Find The 4th Term Of The Binomial Expansion Of (6a - B)6. The 4th Term Is (Simplify Your Answer.)

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Introduction

Understanding binomial expansions is fundamental in algebra, especially when dealing with polynomial expressions raised to powers. These expansions are crucial in numerous fields such as mathematics, physics, engineering, and computer science, where polynomial approximations and combinatorial calculations are common. In this article, we will focus on a specific binomial expansion problem: finding the 4th term of the expansion of $(6a - B)^6$. This problem not only tests your understanding of binomial theorem but also emphasizes the importance of binomial coefficients, algebraic manipulation, and simplification skills.

The binomial theorem provides a systematic way to expand expressions of the form $(x + y)^n$. Although our given expression involves $(6a - B)^6$, the principles remain the same, with particular attention paid to the negative sign and the coefficients involved. By understanding how to identify each term, especially the 4th term in this case, you can confidently tackle similar problems and deepen your grasp of polynomial expansions.

Understanding the Binomial Theorem

What Is the Binomial Theorem?

The binomial theorem states that for any positive integer n,

$$
(x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^{k}
$$

where

- $\binom{n}{k}$ is the binomial coefficient, calculated as $\frac{n!}{k!(n-k)!}$,
- x and y are any algebraic expressions.

Expansion Pattern

The expansion results in $(n + 1)$ terms, each comprising:

- A binomial coefficient,
- Powers of x decreasing from n to 0,
- Powers of y increasing from 0 to n.

Application to Our Expression

In our case, the expression is $(6a - B)^6$. Notice the negative sign, which affects the binomial expansion as follows:

$$
(6a - B)^6 = \sum_{k=0}^6 \binom{6}{k} (6a)^{6 - k} (-B)^k
$$

This formula allows us to find any term in the expansion, including the 4th term.

Step-by-Step Solution to Find the 4th Term

Step 1: Identify the General Term

The general term (the $(k+1)^{th}$ term) in the expansion of $(6a - B)^6$ is:

$$
T_{k+1} = \binom{6}{k} (6a)^{6 - k} (-B)^k
$$

where k ranges from 0 to 6.

Step 2: Determine the Value of k for the 4th Term

Since the first term corresponds to $k=0$, the 4th term corresponds to:

$$
k = 3
$$

because:

- 1st term: $k=0$,
- 2nd term: $k=1$,
- 3rd term: $k=2$,
- 4th term: $k=3$.

Step 3: Substitute $k=3$ into the General Term

Plugging in $k=3$:

$$
T_{4} = \binom{6}{3} (6a)^{6 - 3} (-B)^3
$$

Calculations involve each component:

- Binomial coefficient: $\binom{6}{3}$,
- Power of $6a$: $(6a)^3$,
- Power of $-B$: $(-B)^3$.

Step 4: Calculate the Binomial Coefficient

$$
\binom{6}{3} = \frac{6!}{3! \times 3!} = \frac{720}{6 \times 6} = \frac{720}{36} = 20
$$

Step 5: Calculate $(6a)^3$

$$
(6a)^3 = 6^3 \times a^3 = 216a^3
$$

Step 6: Calculate $(-B)^3$

$$
(-B)^3 = -B^3
$$

because raising a negative to an odd power preserves the negative sign.

Step 7: Combine All Components

Putting it all together:

$$
T_{4} = 20 \times 216a^3 \times (-B^3)
$$

which simplifies to:

$$
T_{4} = 20 \times 216a^3 \times -B^3
$$

Simplifying the 4th Term

Step 8: Multiply Numerical Coefficients

Calculate 20×216:

$$
20 \times 216 = (20 \times 200) + (20 \times 16) = 4000 + 320 = 4320
$$

Step 9: Write the Final Expression

Multiplying everything:

$$
T_{4} = 4320a^3 \times (-B^3) = -4320a^3 B^3
$$

Final Answer:

$$
\boxed{
\textbf{The 4th term of the expansion of } (6a - B)^6 \text{ is } -4320a^3 B^3
}
$$

Additional Insights and Tips

Handling Negative Signs

Whenever you see a negative sign in the binomial, remember that:

- The sign alternates depending on the power of the negative term,
- For odd powers, the sign remains negative,
- For even powers, the sign becomes positive.

