putting it all together:
$$
\frac{dy}{dx} = \frac{1}{x^2 - 4x + 1} \times (2x - 4)
$$
or simply:
$$
\boxed{
\frac{dy}{dx} = \frac{2x - 4}{x^2 - 4x + 1}
}
$$

Evaluating the Derivative at the Point $(4, 0)$

To find the slope of the tangent line at $x = 4$, substitute $x = 4$ into the derivative:

Calculate numerator:

$$
2 \times 4 - 4 = 8 - 4 = 4
$$

Calculate denominator:

$$
(4)^2 - 4 \times 4 + 1 = 16 - 16 + 1 = 1
$$

Compute the slope m:

$$
m = \frac{4}{1} = 4
$$

So, the slope of the tangent line at $(4, 0)$ is 4.

Formulating the Equation of the Tangent Line

The general form of the equation of a line with slope m passing through point (x_0, y_0) is:

$$
y - y_0 = m(x - x_0)
$$

Using the point $(4, 0)$ and slope 4:

$$
y - 0 = 4(x - 4)
$$

Simplify:

$$
y = 4(x - 4)
$$

which expands to:

$$
\boxed{
y = 4x - 16
}
$$

This is the equation of the tangent line to the curve at the point $(4, 0)$.

Summary of the Process

- Verify that the given point lies on the curve by substituting x into the function and checking the y-value.
- Differentiate the function $y = \ln(x^2 - 4x + 1)$ using the chain rule to find $\frac{dy}{dx}$.
- Evaluate the derivative at $x = 4$ to find the slope of the tangent line at that point.
- Use the point-slope form of a line to write the equation of the tangent line using the known point and the slope.

Additional Insights and Considerations

Domain Restrictions

Since the function involves a natural logarithm, the argument inside must be positive:

$$
x^2 - 4x + 1 > 0
$$

Solve the quadratic inequality:

$$
x^2 - 4x + 1 > 0
$$

Find roots:

$$
x = \frac{4 \pm \sqrt{(-4)^2 - 4 \times 1 \times 1}}{2} = \frac{4 \pm \sqrt{16 - 4}}{2} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}
$$

Since the quadratic opens upwards, the expression is positive outside the roots:

$$
x < 2 - \sqrt{3} \quad \text{or} \quad x > 2 + \sqrt{3}
$$

The point $x=4$ is within the domain because:

$$
4 > 2 + \sqrt{3} \quad (\text{since } \sqrt{3} \approx 1.732, \quad 2 + 1.732 \approx 3.732)
$$

Thus, the point is valid within the domain.

Implications for Calculus and Analysis

Knowing the slope at a point provides insights into the behavior of the function, including increasing/decreasing intervals, concavity, and potential points of inflection. The tangent line is also essential in approximation techniques like linearization.

Conclusion

Deriving the equation of a tangent line to a curve at a given point involves several systematic steps: verifying the point's position on the curve, computing the derivative to find the slope, evaluating the derivative at the point, and then applying the point-slope form to write the line's equation. For the function $y = \ln(x^2 - 4x + 1)$ at $(4, 0)$, these steps lead us to the tangent line:

$$
\boxed{
y = 4x - 16
}
$$

This process exemplifies how calculus tools enable us to understand the local behavior of functions through tangent lines, providing both theoretical insights and practical applications in various fields such as physics, engineering, and economics.

Frequently Asked Questions

How do you find the equation of the tangent line to the curve $y = \ln(x^2 - 4x + 1)$ at the point (4, 0)?

First, find the derivative y' using the chain rule to get the slope at $x=4$, then use point-slope form with the point (4, 0).

What is the derivative of $y = \ln(x^2 - 4x + 1)$?

The derivative is $y' = (2x - 4) / (x^2 - 4x + 1)$, obtained by applying the chain rule.

How do you evaluate the slope of the tangent line at $x = 4$ for the given function?

Substitute $x=4$ into the derivative $y' = (2x - 4) / (x^2 - 4x + 1)$.

