Find The Center And Radius Of The Circle Given By The Equation: X^2 + Y^2 + 6x - 4y - 12 = 0.

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Understanding the properties of circles is fundamental in geometry, particularly when analyzing their equations and identifying key features such as the center and radius. Circles are among the most straightforward yet significant shapes studied in geometry because of their symmetry and unique properties. Determining the center and radius from a general quadratic equation involving x and y enables students and professionals to analyze geometric problems efficiently, perform transformations, and solve real-world applications.

In this comprehensive guide, we will explore the process of finding the center and radius of a circle given by the equation:

$$ x^2 + y^2 + 6x - 4y - 12 = 0 $$

We'll cover the theoretical background, step-by-step procedures, relevant formulas, and practical tips to help you master this important skill.

Understanding the Standard Equation of a Circle

Before diving into solving the specific problem, it's crucial to understand the standard form of a circle's equation and its components.

The Standard Form of a Circle

The equation of a circle with center (h, k) and radius r is expressed as:

$$
(x - h)^2 + (y - k)^2 = r^2
$$

This form makes it easy to identify the center (h, k) and the radius r directly.

Key components:
- The center of the circle is at (h, k).
- The radius is r.

The General Form of a Circle Equation

Circles can also be expressed in the expanded quadratic form:

$$
x^2 + y^2 + Dx + Ey + F = 0
$$

where D, E, F are real numbers.

Important note: Not all quadratic equations of this form represent circles; some could represent other conic sections. To confirm that the given equation is a circle, the coefficients of x^2 and y^2 must be equal, and the equation must satisfy certain conditions, which we'll verify.

Verifying the Equation Represents a Circle

Let's analyze the given equation:

$$
x^2 + y^2 + 6x - 4y - 12 = 0
$$

- The coefficients of x^2 and y^2 are both 1, which satisfies the necessary condition for representing a circle.
- Our goal now is to rewrite this equation in the standard form to easily identify the center and radius.

Methodology for Finding the Center and Radius

The most effective approach for extracting the center and radius from the given quadratic form is completing the square. This method involves reorganizing the equation into the standard circle form.

Step-by-step process:

1. Group the x and y terms.
2. Complete the square for both the x and y groups.
3. Rewrite the equation as a perfect square trinomial plus a constant.
4. Express the equation in the standard form $(x - h)^2 + (y - k)^2 = r^2$.
5. Identify the center (h, k) and the radius r.

Step-by-Step Solution

Let's apply this systematic process to the given equation.

Step 1: Write the Equation

Original Equation:

$$
x^2 + y^2 + 6x - 4y - 12 = 0
$$

Step 2: Move the Constant to the Other Side

Add 12 to both sides:

$$
x^2 + 6x + y^2 - 4y = 12
$$

Step 3: Complete the Square for x and y

- For $x^2 + 6x$:

- Take half of the coefficient of x (which is 6), divide by 2: $6/2 = 3$.
- Square it: $3^2 = 9$.

- For $y^2 - 4y$:

- Take half of the coefficient of y (which is -4), divide by 2: $-4/2 = -2$.
- Square it: $(-2)^2 = 4$.

Now, add these squares inside the equation, but since we are adding numbers to the left side, we must also add the same to the right side to keep the equation balanced.

Step 4: Add the Squares and Adjust the Equation

Add 9 and 4 to both sides:

$$
x^2 + 6x + 9 + y^2 - 4y + 4 = 12 + 9 + 4
$$

Simplify:

$$
(x^2 + 6x + 9) + (y^2 - 4y + 4) = 12 + 13
$$

$$
(x + 3)^2 + (y - 2)^2 = 25
$$

Identifying the Center and Radius

Now, the equation is in the standard circle form:

$$
(x + 3)^2 + (y - 2)^2 = 25
$$

Comparing to $(x - h)^2 + (y - k)^2 = r^2$:

- $h = -3$
- $k = 2$
- $r^2 = 25 \Rightarrow r = \sqrt{25} = 5$

Therefore:

- Center of the circle: $(-3, 2)$
- Radius of the circle: 5

Summary of the Findings

Feature	Value
Center	$(-3, 2)$
Radius	5

This process demonstrates how algebraic techniques like completing the square are vital tools in coordinate geometry. It allows us to convert quadratic equations into their geometric representations, facilitating analysis and problem-solving.

