Find The Absolute Maximum Value For The Function F(x) = X2 4, On The Interval [3, 0) U (0, 2].

Find The Absolute Maximum Value For The Function F(x) = X2 4, On The Interval [3, 0) U (0, 2]. Determining the absolute maximum of a function over a specified interval is a fundamental concept in calculus, often involving the analysis of critical points and boundary values. In this article, we will explore the process of finding the maximum value of the given function, F(x) = x^2 + 4, over the combined interval [3, 0) U (0, 2]. We will discuss the method systematically and provide detailed explanations to ensure clarity.

Understanding the Function and the Interval

The Function F(x) = x^2 + 4

The function in question is a quadratic function:
- Form: F(x) = x^2 + 4
- Type: Parabola opening upward
- Vertex: At x = 0, since the quadratic term is positive
- Range: For all real x, F(x) ≥ 4

This is a smooth, continuous function with no discontinuities, which simplifies the analysis of its extrema.

Interval Specification and Its Implications

The interval provided is [3, 0) U (0, 2]. Notice:
- It is a union of two parts, excluding the point x=0.
- The first interval is from 3 down to just before 0, i.e., [3, 0).
- The second interval is from just after 0 to 2, i.e., (0, 2].

This structure indicates that the function's maximum might occur at boundary points of these intervals or at critical points within the intervals.

Analyzing the Function on the Specified Intervals

Continuity and Behavior of F(x)

Since $F(x) = x^2 + 4$ is continuous everywhere, the maximum value over a closed interval typically occurs at critical points or at the endpoints. However, our interval includes open intervals at 0, so we must consider limits at that point.

Important observations:
- At x=3, $F(3) = 3^2 + 4 = 9 + 4 = 13$
- As x approaches 0 from the left, F(x) approaches 4 (since $\lim_{x \to 0^-} x^2 + 4 = 4$)
- As x approaches 0 from the right, F(x) also approaches 4
- At x=2, $F(2) = 4 + 4 = 8$

Because the intervals are open at 0, the value of F(x) at x=0 is not included, but the limits as x approaches 0 are relevant for the maximum.

Finding Critical Points

To find potential maximum points within the intervals, we analyze the derivative:

$$F'(x) = 2x$$

Critical points occur where $F'(x) = 0$:
- $2x = 0 \Rightarrow x=0$

Since x=0 is a critical point, but it's excluded from the interval (open at 0), we must consider the behavior approaching 0 from both sides.

Determining the Absolute Maximum Value

Step 1: Evaluate Endpoints and Critical Points

- At x=3: F(3)=13
- Approaching x=0 from the left: F(x) approaches 4
- Approaching x=0 from the right: F(x) approaches 4
- At x=2: F(2)=8

Since the point x=0 is not included, the function approaches 4 but never attains it at x=0.

Step 2: Consider the Behavior at the Endpoints of the Intervals
- For [3, 0):
- The maximum value is at x=3, which is 13
- As x approaches 0 from the left, F(x) approaches 4, which is less than 13
- For (0, 2]:
- The maximum value is at x=2, which is 8
- As x approaches 0 from the right, F(x) approaches 4, which is less than 8

Step 3: Summarize the Maximum Values
- On [3, 0): maximum is at x=3, with F(3)=13
- On (0, 2]: maximum is at x=2, with F(2)=8

Since the intervals are disjoint and the function is continuous (except at 0, which is not included), the overall absolute maximum over the combined domain is the highest among these maximums:

$$ \boxed{
\text{Absolute Maximum} = \max\{13, 8\} = 13
} $$

This maximum is attained at x=3.

Conclusion: The Absolute Maximum Value

The absolute maximum value of the function $F(x) = x^2 + 4$ on the combined interval $[3, 0) \cup (0, 2]$ is 13, which occurs at x=3. The function approaches 4 near x=0 but does not attain it, and the maximum within the second interval at x=2 is only 8, which is less than 13.

Additional Insights and Tips for Similar Problems

1. Always Check Critical Points and Endpoints
- For continuous functions on closed intervals, potential extrema are at critical points or endpoints.
- For open intervals, consider limits approaching boundary points.

2. Consider the Effect of Open Intervals

- When the interval excludes boundary points, analyze the behavior of the function as it approaches those points.
- Use limits to determine the supremum or infimum.

3. Use Derivatives for Critical Points

- Find where the derivative is zero or undefined to locate potential extrema inside the interval.

4. Compare Values at These Points

- The maximum value is the largest among the critical points and boundary points.

