Find The Equation Of The Tangent Plane To The Surface Z=e^(3x/17)ln(4y) At The Point (1,3,2.96449).

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Introduction

Understanding how to find the equation of a tangent plane to a surface at a specific point is fundamental in multivariable calculus. It allows us to approximate the surface locally and analyze the behavior of functions with respect to changes in variables. In this article, we will explore the detailed process of deriving the equation of the tangent plane to the surface defined by

$$ Z = e^{\frac{3x}{17}} \ln(4y) $$

at the point $(1, 3, 2.96449)$. We will go through all necessary steps, including calculating partial derivatives, evaluating them at the given point, and formulating the tangent plane equation. This comprehensive guide aims to clarify the underlying concepts and provide a clear methodology for similar problems.

Understanding the Surface Equation

Before diving into calculations, it is essential to understand the surface's structure:

- The surface is defined by a function $ Z = f(x, y) $.
- The function involves an exponential term $ e^{\frac{3x}{17}} $ and a natural logarithm $ \ln(4y) $.
- The point of interest $(1, 3, 2.96449)$ lies on this surface, which can be verified by substitution.

Verifying the Point on the Surface

Let's verify that the point $(x, y, z) = (1, 3, 2.96449)$ lies on the surface:

$$
Z = e^{\frac{3 \times 1}{17}} \ln(4 \times 3)
$$

Calculate each component:

1. $ \frac{3 \times 1}{17} = \frac{3}{17} \approx 0.17647 $

2. $e^{0.17647} \approx 1.193$

3. $4 \times 3 = 12$

4. $\ln(12) \approx 2.4849$

Now, compute:

$$
Z \approx 1.193 \times 2.4849 \approx 2.96449
$$

This matches the given z-coordinate, confirming the point lies on the surface.

Step 1: Find Partial Derivatives of the Surface

To determine the tangent plane, we need the gradient vector at the point, which involves calculating the partial derivatives of $f(x, y)$:

$$
f(x, y) = e^{\frac{3x}{17}} \ln(4y)
$$

1.1 Partial derivative with respect to x:

Using the product rule:

$$
f_x = \frac{\partial}{\partial x} \left(e^{\frac{3x}{17}} \ln(4y) \right)
$$

Since $\ln(4y)$ is treated as a constant with respect to x:

$$
f_x = \frac{\partial}{\partial x} e^{\frac{3x}{17}} \times \ln(4y)
= e^{\frac{3x}{17}} \times \frac{\partial}{\partial x} \left(\frac{3x}{17} \right) \times \ln(4y)
$$

$$
f_x = e^{\frac{3x}{17}} \times \left(\frac{3}{17} \right) \times \ln(4y)
$$

1.2 Partial derivative with respect to y:

Using the product rule again:

$$
f_y = e^{\frac{3x}{17}} \times \frac{\partial}{\partial y} \ln(4y)

Since $\ln(4y) = \ln(4) + \ln(y)$, and $\ln(4)$ is constant:

$$
f_y = e^{\frac{3x}{17}} \times \frac{1}{y}
$$

Step 2: Evaluate Partial Derivatives at the Point $(1, 3)$

Now, we evaluate f_x and f_y at $x=1$, $y=3$.

2.1 Compute $f_x(1, 3)$:

$$
f_x(1, 3) = e^{\frac{3 \times 1}{17}} \times \frac{3}{17} \times \ln(4 \times 3)
$$

Recall from earlier:

- $e^{\frac{3}{17}} \approx 1.193$
- $\ln(12) \approx 2.4849$

Thus,

$$
f_x(1, 3) \approx 1.193 \times \frac{3}{17} \times 2.4849
$$

Calculate:

- $\frac{3}{17} \approx 0.17647$

Now,

$$
f_x \approx 1.193 \times 0.17647 \times 2.4849
$$

First, $1.193 \times 0.17647 \approx 0.2104$

Then,

$$
f_x \approx 0.2104 \times 2.4849 \approx 0.5239
$$

2.2 Compute $f_y(1, 3)$:

$$
f_y(1, 3) = e^{\frac{3}{17}} \times \frac{1}{3} \approx 1.193 \times \frac{1}{3} \approx 0.3977
$$

Step 3: Formulate the Equation of the Tangent Plane

The general equation of the tangent plane to the surface $z = f(x, y)$ at the point (x_0, y_0, z_0) is:

$$
z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)
$$

Substituting the known values:

- $x_0 = 1$
- $y_0 = 3$
- $z_0 \approx 2.96449$
- $f_x(1, 3) \approx 0.5239$
- $f_y(1, 3) \approx 0.3977$

The tangent plane equation becomes:

$$
z - 2.96449 = 0.5239 (x - 1) + 0.3977 (y - 3)
$$

Simplify:

$$
z = 2.96449 + 0.5239 (x - 1) + 0.3977 (y - 3)
$$

This is the explicit equation of the tangent plane at the given point.