Additional Tips and Considerations

- Always verify that the given quadratic equation genuinely represents a circle before proceeding.
- When completing the square, carefully add and subtract the same values to maintain equality.
- Remember that the signs of the numbers inside the parentheses indicate the position relative to the axes: positive for left/up, negative for right/down.
- The radius must be a positive value; if you obtain a negative under the square root, it indicates no real circle exists with that equation.

Applications of Finding the Center and Radius

Understanding how to find the center and radius is essential in various fields and applications:

- Computer Graphics: Designing objects with circular components.
- Navigation: Determining the location and range of signals or zones.
- Engineering: Analyzing mechanical parts with circular features.
- Astronomy: Calculating orbits and celestial object positions.
- Robotics: Path planning involving circular trajectories.

Practice Problems for Mastery

To reinforce your understanding, try solving these problems:

1. Find the center and radius of the circle given by $x^2 + y^2 - 4x + 2y + 1 = 0$.
2. Given the circle's center at $(4, -3)$ and radius 7, write its equation in standard form.
3. Determine whether the equation $x^2 + y^2 + 8x + 6y + 10 = 0$ represents a real circle and find its center and radius if it does.

Conclusion

Finding the center and radius of a circle from its algebraic equation is a fundamental skill in geometry that combines algebraic manipulation with geometric interpretation. By mastering the process of completing the square, you can efficiently convert a quadratic form into the standard circle equation, revealing the circle's geometric features.

In the specific example of $x^2 + y^2 + 6x - 4y - 12 = 0$, we've demonstrated a clear, step-by-step approach to determine that the circle's center is at $(-3, 2)$ and its radius is 5 units. This technique is broadly applicable, empowering you to analyze and interpret a wide range of problems involving circles and other conic sections.

Mastery of these concepts not only enhances your mathematical toolkit but also prepares you for advanced studies and practical applications across diverse scientific and engineering disciplines.

Frequently Asked Questions

How do you find the center and radius of the circle given by the equation $X^2 + Y^2 + 6x - 4y - 12 = 0$?

First, rewrite the equation in standard form by completing the square for both x and y terms, then identify the center (h, k) and radius r from the standard form $(x - h)^2 + (y - k)^2 = r^2$.

What is the first step in converting the given circle equation to standard form?

Group the x and y terms together and move the constant to the other side: $X^2 + 6x + Y^2 - 4y = 12$.

How do you complete the square for the x-terms in the equation?

Take half of the coefficient of x (which is 6), square it ($3^2=9$), and add this inside the equation to complete the square for x.

What is the process for completing the square for the y-terms?

Take half of the coefficient of y (-4), square it ($(-2)^2=4$), and add this to the y-terms to complete the square.

After completing the square, how do you find the radius of the circle?

Once the equation is in the form $(x - h)^2 + (y - k)^2 = r^2$, take the square root of the right side to find the radius r.

Can you demonstrate the complete process to find the center and radius from the given equation?

Yes. Rewrite as $X^2 + 6x + Y^2 - 4y = 12$. Complete the square: $(X^2 + 6X + 9) + (Y^2 - 4Y + 4) = 12 + 9 + 4$. Simplify to $(X + 3)^2 + (Y - 2)^2 = 25$. Therefore, the center is (-3, 2) and the radius is 5.

What is the importance of rewriting the given equation in standard form?

Rewriting in standard form makes it straightforward to identify the circle's center and radius directly from the equation.

What are the values of the center and radius for the circle given by the equation $X^2 + Y^2 + 6x - 4y - 12 = 0$?

The center is at (-3, 2) and the radius is 5.

Why is completing the square necessary in this problem?

Completing the square transforms the general quadratic form into the standard form of a circle, enabling easy identification of the center and radius.

Are there alternative methods to find the center and radius besides completing the square?

While completing the square is the most straightforward, one could also use the method of converting the equation to standard form algebraically or graphically, but completing the square is most direct for this type of problem.