What is the slope of the tangent line at the point

(4, 0)?

Plugging in x=4, the slope y' = (8 - 4) / (16 - 16 + 1) = 4/1 = 4.

How is the tangent line equation formulated once the slope is known?

Using point-slope form: $y - y_1 = m(x - x_1)$, where m is the slope and (x_1, y_1) is the point.

What is the equation of the tangent line to the curve at (4, 0)?

Using slope m=4 and point (4, 0), the tangent line is y - 0 = 4(x - 4), simplifying to y = 4x - 16.

Why is it important to verify the point lies on the curve before finding the tangent line?

To ensure the point is on the curve, confirming the tangent line is correctly associated with that point.

Can you summarize the steps to find the tangent line at a given point on a logarithmic curve?

Yes: 1) Find the derivative for the slope, 2) evaluate the derivative at the point, 3) use point-slope form with the point and slope to write the tangent line equation.

Additional Resources

Find An Equation Of The Tangent Line To The Curve At The Given Point. Y = Ln(x^2 - 4x + 1), (4, 0)

Introduction

Mathematics often challenges students and professionals alike to understand the subtle relationships between a curve and its tangent lines. One of the core skills in calculus involves finding the equation of a tangent line at a particular point on a given curve. This process not only enhances comprehension of derivatives but also deepens understanding of the geometric interpretation of functions. In this article, we will explore the step-by-step procedure to find the tangent line to the curve defined by \(y = \ln(x^2 - 4x + 1) \) at the point \((4, 0) \). We will delve into the mathematical concepts involved, analyze the function's behavior around the specified point, and interpret the significance of the tangent line within the context of the curve.

Understanding the Problem

The Curve and the Point

The core problem involves the function:

$$
y = \ln(x^2 - 4x + 1)
$$

and the point:

$$
(4, 0)
$$

The goal is to find the equation of the tangent line to this curve at $x = 4$.

Why is this important?

Tangent lines serve as linear approximations of functions at specific points. They are fundamental in calculus for understanding the local behavior of functions, analyzing instantaneous rates of change, and solving optimization problems. Finding the tangent line requires both differentiating the function and substituting specific values, making it a practical illustration of derivative application.

Step 1: Verify the Point Lies on the Curve

Before proceeding, it’s essential to confirm that the given point $(4, 0)$ indeed lies on the curve $y = \ln(x^2 - 4x + 1)$.

- Calculate y at $x = 4$:

$$
x^2 - 4x + 1 = (4)^2 - 4 \times 4 + 1 = 16 - 16 + 1 = 1
$$

- Compute y:

$$
y = \ln(1) = 0
$$

Since the computed y matches the given point's y-coordinate, $(4, 0)$ indeed lies on the curve.

Step 2: Find the Derivative $\frac{dy}{dx}$

To find the tangent line, we need the slope of the curve at $x = 4$, which is given by the derivative $\frac{dy}{dx}$. Differentiating $y = \ln(x^2 - 4x + 1)$ requires the chain rule.

Differentiation Process

Recall that:

$$
\frac{d}{dx} \ln(u) = \frac{1}{u} \frac{du}{dx}
$$

where $u = x^2 - 4x + 1$.

- Compute $\frac{du}{dx}$:

$$
\frac{d}{dx} (x^2 - 4x + 1) = 2x - 4
$$

- Express $\frac{dy}{dx}$:

$$
\frac{dy}{dx} = \frac{1}{x^2 - 4x + 1} \times (2x - 4)
$$

This simplifies to:

$$
\boxed{
\frac{dy}{dx} = \frac{2x - 4}{x^2 - 4x + 1}
}
$$

Step 3: Calculate the Slope at $x = 4$

Substituting $x = 4$:

- Numerator:

$$
2(4) - 4 = 8 - 4 = 4
$$

- Denominator:

$$
(4)^2 - 4 \times 4 + 1 = 16 - 16 + 1 = 1
$$

- Derivative at $x = 4$:

$$
\frac{dy}{dx} \bigg|_{x=4} = \frac{4}{1} = 4
$$

Thus, the slope of the tangent line at $x = 4$ is 4.

