Find Sin X 2 , Cos X 2 , And Tan X 2 From The Given Information. Sec(x) = 6 5 , 270 < X < 360

Find Sin X 2 , Cos X 2 , And Tan X 2 From The Given Information. Sec(x) = 6 5 , 270 < X < 360

Introduction

Trigonometry is an essential branch of mathematics that deals with the relationships between the angles and sides of triangles. It plays a vital role in various disciplines such as physics, engineering, astronomy, and architecture. One common problem in trigonometry involves finding the values of the sine, cosine, and tangent of a double angle, given certain information about the original angle.

In this article, we will explore how to determine sin 2x, cos 2x, and tan 2x when given specific data about sec x and the range of x. The problem statement is:

> Find sin 2x, cos 2x, and tan 2x from the given information: sec x = 6/5, where 270° < x < 360°.

This problem involves understanding the relationships between the basic trigonometric functions and their double angles, as well as interpreting the given information within the specified quadrant.

Understanding the Given Information

What is Sec x?

The secant function, sec x, is defined as the reciprocal of cosine:

$$
\sec x = \frac{1}{\cos x}
$$

Given:

$$
\sec x = \frac{6}{5}
$$

This implies:

$$
\cos x = \frac{1}{\sec x} = \frac{5}{6}
$$

The Range of x

The problem states:

$$
270^\circ < x < 360^\circ
$$

This range indicates x is in the fourth quadrant of the unit circle.

Quadrant IV is characterized by:

- $\cos x > 0$
- $\sin x < 0$

Since cos x is positive (which aligns with the given cos x = 5/6), and x is in the fourth quadrant, sin x must be negative.

Step 1: Find sin x

Using the Pythagorean identity:

$$
\sin^2 x + \cos^2 x = 1
$$

Substitute:

$$
\cos x = \frac{5}{6}
$$

Calculating:

$$
\sin^2 x = 1 - \left(\frac{5}{6}\right)^2 = 1 - \frac{25}{36} = \frac{36}{36} - \frac{25}{36} = \frac{11}{36}
$$

Thus,

$$
\sin x = \pm \sqrt{\frac{11}{36}} = \pm \frac{\sqrt{11}}{6}
$$

Since x is in the Fourth Quadrant, where $\sin x < 0$, we take:

$$
\sin x = - \frac{\sqrt{11}}{6}
$$

\]

Step 2: Find the Double Angle Values

Now that we have sin x and cos x, we can find sin 2x, cos 2x, and tan 2x using the double angle formulas.

Double Angle Formulas Recap

- \(\sin 2x = 2 \sin x \cos x\)
- \(\cos 2x = \cos^2 x - \sin^2 x\)
- \(\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}\)

Alternatively, tan 2x can also be expressed directly in terms of sin 2x and cos 2x:

\[
\tan 2x = \frac{\sin 2x}{\cos 2x}
\]

Step 3: Calculate sin 2x

Using:

\[
\sin 2x = 2 \sin x \cos x
\]

Substitute:

\[
\sin x = - \frac{\sqrt{11}}{6}, \quad \cos x = \frac{5}{6}
\]

Calculating:

\[
\sin 2x = 2 \times \left(- \frac{\sqrt{11}}{6}\right) \times \frac{5}{6} = 2 \times - \frac{\sqrt{11} \times 5}{36} = - \frac{10 \sqrt{11}}{36} = - \frac{5 \sqrt{11}}{18}
\]

Answer:

\[
\boxed{
\sin 2x = - \frac{5 \sqrt{11}}{18}
}
\]

Step 4: Calculate cos 2x

Using:

$$
\cos 2x = \cos^2 x - \sin^2 x
$$

Substitute:

$$
\cos^2 x = \left(\frac{5}{6}\right)^2 = \frac{25}{36}
$$
$$
\sin^2 x = \left(- \frac{\sqrt{11}}{6}\right)^2 = \frac{11}{36}
$$

Calculating:

$$
\cos 2x = \frac{25}{36} - \frac{11}{36} = \frac{14}{36} = \frac{7}{18}
$$

Answer:

$$
\boxed{
\cos 2x = \frac{7}{18}
}
$$

Step 5: Calculate tan 2x

Using:

$$
\tan 2x = \frac{\sin 2x}{\cos 2x}
$$

Substitute the values found:

$$
\tan 2x = \frac{- \frac{5 \sqrt{11}}{18}}{\frac{7}{18}} = - \frac{5 \sqrt{11}}{18} \times \frac{18}{7} = - \frac{5 \sqrt{11}}{7}
$$

Answer:

$$
\boxed{
\tan 2x = - \frac{5 \sqrt{11}}{7}
}
$$

Summary of Results

Function	Value
$\sin 2x$	$- \frac{5 \sqrt{11}}{18}$
$\cos 2x$	$\frac{7}{18}$
$\tan 2x$	$- \frac{5 \sqrt{11}}{7}$

Additional Insights and Applications

Importance of Quadrant Identification

Knowing the quadrant where x lies is critical because it determines the signs of the sine and cosine functions. For $270° < x < 360°$, the fourth quadrant, cos x is positive, and sin x is negative. This affects the signs of all derived double angle functions.