Binomial Coefficient Calculations

Use factorial formulas or Pascal's triangle for quick binomial coefficient calculations, especially for larger n.

Simplification Strategies

- Always expand step-by-step,
- Simplify numerical coefficients early,
- Be mindful of negative signs and powers.

Common Mistakes to Avoid

- Mixing up the value of k when identifying the term,
- Forgetting the negative sign when raising $(-B)$ to a power,
- Incorrectly calculating binomial coefficients.

Applications of Binomial Expansion

Understanding how to find specific terms in a binomial expansion has practical applications, including:

- Calculating probabilities in binomial distributions,
- Developing polynomial approximations (e.g., binomial series),
- Solving algebraic equations involving powers,
- Modeling real-world phenomena in science and engineering.

Conclusion

Mastering binomial expansion and understanding how to extract specific terms is a vital skill in algebra. In this article, we carefully dissected the process of finding the 4th term of $(6a - B)^6$, emphasizing the importance of binomial coefficients, algebraic manipulation, and sign management. The final simplified form of the 4th term is $-4320a^3 B^3$. By applying these systematic steps, you can confidently approach similar problems involving binomial expansions, enabling you to solve complex algebraic expressions efficiently and accurately.

Keywords for SEO Optimization

- Binomial expansion
- Find 4th term of binomial
- Expansion of $(6a - B)^6$
- Binomial theorem tutorial
- Simplify binomial terms
- Algebra polynomial expansion
- Binomial coefficient calculation
- How to expand binomial expressions
- Polynomial term extraction
- Algebraic expression simplification

Remember: Practice makes perfect. Regularly solving binomial problems will enhance your understanding and speed, making complex algebraic expressions more manageable and intuitive.

Frequently Asked Questions

What is the general term in the binomial expansion of $(6a - b)^6$?

The general term is $T_{k+1} = C(6, k) (6a)^{6 - k} (-b)^k$, where k ranges from 0 to 6.

How do I find the 4th term in the binomial expansion of $(6a - b)^6$?

The 4th term corresponds to $k = 3$ in the general term, so $T_4 = C(6, 3) (6a)^{3} (-b)^{3}$.

Can you help me simplify the 4th term of $(6a - b)^6$?

Yes. $T_4 = C(6, 3) (6a)^3 (-b)^3 = 20\ 216a^3 (-b)^3 = 20\ 216a^3 (-b^3) = -4320a^3b^3$.

What is the value of the binomial coefficient C(6, 3)?

$C(6, 3) = 20$.

What is the simplified form of the 4th term in the binomial expansion of $(6a - b)^6$?

The 4th term simplifies to $-4320a^3b^3$.

How do I verify the correctness of the 4th term in the binomial expansion?

By calculating the general term for $k=3$, substituting the values, and simplifying as shown, you confirm the 4th term is $-4320a^3b^3$.

Additional Resources

Find The 4th Term Of The Binomial Expansion Of $(6a - B)^6$. The 4th Term Is (Simplify Your Answer.) is a classic problem in algebra that tests understanding of binomial expansion, combinatorial coefficients, and algebraic simplification. This problem is commonly encountered in high school and early college-level mathematics courses, particularly within topics related to binomial theorem and polynomial expansion. It offers a practical application of the binomial theorem, which is a fundamental principle for expanding expressions raised to a power without multiplying the binomial multiple times manually. This article aims to provide a comprehensive explanation of how to identify and simplify the 4th term in the expansion of $(6a - B)^6$, including detailed steps, key concepts, and tips for solving similar problems efficiently.

Understanding the Binomial Theorem

What is the Binomial Theorem?