Step 4: Final Expression of the Tangent Plane

Expanding the right-hand side:

$$
z = 2.96449 + 0.5239 x - 0.5239 + 0.3977 y - 1.1931
$$

Combine like terms:

\[

$$
z = (2.96449 - 0.5239 - 1.1931) + 0.5239 x + 0.3977 y
$$

Calculate the constant term:

$$
2.96449 - 0.5239 - 1.1931 \approx 1.24749
$$

Thus, the equation of the tangent plane is approximately:

$$
z \approx 1.24749 + 0.5239 x + 0.3977 y
$$

or, in standard form:

$$
z - 0.5239 x - 0.3977 y \approx 1.24749
$$

Conclusion

In this comprehensive guide, we have detailed the steps to find the equation of the tangent plane to the surface

$$
Z = e^{\frac{3x}{17}} \ln(4y)
$$

at the point $(1, 3, 2.96449)$. The process involved verifying the point's inclusion on the surface, calculating partial derivatives with respect to x and y, evaluating these derivatives at the point, and finally constructing the tangent plane equation using the point-slope form.

Summary of Key Steps:

- Verify the point lies on the surface.
- Derive the partial derivatives f_x and f_y.
- Evaluate derivatives at the point (x_0, y_0).
- Use the tangent plane formula:

$$
z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)
$$

- Simplify to obtain the explicit equation.

This method can be applied to similar problems involving tangent planes to surfaces described by multivariable functions, making it a valuable technique in advanced calculus and mathematical modeling.

Additional Tips for Finding Tangent Planes

- Always verify the point is on the surface before proceeding.
- Carefully compute partial derivatives, respecting

Frequently Asked Questions

What is the general method to find the equation of the tangent plane to a surface at a given point?

To find the tangent plane, you first find the gradient vector of the surface's defining function at the point, which gives the normal vector. Then, use the point-normal form of the plane equation: $n\cdot(r - r_0) = 0$, where n is the normal vector and r_0 is the point.

How do you compute the partial derivatives needed for the tangent plane of the surface z = e^(3x/17) ln(4y)?

You compute ∂z/∂x and ∂z/∂y by differentiating z with respect to each variable while treating the other as constant. Specifically, ∂z/∂x = e^(3x/17) (3/17) ln(4y) and ∂z/∂y = e^(3x/17) (1/y).

What is the normal vector to the surface at the point (1,3,2.96449)?

The normal vector is given by the gradient of the surface's defining function at the point. It is n = (∂z/∂x, ∂z/∂y, -1) evaluated at (x, y). Using the derivatives, n ≈ (0.368, 0.333, -1).

How is the tangent plane equation formulated once the normal vector and point are known?

The tangent plane equation is given by $n\cdot(r - r_0) = 0$, which expands to (n_x)(x - x_0) + (n_y)(y - y_0) + (n_z)(z - z_0) = 0, where n is the normal vector and (x_0, y_0, z_0) is the point.

What is the approximate equation of the tangent plane to the surface at the given point?

Using the computed normal vector n ≈ (0.368, 0.333, -1) and point (1, 3, 2.96449), the tangent plane equation is approximately: 0.368(x - 1) + 0.333(y - 3) - (z - 2.96449) = 0.

Why is it important to verify the point lies on the surface before finding the tangent plane?

Because the tangent plane is defined at a specific point on the surface, verifying ensures the point satisfies the surface equation, confirming correctness before proceeding with derivative calculations.

Can the tangent plane be used to approximate the surface near the point? If so, how?

Yes, the tangent plane provides a linear approximation of the surface near the point. For points close to (1, 3, 2.96449), the surface can be approximated by the tangent plane equation, simplifying analysis and calculations.