Double Angle Formulas in Practice

Double angle formulas are powerful tools in simplifying complex trigonometric expressions. They are especially useful in solving equations, analyzing wave functions, and in calculus when dealing with derivatives and integrals involving trigonometric functions.

Real-World Applications

Understanding these calculations is vital in engineering fields such as signal processing, where waveforms are studied using trigonometric functions. In navigation and astronomy, calculating angles and their double angles helps in triangulation and positioning.

Final Remarks

This comprehensive approach demonstrates how to determine the double angle sine, cosine, and tangent given a specific secant value and quadrant information. Mastery of these techniques enhances problem-solving skills in trigonometry and deepens understanding of the relationships between trigonometric functions.

Always remember to verify the quadrant and signs when working with inverse functions or given ranges to ensure accurate results.

Frequently Asked Questions

Given sec(x) = 65 and 270° < x < 360°, what is the value of cos(x)?

Since $\sec(x) = 65$, then $\cos(x) = 1/\sec(x) = 1/65$.

With sec(x) = 65, how can we find sin(x) given the quadrant?

In the fourth quadrant ($270° < x < 360°$), sin(x) is negative. Using the identity $\sec^2(x) = 1 + \tan^2(x)$, we can find tan(x) and then sin(x).

What is the value of tan(x) given sec(x) = 65?

First, find $\cos(x) = 1/65$. Then, $\sin(x) = -\sqrt{(1 - \cos^2(x))} = -\sqrt{(1 - (1/65)^2)}$. So, $\tan(x) = \sin(x)/\cos(x)$.

How do we calculate sin(x/2) and cos(x/2) from the given information?

Once sin(x) and cos(x) are known, use the half-angle formulas: $\sin(x/2) = \pm\sqrt{[(1 - \cos x)/2]}$, $\cos(x/2) = \pm\sqrt{[(1 + \cos x)/2]}$, considering the quadrant to determine signs.

What is the value of sin(2x) in terms of sin(x) and cos(x)?

Using the double angle formula: $\sin(2x) = 2 \sin(x) \cos(x)$.

How do we find cos(2x) from the given information?

Use the double angle formula: $\cos(2x) = 2 \cos^2(x) - 1$ or $\cos(2x) = 1 - 2 \sin^2(x)$, once sin(x) and cos(x) are known.

What is the value of tan(2x) based on the given data?

Calculate $\tan(2x) = 2 \tan(x) / (1 - \tan^2(x))$, after finding tan(x) from the previous steps.

Why is the sign of sin(x) negative in this problem?

Because x is in the fourth quadrant ($270° < x < 360°$), where sine is negative, while cosine is positive.

Additional Resources

Find Sin X 2, Cos X 2, and Tan X 2 from the Given Information: Sec(X) = 6/5, 270° < X < 360°

Introduction

Understanding the relationships between trigonometric functions and their values is fundamental in mathematics, especially in solving problems involving angles and their respective ratios. In this comprehensive analysis, we explore how to find the values of $\sin 2X$, $\cos 2X$, and $\tan 2X$ given that $\sec X = \frac{6}{5}$ and $270^\circ < X < 360^\circ$.

This problem is particularly intriguing because it combines inverse trigonometric functions, angle ranges, and double-angle formulas. It offers a practical example of how to manipulate and interpret trigonometric identities within specific quadrants, making it valuable for students, educators, and professionals engaged in mathematical problem-solving.

Understanding the Given Data

The value of $\sec X$

- $\sec X = \frac{6}{5}$

Recall that $\sec X = \frac{1}{\cos X}$, so:

$$
\cos X = \frac{1}{\sec X} = \frac{5}{6}
$$

Since $\sec X$ is positive, $\cos X$ is also positive or negative depending on the quadrant.

The angle range $270^\circ < X < 360^\circ$

- This range situates X in the fourth quadrant of the Cartesian plane.

- Quadrant IV: $\cos X > 0$, $\sin X < 0$

Therefore, $\cos X > 0$ and $\sin X < 0$ in this range, which guides the sign conventions for the subsequent calculations.

Step 1: Find $\sin X$

Using the Pythagorean identity:

$$
\sin^2 X + \cos^2 X = 1
$$

Plugging in $\cos X = \frac{5}{6}$:

$$
\sin^2 X + \left(\frac{5}{6}\right)^2 = 1

\]
\[
\sin^2 X + \frac{25}{36} = 1
\]
\[
\sin^2 X = 1 - \frac{25}{36} = \frac{36}{36} - \frac{25}{36} = \frac{11}{36}
\]

Taking the square root:

\[
\sin X = \pm \frac{\sqrt{11}}{6}
\]

Because \(X\) is in the fourth quadrant where \(\sin X < 0\), we select the negative root:

\[
\boxed{
\sin X = - \frac{\sqrt{11}}{6}
}
\]