Step 4: Write the Equation of the Tangent Line

The equation of a line with slope m passing through point (x_0, y_0) is:

$$
y - y_0 = m (x - x_0)
$$

- Given:

$$
x_0 = 4, \quad y_0 = 0, \quad m = 4
$$

- Equation:

$$
y - 0 = 4 (x - 4)
$$

$$
\boxed{
y = 4x - 16
}
$$

This line is the tangent to the curve at the point $(4, 0)$.

Analytical Insights and Contextual Significance

Geometric Interpretation

The tangent line $y = 4x - 16$ provides a linear approximation of the curve $y = \ln(x^2 - 4x + 1)$ near $x = 4$. It represents the best linear estimate of the function's behavior at that point. For values of x close to 4, the function's graph will be very close to this line, enabling predictions and analyses of the curve's local behavior.

Behavior of the Function Near the Point

- Concavity and Derivative Sign:

The positive slope (4) indicates that immediately to the right of $x=4$, the function is increasing.

- Implications for Function Growth:

Since the derivative is positive and constant (at the point), the function is increasing at $x=4$. The tangent line's steepness suggests a relatively rapid increase in y with respect to x near this point.

Practical Applications

- Approximate Calculations:

Engineers and scientists often use tangent lines for linear approximations, simplifying complex functions into manageable calculations, especially in physics or economics.

- Optimization and Marginal Analysis:

Understanding the slope at a specific point can help determine whether a function is increasing or decreasing, crucial for optimization problems.

- Graph Sketching:

The tangent line aids in sketching the graph of the original function, especially in regions where the function's behavior is complex or not easily visualized.

Broader Mathematical Context

The Role of Derivatives in Geometry

Derivatives serve as a bridge between algebraic functions and their geometric representations. They quantify how a function changes at a specific point, translating into the slope of the tangent line. This connection is fundamental in calculus, underpinning concepts like instantaneous velocity in physics and marginal cost in economics.

The Chain Rule and Its Significance

The differentiation process highlighted the chain rule, a vital technique in calculus for dealing with composite functions. Recognizing that $y = \ln(u)$ where $u = x^2 - 4x + 1$, exemplifies how complex functions can be systematically decomposed to facilitate differentiation.

Logarithmic Functions and Their Properties

Since the function involves a natural logarithm, understanding its properties enhances comprehension of the behavior of the function:

- $\ln(x)$ is defined for $x > 0$ and increases monotonically.
- Its derivative $\frac{1}{x}$ indicates a decreasing rate of increase for larger x.

In this context, the argument $x^2 - 4x + 1$ impacts the domain and the function's shape.

Final Remarks and Summary

The process of finding the tangent line to the curve $y = \ln(x^2 - 4x + 1)$ at the point $(4, 0)$ encapsulates several core calculus principles:

- Confirming the point lies on the curve.
- Differentiating the function accurately using the chain rule.
- Substituting the point's coordinates to find the slope.
- Applying the point-slope form to derive the tangent line's equation.

The resulting tangent line $y = 4x - 16$ not only provides a local linear approximation but also deepens understanding of the function's behavior at $x=4$. Such mathematical techniques find applications across sciences, engineering, and economics, showcasing the versatility and importance of calculus in analyzing real-world phenomena.

By mastering these steps, students and professionals alike can enhance their analytical toolkit, enabling them to interpret and predict complex behaviors with clarity and precision.

In conclusion, the derivation of the tangent line to the specified curve at the given point exemplifies the elegance and utility of calculus. It illustrates how derivative concepts translate into geometric insights, empowering a wide array of analytical and practical applications. As a foundational skill, it paves the way for more advanced explorations in mathematical analysis, modeling, and problem-solving.

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