Final Thoughts

Understanding how to analyze functions over complex intervals, especially those involving open and closed parts, is crucial in calculus. By systematically evaluating function values at critical points and boundary limits, you can accurately determine the absolute maximum or minimum values. In this case, recognizing that the maximum occurs at x=3 simplifies the problem, as the function's parabola opens upward and increasing x increases the value of F(x).

Summary:
- The function $F(x) = x^2 + 4$ reaches its maximum value of 13 at x=3 within the interval $[3, 0) \cup (0, 2]$.
- The behavior near x=0 approaches 4 but does not include this value, so it cannot be considered the maximum.
- Always analyze critical points and boundary behaviors when dealing with piecewise or union intervals to accurately find extrema.

Keywords for SEO Optimization:
- Find the maximum of quadratic function
- Absolute maximum value calculus
- Analyzing functions over union intervals
- Critical points and boundary analysis
- Calculus maximum value problems
- Piecewise function maximum determination
- Limits at open interval boundaries
- How to find maximum value of a function

Frequently Asked Questions

What is the function we are analyzing for its absolute maximum value?

The function is F(x) = x^2 + 4.

What is the domain of the function F(x) in this problem?

The domain is the union of the intervals [3, 0) and (0, 2).

Are the intervals in the domain open or closed, and what does that imply for the analysis?

The intervals are partly open and partly closed; specifically, [3, 0) is closed at 3 and open at 0, while (0, 2) is open at 0 and 2, which affects whether endpoints are included in the maximum search.

Does the function F(x) = x^2 + 4 have any critical points within the domain?

Yes, the derivative F'(x) = 2x is zero at x=0, but 0 is not included in the domain intervals, so there are no critical points within the domain.

How do we evaluate the absolute maximum of F(x) on the given domain?

We evaluate the function at the endpoints of the intervals, considering the limits approaching any open endpoints, and compare these values to find the maximum.

What are the function values at the endpoints and critical points relevant to the domain?

At x=3, F(3)= 3^2 + 4=13; at x approaching 0 from the right, F(x) approaches 4; at x approaching 2 from the left, F(2)=8.

Based on the evaluations, where does the absolute maximum of F(x) occur?

The absolute maximum occurs at x=3, where F(3)=13, since this is the highest value among the endpoint evaluations.

Is the maximum at x=3 included in the domain, and how does that affect our conclusion?

Yes, x=3 is included in the domain [3, 0), so the maximum value of 13 at x=3 is valid and is the absolute maximum for the function on the domain.

Additional Resources

Find The Absolute Maximum Value For The Function $F(x) = x^2 - 4$, On The Interval [3, 0) U (0, 2]. – this is a classic problem in calculus that involves analyzing the behavior of a function over a specified domain, especially when the domain is disconnected or contains points of discontinuity. Such problems are essential for understanding how functions behave across different regions and how to identify their maximum and minimum values, particularly the absolute maximum.

In this comprehensive guide, we will explore the step-by-step process to find the absolute maximum value for the function $F(x) = x^2 - 4$ over the given domain, which is the union of two intervals: [3, 0) and (0, 2]. We'll clarify what challenges this domain presents, analyze the function's behavior, and systematically determine where the maximum occurs.

Understanding the Problem

Before diving into calculations, let's clarify the key components of the problem:

- Function: $F(x) = x^2 - 4$
- Domain: The union of two intervals, [3, 0) and (0, 2].

At first glance, the notation [3, 0) might seem unconventional because, typically, interval notation has the lower number first. Assuming this is a typo or a misinterpretation, the intended domain likely is:

- [0, 3] and (0, 2), or perhaps the problem means [3, 0), which would be invalid, or perhaps it means [0, 3] with some points excluded.

However, based on standard conventions and typical problem structures, the domain is probably:

Domain D = [3, 0) U (0, 2)

which is a union of two intervals:

- From 3 down to just above 0 (excluding 0): [3, 0)
- From just above 0 up to 2: (0, 2)

But because the interval [3, 0) is decreasing, and the notation is non-standard, it's more plausible that the domain is:

D = [0, 3] \ {0}, or perhaps the domain excludes 0, but given the problem, the most logical interpretation is that the domain is:

D = [0, 3] \ {0}, or specifically:

D = [0, 3] \ {0}, with the union of two parts: (0, 2) and (2, 3].

Alternatively, perhaps the intended domain is [0, 3], excluding 0, i.e., (0, 3].

For clarity, let's assume the domain is:

- [0, 2) U (2, 3]

which is a union of two intervals, excluding the point 2. Or possibly:

- [3, 0) U (0, 2)

which would make the domain from 0 to 3, excluding 0, with the intervals specified.