Step 2: Find \(\sin 2X\), \(\cos 2X\), and \(\tan 2X\)

Double-angle identities are crucial:

- \(\sin 2X = 2 \sin X \cos X\)
- \(\cos 2X = \cos^2 X - \sin^2 X\)
- \(\tan 2X = \frac{2 \tan X}{1 - \tan^2 X}\)

But before applying these, it’s helpful to determine \(\tan X\):

\[
\tan X = \frac{\sin X}{\cos X} = \frac{- \frac{\sqrt{11}}{6}}{\frac{5}{6}} = - \frac{\sqrt{11}}{5}
\]

Step 3: Calculate \(\sin 2X\)

\[
\sin 2X = 2 \sin X \cos X
\]
\[
= 2 \times \left(- \frac{\sqrt{11}}{6}\right) \times \frac{5}{6}
\]
\[
= 2 \times - \frac{\sqrt{11} \times 5}{36}
\]

\[
= - \frac{10 \sqrt{11}}{36}
\]

Simplify numerator and denominator:

\[
\sin 2X = - \frac{5 \sqrt{11}}{18}
\]

Result:

\[
\boxed{
\sin 2X = - \frac{5 \sqrt{11}}{18}
}
\]

Step 4: Calculate \(\cos 2X\)

Using the identity:

\[
\cos 2X = \cos^2 X - \sin^2 X
\]
\[
= \left(\frac{5}{6}\right)^2 - \left(- \frac{\sqrt{11}}{6}\right)^2
\]
\[
= \frac{25}{36} - \frac{11}{36} = \frac{14}{36} = \frac{7}{18}
\]

Result:

\[
\boxed{
\cos 2X = \frac{7}{18}
}
\]

Step 5: Calculate \(\tan 2X\)

Using the double-angle formula for tangent:

\[
\tan 2X = \frac{2 \tan X}{1 - \tan^2 X}
\]

Recall $\tan X = - \frac{\sqrt{11}}{5}$:

$$
\tan^2 X = \left(- \frac{\sqrt{11}}{5} \right)^2 = \frac{11}{25}
$$

Now compute numerator and denominator:

$$
2 \tan X = 2 \times - \frac{\sqrt{11}}{5} = - \frac{2 \sqrt{11}}{5}
$$

$$
1 - \tan^2 X = 1 - \frac{11}{25} = \frac{25}{25} - \frac{11}{25} = \frac{14}{25}
$$

Thus,

$$
\tan 2X = \frac{- \frac{2 \sqrt{11}}{5}}{\frac{14}{25}} = - \frac{2 \sqrt{11}}{5} \times \frac{25}{14}
$$

Simplify:

$$
= - \frac{2 \sqrt{11} \times 25}{5 \times 14} = - \frac{50 \sqrt{11}}{70}
$$

Reduce numerator and denominator:

$$
= - \frac{5 \sqrt{11}}{7}
$$

Result:

$$
\boxed{
\tan 2X = - \frac{5 \sqrt{11}}{7}
}
$$

Summary of Results

Trigonometric Function	Value
$\sin 2X$	$- \frac{5 \sqrt{11}}{18}$
$\cos 2X$	$\frac{7}{18}$

$\tan 2X$	$- \frac{5 \sqrt{11}}{7}$

Additional Insights and Practical Applications

Understanding how these values are derived exemplifies the interconnectedness of trigonometric identities and the importance of quadrant considerations. These calculations have practical implications in physics, engineering, and computer science, where wave functions, oscillations, and rotations often depend on precise angle measurements and their double angles.

In particular:

- Quadrant considerations influence the sign of sine and cosine, crucial for accurate results.
- Double-angle formulas simplify the process of finding related functions without needing to revert to original angles.
- Simplified radical forms maintain precision, especially when dealing with irrational numbers.

Final Remarks

This investigation demonstrates the elegance and utility of fundamental trigonometric identities. Given the information $\sec X = \frac{6}{5}$ and the specified angle range, we've systematically derived the following key double-angle values:

- $\sin 2X = - \frac{5 \sqrt{11}}{18}$
- $\cos 2X = \frac{7}{18}$
- $\tan 2X = - \frac{5 \sqrt{11}}{7}$

These results are consistent with the properties of functions in the fourth quadrant and illustrate the power of combining identities with quadrant analysis to solve complex trigonometric problems.

References:

- Stewart, J. (2015). Calculus: Concepts and Contexts. Cengage Learning.
- Larson, R., & Hostetler, R. (2012). Precalculus with Limits. Brooks Cole.
- Trigonometry identities and formulas. (n.d.). In MathWorld. Wolfram Research.

Disclaimer: This analysis assumes familiarity with basic trigonometric identities and algebraic manipulation.

Find Sin X 2 Cos X 2 And Tan X 2 From The Given Information

[Sec X 6 5 270 Lt X Lt 360]

Find Sin X 2 Cos X 2 And Tan X 2 From The Given Information Sec X 6 5 270 Lt X Lt 360

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