Find Parametric Equations For The Normal Line To The Surface Z = Y - 27 At The Point P(1, 1,-1)?

Find Parametric Equations For The Normal Line To The Surface Z = Y - 27 At The Point P(1, 1,-1)?

Understanding the geometric properties of surfaces in three-dimensional space is a fundamental aspect of multivariable calculus. Among these properties, the normal line to a surface at a given point provides critical insights into the surface's orientation and behavior. In this article, we will explore how to find the parametric equations for the normal line to the surface defined by $Z = Y - 27$ at the specific point $P(1, 1, -1)$. This comprehensive guide aims to clarify the underlying concepts, detailed steps, and methods involved in deriving the normal line, making it accessible to students and enthusiasts interested in multivariable calculus.

Understanding the Surface $Z = Y - 27$

Before delving into the calculation process, it's essential to understand the nature of the surface itself.

What is the Surface $Z = Y - 27$?

The surface is given by the equation:

$$
Z = Y - 27
$$

This is a plane in three-dimensional space, expressed explicitly in terms of the variables Y and Z, with Z depending linearly on Y. To analyze it effectively, we can think of the surface as a plane with a specific orientation and slope.

Step 1: Recognize the Surface as a Level Surface of a Function

To find the normal line, it's often helpful to treat the surface as a level surface of a function \(F(x, y,

z) \).

Expressing the Surface as a Level Surface

Rearranging the equation:

$$
Z = Y - 27
$$

we can write:

$$
F(x, y, z) = Z - Y + 27 = 0
$$

Here:

$$
F(x, y, z) = z - y + 27
$$

This formulation allows us to utilize gradient vectors to find the normal direction.

Step 2: Compute the Gradient of $F(x, y, z)$

The gradient vector of F at any point provides the direction of the maximum rate of increase of F and is perpendicular (normal) to the level surface at that point.

Calculating the Gradient ∇F

Given:

$$
F(x, y, z) = z - y + 27
$$

the partial derivatives are:

- $\frac{\partial F}{\partial x} = 0$
- $\frac{\partial F}{\partial y} = -1$
- $\frac{\partial F}{\partial z} = 1$

Therefore, the gradient vector is:

$$
\nabla F(x, y, z) = \left(0, -1, 1 \right)
$$

Step 3: Evaluate the Gradient at Point $P(1, 1, -1)$

Since the gradient is constant for this linear surface (the surface is a plane), the normal vector at any point on the surface is:

$$
\vec{n} = \nabla F = (0, -1, 1)
$$

This vector points in the direction perpendicular to the surface at P.

Step 4: Write the Parametric Equations for the Normal Line

The parametric equations for a line passing through a point $P(x_0, y_0, z_0)$ with direction vector $\vec{d} = (a, b, c)$ are:

$$
\begin{cases}
x = x_0 + a t \\
y = y_0 + b t \\
z = z_0 + c t
\end{cases}
$$

where t is a parameter.

Applying to Our Point and Normal Vector

Given:

- Point $P(1, 1, -1)$
- Normal vector $\vec{n} = (0, -1, 1)$

The parametric equations of the normal line are:

$$
\begin{cases}
x(t) = 1 + 0 \times t = 1 \\
y(t) = 1 - 1 \times t = 1 - t \\
z(t) = -1 + 1 \times t = -1 + t
\end{cases}
$$

Final Parametric Equations of the Normal Line

$$
\boxed{
\begin{cases}
x(t) = 1 \\
y(t) = 1 - t \\
z(t) = -1 + t
\end{cases}
}
$$

This set of equations describes the line passing through $P(1, 1, -1)$ and perpendicular to the surface $Z = Y - 27$.

Understanding the Significance of the Normal Line

The normal line is an essential concept in differential geometry and multivariable calculus because it helps in understanding the orientation of the surface at a point. It is perpendicular to the tangent plane, which itself is tangent to the surface at that point.

Additional Insights and Applications

1. Tangent Plane Equation

Using the normal vector, we can also find the equation of the tangent plane at P:

$$
(\text{Normal vector}) \cdot (\textbf{r} - \textbf{r}_0) = 0
$$

where $\textbf{r}_0 = (x_0, y_0, z_0)$.

Plugging in:

$$
(0, -1, 1) \cdot (x - 1, y - 1, z + 1) = 0
$$

which simplifies to:

$$
0 \times (x - 1) - 1 \times (y - 1) + 1 \times (z + 1) = 0
$$

$$
-(y - 1) + (z + 1) = 0
$$

$$
- y + 1 + z + 1 = 0
$$

$$
z - y + 2 = 0
$$

This is the equation of the tangent plane at P.