Clarifying the Domain

Given the ambiguity, the most probable intended domain is:

- [0, 2] (excluding 0? or including it?)
- [3, 0) is probably a typo or misstatement, and the intended domain is:

D = [0, 3], but with certain points excluded, or perhaps the domain is [0, 2] ∪ [3, ∞), but given the problem's wording, the domain is:

D = [0, 2) ∪ [3, ∞)

or perhaps:

D = [0, 2) U [3, ∞)

Making an Assumption

To proceed logically, let's assume the domain is the union of two intervals:

D = [0, 2) U [3, ∞)

or perhaps:

$D = [0, 3]$, with the point 0 excluded.

However, the problem states "[3, 0) U (0, 2]", which suggests the domain is:

- From 3 down to just above 0 (excluding 0): [3, 0) — which is decreasing from 3 to 0, but interval notation should be [0, 3] if ascending.

- From just above 0 up to 2: (0, 2).

Given the ambiguity, to provide a meaningful analysis, we will interpret the domain as:

$D = [0, 2) \cup [3, \infty)$

and the function is defined for all $x \geq 0$, with the focus on these intervals.

The Function Behavior and Domain

Given the function:

$F(x) = x^2 - 4$

and the domain:

- $x \in [0, 2)$ and
- $x \in [3, \infty)$

Our goal:

Find the absolute maximum value of $F(x)$ over this domain.

Step 1: Analyze the Function

The function:

$F(x) = x^2 - 4$

is a parabola opening upward, with its vertex at $x = 0$ (minimum point), where:

$F(0) = -4$

As x increases, the function increases quadratically.

Step 2: Determine Critical Points

Since the function is continuous and differentiable everywhere, but our domain is restricted, the potential points for maximum are:

- Endpoints of the domain intervals.
- Critical points within the domain (where derivative is zero).

Derivative:

$F'(x) = 2x$

- Critical point at $x = 0$ (since $F'(0) = 0$).

But $x=0$ is within the domain $[0, 2)$.

Step 3: Evaluate Endpoints and Critical Points

Endpoint evaluations:

- At $x=0$:

$F(0) = 0^2 - 4 = -4$

- At x approaching 2 from the left:

Since 2 is not included, but since the function is continuous:

$F(2) = 4 - 4 = 0$

But because $(0, 2)$ is open, the maximum value at x approaching 2 from the left is approaching 0.

- At $x=3$:

$F(3) = 9 - 4 = 5$

- At x approaching infinity:

$F(x)$ tends to infinity; thus, on $[3, \infty)$, the maximum is unbounded and tends to infinity.

Step 4: Find Absolute Maximum

Over the interval $[0, 2)$:

- The maximum value of $F(x)$ on $[0, 2)$ is approaching $F(2) = 0$, but since 2 is not included, the maximum is approaching 0 from below.

- The maximum at the critical point $x=0$ is $F(0) = -4$.

- Therefore, the absolute maximum on [0, 2) is approaching 0 as x approaches 2 from the left.

Over the interval [3, ∞):

- Since $F(x) = x^2 - 4$ increases without bound as x increases, the absolute maximum on this interval is infinity.

Conclusion:

- On [0, 2): The function approaches 0 as x approaches 2 from the left, but since 2 is not included, there is no point at which the function attains 0 exactly within the domain. Therefore, the supremum (least upper bound) is 0, but the maximum value is not attained.

- On [3, ∞): The function increases without bound, so there is no finite maximum; the maximum tends to infinity.

Final statement:

The absolute maximum value of $F(x) = x^2 -4$ on the domain $[0, 2) \cup [3, \infty)$ is unbounded, tending to infinity.

However, if the domain is limited to [0, 2] and [3, 3], then:

- The maximum on [0, 2] is at x approaching 2, with value approaching 0.
- The maximum at $x=3$ is 5, which is greater than any value in [0, 2].

Thus, the absolute maximum over the entire domain is:

$F(3) = 5$

since this is the highest finite value obtained at $x=3$.

Summary and Key Takeaways

1. Critical Points and Endpoints
- For functions like $F(x) = x^2 - 4$, critical points occur where the derivative is zero.
- Endpoints of the domain are crucial in determining maximum and minimum values.

2. Behavior at Infinity
- If the domain extends to infinity, the function's behavior at large x (or negative x) determines whether maxima or minima are finite or infinite.

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3. Domain Considerations
- When the domain is disconnected or contains open intervals, maxima and minima may not be attained but only approached.

4. Practical Approach:
- Evaluate the function at critical points within the domain.
- Evaluate at boundary points or points approaching boundaries.
- Compare these values to identify the absolute maximum and minimum.

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