Additional Resources

Find The Center And Radius Of The Circle Given By The Equation: $X^2 + Y^2 + 6x - 4y - 12 = 0$

Understanding the geometric properties of a circle, particularly its center and radius, is fundamental in coordinate geometry. The equation provided, $X^2 + Y^2 + 6x - 4y - 12 = 0$, is a standard form of a circle's equation, and deriving its center and radius involves several algebraic steps rooted in completing the square. This process not only clarifies the geometric interpretation of the equation but also enhances one's ability to analyze and manipulate quadratic equations in two variables. In this detailed review, we will explore the methods used to find the center and radius from the given equation, discuss the underlying concepts, and provide a comprehensive guide to solving similar problems.

Understanding the Standard Equation of a Circle

Before diving into the specific problem, it’s essential to understand the general form of a circle's equation. The standard form is:

$$ (x - h)^2 + (y - k)^2 = r^2 $$

where:

- (h, k) is the center of the circle.
- r is the radius.

Any quadratic equation representing a circle can often be rewritten into this form through algebraic manipulation, specifically completing the square for both the x and y terms.

Analyzing the Given Equation

The given equation:

$$ X^2 + Y^2 + 6x - 4y - 12 = 0 $$

contains quadratic terms and linear terms in both x and y, along with a constant. To find the circle’s center and radius, the goal is to rewrite this equation in the standard form. The process involves:

- Grouping x and y terms separately
- Completing the square for each group
- Isolating the constant on one side

Step-by-Step Solution

Let's proceed systematically:

1. Rewrite the equation with aligned variables:

$$ x^2 + 6x + y^2 - 4y = 12 $$

Notice that we moved the constant term (-12) to the right side to prepare for completing the square.

2. Complete the square for the x-terms:

- Take the coefficient of x, which is 6.
- Half of 6 is 3.
- Square it, resulting in $3^2 = 9$.

Add and subtract 9:

$$ x^2 + 6x + 9 - 9 $$

Similarly, for the y-terms:

- The coefficient of y is -4.
- Half of -4 is -2.
- Square it to get 4.

Add and subtract 4:

$$ y^2 - 4y + 4 - 4 $$

3. Rewrite the equation incorporating these:

$$ (x^2 + 6x + 9) + (y^2 - 4y + 4) = 12 + 9 + 4 $$

which simplifies to:

$$ (x + 3)^2 + (y - 2)^2 = 25 $$

4. Interpret the standard form:

$$ (x + 3)^2 + (y - 2)^2 = 25 $$

This is now in the standard circle form, where:

- The center is at (-3, 2)
- The radius is $\sqrt{25} = 5$

Final Results and Interpretation

From the algebraic manipulation above, the circle's center and radius are:

- Center: (-3, 2)
- Radius: 5

This information fully characterizes the circle in the coordinate plane.

Features and Significance of the Result

Understanding how to find the center and radius from a general quadratic equation is crucial for various applications in geometry, physics, engineering, and computer graphics. Here are some features and significance:

Pros:
- Versatility: The method applies to any circle equation in quadratic form, enabling quick identification

of geometric properties.
- Insightful: Reveals the geometric meaning behind algebraic coefficients.
- Foundation: Strengthens understanding of completing the square, a fundamental algebraic technique.

Cons:
- Complexity with Higher Degrees: More complex equations involving higher powers or multiple variables may not be straightforwardly reducible.
- Potential for Errors: Mistakes in completing the square or algebraic manipulation can lead to incorrect centers or radii.

Additional Tips for Solving Similar Problems

- Always carefully group x and y terms before completing the square.
- Remember to balance the added squares by also adding their constants to the other side of the equation.
- Double-check algebraic steps to avoid sign errors.
- When in doubt, expand the completed square form to verify it matches the original equation.

Conclusion

Finding the center and radius of a circle given its general quadratic equation is a fundamental skill in coordinate geometry. The process of completing the square transforms a complex quadratic form into the familiar standard form, revealing the circle's geometric properties. The example X^2 + Y^2 + 6x - 4y - 12 = 0 demonstrates how algebraic techniques directly translate into geometric insights—specifically, that the circle is centered at (-3, 2) with a radius of 5. Mastery of this method enhances problem-solving skills and deepens understanding of the relationship between algebra and geometry, making it an essential tool for students and professionals alike.

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