The binomial theorem provides a formula for expanding expressions of the form $(x + y)^n$, where n is a non-negative integer. It states that:

$$
(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k
$$

Here, $\binom{n}{k}$ is the binomial coefficient, calculated as:

$$
\binom{n}{k} = \frac{n!}{k! (n-k)!}
$$

This theorem simplifies the process of expansion by providing a systematic way to determine each term's coefficient and powers.

Application to $(6a - B)^6$

In the given problem, the expression is $(6a - B)^6$. We can rewrite it as:

$$
(6a + (-B))^6
$$

which aligns with the binomial form $(x + y)^n$ with:

- $x = 6a$
- $y = -B$
- $n = 6$

Our goal is to find the 4th term in this expansion.

Identifying the Terms in the Expansion

General Term in Binomial Expansion

The general term (the $(k+1)^{th}$ term) in the binomial expansion of $(x + y)^n$ is:

$$
T_{k+1} = \binom{n}{k} x^{n-k} y^k
$$

where k ranges from 0 to n.

Determining the 4th Term

Since the terms are indexed starting from $k=0$:

- 1st term corresponds to $k=0$
- 2nd term corresponds to $k=1$
- 3rd term corresponds to $k=2$
- 4th term corresponds to $k=3$

Therefore, for the 4th term:

$$
k=3
$$

We can now substitute into the general formula:

$$
T_{4} = \binom{6}{3} (6a)^{6-3} (-B)^3
$$

Calculating the 4th Term

Step 1: Calculate the Binomial Coefficient $\binom{6}{3}$

Using the formula:

$$
\binom{6}{3} = \frac{6!}{3! (6-3)!} = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20
$$

Step 2: Compute the Powers of x and y

- $x^{6-3} = (6a)^3 = 6^3 a^3 = 216 a^3$
- $y^3 = (-B)^3 = - B^3$

Step 3: Combine All Components

Putting everything together:

$$
T_{4} = 20 \times 216 a^3 \times (- B^3)
$$

Calculating the coefficient:

$$
20 \times 216 = 4320
$$

Thus,

$$
T_{4} = 4320 a^3 \times (- B^3) = -4320 a^3 B^3
$$

Final Simplified Answer

The 4th term of the binomial expansion of $(6a - B)^6$ is:

$$
\boxed{-4320 a^3 B^3}
$$

This is the simplified form, combining the binomial coefficient, powers, and the negative sign.

Features and Tips for Binomial Expansion Problems

Features:

- Systematic approach using the binomial theorem avoids manual multiplication of large polynomials.
- The method applies to any binomial raised to a positive integer power.
- The ability to quickly compute binomial coefficients is crucial.

Tips:

- Always identify the correct value of k for the term you wish to find.
- Use factorial formulas or Pascal’s triangle for binomial coefficients to save time.
- Simplify powers before multiplying coefficients to avoid errors.
- Pay attention to signs, especially when dealing with negative terms.

Pros and Cons of Using Binomial Theorem for Expansion

Pros:

- Efficiency: Significantly reduces computation time for large powers.
- Accuracy: Minimize manual multiplication errors.
- Versatility: Can be applied to any binomial expression with positive integer exponents.
- Foundation: Strengthens understanding of combinatorics and algebraic manipulation.

Cons:

- Complexity for Large n: Manual calculation of binomial coefficients becomes cumbersome for large exponents.
- Limited to Positive Integer Powers: Not directly applicable to non-integer or negative powers without more advanced techniques.
- Requires Memorization or References: Knowing the binomial coefficient formulas or Pascal’s triangle is essential.

Conclusion

Finding the 4th term in the binomial expansion of $(6a - B)^6$ involves understanding the binomial theorem, correctly identifying the term’s index, calculating the binomial coefficient, and simplifying the algebraic expression. The final answer, $-4320 a^3 B^3$, demonstrates the power of systematic expansion and algebraic manipulation. Mastery of this process enhances problem-solving efficiency and prepares students for more complex algebraic and combinatorial challenges. Whether for academic exams or practical applications, the binomial theorem remains an invaluable tool in the mathematician’s toolkit.

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