2. Geometric Interpretation

Since the surface is a plane, the normal line is straightforward. For more complex surfaces defined by nonlinear functions, the process involves calculating the gradient of the defining function at the point of interest, as we've done here.

3. Practical Applications

Understanding normal lines is vital in fields such as:

- Computer Graphics: For shading and lighting calculations.
- Physics: In analyzing surface interactions.
- Engineering: For stress analysis on surfaces.
- Mathematics: In optimization and differential geometry.

Summary

To recap, the process of finding the parametric equations for the normal line to the surface $Z = Y - 27$ at point $P(1, 1, -1)$ involves:

1. Expressing the surface as a level surface $F(x, y, z) = 0$.
2. Computing the gradient ∇F, which gives the normal vector.
3. Evaluating the gradient at the point P.
4. Using the point-normal form of a line to write parametric equations.

This systematic approach ensures clarity and accuracy, especially when dealing with more complex surfaces where the normal vector varies across the surface.

Conclusion

Finding the parametric equations for the normal line to a surface is a fundamental skill in multivariable calculus, crucial for understanding surface orientation and related concepts. By converting the surface equation into a level surface and utilizing the gradient, we obtain an elegant and straightforward method to derive the normal line. In the case of the surface $Z = Y - 27$, which is a plane, this process is simplified by the constant nature of the gradient, resulting in a clear and precise normal line equation.

Whether you're studying for calculus exams, working on geometric modeling, or exploring surface properties in advanced mathematics, mastering the technique of finding normal lines enhances your understanding and analytical capabilities in three-dimensional space.

Keywords: parametric equations, normal line, surface $Z = Y - 27$, gradient, level surface, tangent plane, multivariable calculus, three-dimensional geometry

Frequently Asked Questions

What is the general approach to find the equation of the normal line to a surface at a given point?

To find the normal line, first compute the gradient vector of the surface at the point, which is perpendicular to the surface. Then, use this gradient vector as the direction vector to parametrize the line passing through the point.

How do you find the gradient vector for the surface Z = Y - 27?

Treat Z as a function of X and Y (even if it doesn't explicitly depend on X, it's considered constant with respect to X). The gradient vector is given by the partial derivatives: $\nabla Z = (\partial Z/\partial X, \partial Z/\partial Y, -1)$, but since $Z = Y - 27$, $\partial Z/\partial Y = 1$ and $\partial Z/\partial X = 0$.

What is the gradient vector of the surface Z = Y - 27 at the point P(1, 1, -1)?

The gradient vector is (0, 1, -1) because $\partial Z/\partial X = 0$, $\partial Z/\partial Y = 1$, and the partial derivative of Z with respect to Z itself is -1, representing the normal direction.

How do you write the parametric equations of the normal line at point P(1, 1, -1)?

Using the point and the normal vector (0, 1, -1), the parametric equations are: $x = 1$, $y = 1 + t$, $z = -1 - t$, where t is the parameter.

Why is X-coordinate constant in the parametric equations for the normal line?

Because the gradient vector's X-component is zero, indicating the normal line does not change in the X-direction and remains constant at $x = 1$.

Can the parametric equations be written differently for the normal line? If so, how?

Yes, the parametric equations can be written as: $x = 1$, $y = 1 + t$, $z = -1 - t$, or equivalently, as a vector form: $(x, y, z) = (1, 1, -1) + t(0, 1, -1)$.

What is the significance of the normal line in the context of the surface Z = Y - 27?

The normal line represents the line perpendicular to the surface at a specific point, useful for understanding surface orientation, finding tangent planes, or analyzing surface curvature at that point.

Additional Resources

Find Parametric Equations For The Normal Line To The Surface Z = Y - 27 At The Point P(1, 1, -1)

Understanding how to find parametric equations for the normal line to a surface at a specific point is a fundamental skill in multivariable calculus. In this guide, we will walk through the detailed process of finding parametric equations for the normal line to the surface Z = Y - 27 at the point P(1, 1, -1), providing clarity on each step and the underlying concepts involved.

Introduction: The Significance of Normal Lines in Multivariable Calculus

In multivariable calculus, surfaces are often described by functions of two variables, such as $Z = f(X, Y)$. At any given point on this surface, the normal line is a line that is perpendicular to the tangent plane at that point. Finding the parametric equations for this normal line is crucial for understanding surface geometry, analyzing tangent planes, and solving optimization problems constrained to surfaces.

The ability to compute the normal line involves understanding the gradient vector, which points in the direction of the steepest ascent and is perpendicular to the tangent plane. Once the gradient vector at a point is known, it can be used as the direction vector for the normal line.

Step 1: Understanding the Surface and the Point of Interest

Given the surface:

$$ Z = Y - 27 $$

and the point:

$$ P(1, 1, -1) $$

Let's interpret what this means:

- The surface is defined in 3D space, with Z depending on the Y coordinate.
- The point P has coordinates $(X, Y, Z) = (1, 1, -1)$.

Before proceeding, verify that P lies on the surface:

$$ Z_{surface} = Y - 27 = 1 - 27 = -26 $$

But the point's Z-coordinate is -1, not -26. This indicates a discrepancy; perhaps the problem statement has a typo or intends for us to find the normal line at that point assuming the surface's actual equation is $Z = Y - 27$.

Important note: If the point P does not lie on the surface, the normal line at P wouldn't be well-defined with respect to that surface. Therefore, to proceed, we assume that either the point P is intended to be $(1, 1, -26)$, or that the problem's focus is on the surface $Z = Y - 27$ and the point P is a typo.

Assumption: The correct point should be $P(1, 1, -26)$, which lies on the surface $Z=Y-27$.

Step 2: Computing the Gradient Vector of the Surface

The surface $Z = Y - 27$ can be rewritten as an implicit function:

$$ F(X, Y, Z) = Z - Y + 27 = 0 $$

The gradient vector of F:

$$ \nabla F = \left(\frac{\partial F}{\partial X}, \frac{\partial F}{\partial Y}, \frac{\partial F}{\partial Z} \right) $$

Calculating each component:

- $\frac{\partial F}{\partial X} = 0$ (since F doesn't depend on X)
- $\frac{\partial F}{\partial Y} = -1$
- $\frac{\partial F}{\partial Z} = 1$

So,

$$ \nabla F = (0, -1, 1) $$

This gradient vector points in the direction of the maximum rate of increase of F and is perpendicular to the surface at any point. Therefore, the normal vector to the surface at any point is $(0, -1, 1)$.

Step 3: Establishing the Normal Line's Parametric Equations

The parametric equations for a line passing through a point $P(x_0, y_0, z_0)$ with direction vector $\vec{v} = (a, b, c)$ are:

$$
\begin{cases}
x(t) = x_0 + a t \\
y(t) = y_0 + b t \\
z(t) = z_0 + c t
\end{cases}
$$

From our earlier findings:

- Point $P(1, 1, -26)$ (assuming the corrected point)
- Direction vector $\vec{v} = (0, -1, 1)$

Thus, the parametric equations of the normal line are:

$$
\begin{cases}
x(t) = 1 + 0 \cdot t = 1 \\
y(t) = 1 - t \\
z(t) = -26 + t
\end{cases}
$$

Step 4: Final Expression and Interpretation

The normal line to the surface $Z = Y - 27$ at the point $P(1, 1, -26)$ is described parametrically as:

$$\boxed{
\begin{aligned}
x(t) &= 1 \\
y(t) &= 1 - t \\
z(t) &= -26 + t
\end{aligned}
}$$

This provides a complete description of the line in 3D space passing through the point on the surface, perpendicular to it.

Additional Notes: Handling the Original Point $P(1, 1, -1)$

If, instead, the problem intended to use the point $P(1, 1, -1)$, which does not lie on the surface $Z = Y - 27$, then:

- The line passing through $P(1, 1, -1)$ with the same normal vector $(0, -1, 1)$ can still be described parametrically, but it would not be the normal line to the surface at that point (since the point isn't on the surface).

- To make the problem consistent, confirm the point's coordinates or adjust the surface equation accordingly.

Summary: Key Steps for Finding the Normal Line

1. Identify the surface equation and rewrite as an implicit function $F(X, Y, Z)$.
2. Calculate the gradient vector ∇F at the point of interest to find the normal vector.
3. Verify that the point lies on the surface. If not, adjust accordingly.
4. Write the parametric equations using the point and the normal vector.

Conclusion

Finding parametric equations for the normal line to a surface involves understanding the surface's gradient at a point, which provides the perpendicular direction. This process is fundamental in differential geometry, surface analysis, and many applications in physics and engineering. By mastering these steps, you can analyze complex surfaces and their properties with confidence.

Remember: Always verify that the point is on the surface before proceeding, and carefully interpret the surface equation and point coordinates to avoid inconsistencies.

Find Parametric Equations For The Normal Line To The Surface Z Y 27 At The Point P 1 1 1

Find Parametric Equations For The Normal Line To The Surface Z Y 27 At The Point P 1 1 1

Related Articles

- How Does Collection Policy Influence Sales, The Collection Period And The Bad-debt Loss Percentage?
- Factor Quadratics (10Point)x2 18x + 77(x 3) (x 15)(x - 11) (x - 7)(x - 2) (x -9)(x - 11) (x 6)
- Which Details From The Scene With The Scarlet Ibis Foreshadow Doodles Death? Check All That Apply.

Back to Home