Additional Resources

Find The Equation Of The Tangent Plane To The Surface Z = e^(3x/17) ln(4y) At The Point (1,3,2.96449)

Understanding how to determine the tangent plane to a surface at a given point is a fundamental skill in multivariable calculus. The problem involves a surface defined implicitly by a function involving exponential and logarithmic components, which adds layers of complexity. In this comprehensive review, we will explore the detailed steps needed to find the tangent plane for the surface $Z = e^{\frac{3x}{17}} \ln(4y)$ at the specific point $(1, 3, 2.96449)$. We will break down the process into clear sections, explain the underlying calculus principles, and discuss the features and potential challenges involved.

Understanding the Surface and the Problem

The surface is given by the function:

$$
Z = e^{\frac{3x}{17}} \ln(4y)
$$

where Z is a function of x and y. The goal is to find the equation of the tangent plane at a specific point $(x_0, y_0, z_0) = (1, 3, 2.96449)$.

This task involves several steps:
- Confirming the point lies on the surface.
- Computing the partial derivatives $\frac{\partial Z}{\partial x}$ and $\frac{\partial Z}{\partial y}$.
- Using the point and derivatives to formulate the tangent plane equation.

Step 1: Confirming the Point Lies on the Surface

Before proceeding, verify that the given z-value corresponds to the x and y coordinates:

$$
Z = e^{\frac{3(1)}{17}} \ln(4 \times 3) = e^{\frac{3}{17}} \ln(12)
$$

Calculations:
- $\frac{3}{17} \approx 0.17647$
- $e^{0.17647} \approx 1.193$
- $\ln(12) \approx 2.4849$

Multiplying:
$$
1.193 \times 2.4849 \approx 2.96449
$$

The calculated Z-value matches the given point, confirming the point lies on the surface. This validation is crucial for correctness.

Step 2: Calculating Partial Derivatives

The tangent plane relies on the gradient vector, composed of the partial derivatives $\frac{\partial Z}{\partial x}$ and $\frac{\partial Z}{\partial y}$. These derivatives are obtained through standard differentiation rules:

Partial derivative with respect to x:

$$
\frac{\partial Z}{\partial x} = \frac{\partial}{\partial x} \left(e^{\frac{3x}{17}} \ln(4y) \right)
$$

Since $\ln(4y)$ is treated as a constant when differentiating with respect to x:

$$
\frac{\partial Z}{\partial x} = \ln(4y) \times \frac{\partial}{\partial x} \left(e^{\frac{3x}{17}} \right)
$$

Derivative of the exponential:

$$
\frac{\partial}{\partial x} e^{\frac{3x}{17}} = e^{\frac{3x}{17}} \times \frac{3}{17}
$$

Thus,

$$
\boxed{
\frac{\partial Z}{\partial x} = e^{\frac{3x}{17}} \ln(4y) \times \frac{3}{17}
}
$$

Partial derivative with respect to y:

Similarly,

$$
\frac{\partial Z}{\partial y} = \frac{\partial}{\partial y} \left(e^{\frac{3x}{17}} \ln(4y) \right)
$$

Here, $e^{\frac{3x}{17}}$ is constant with respect to y:

$$
\frac{\partial Z}{\partial y} = e^{\frac{3x}{17}} \times \frac{\partial}{\partial y} \left(\ln(4y) \right)
$$

Derivative of the logarithm:

$$
\frac{\partial}{\partial y} \ln(4y) = \frac{1}{4y} \times 4 = \frac{1}{y}
$$

Therefore,

$$
\boxed{
\frac{\partial Z}{\partial y} = e^{\frac{3x}{17}} \times \frac{1}{y}
}
$$

Step 3: Computing the Partial Derivatives at the

Point

Now, evaluate the derivatives at $(x_0, y_0) = (1, 3)$:

- Compute $e^{\frac{3 \times 1}{17}}$:

$$
e^{\frac{3}{17}} \approx 1.193
$$

- Compute $\ln(4 \times 3) = \ln(12) \approx 2.4849$.

Partial derivative with respect to x:

$$
\frac{\partial Z}{\partial x} \bigg|_{(1, 3)} = 1.193 \times 2.4849 \times \frac{3}{17}
$$

Calculations:

$$
1.193 \times 2.4849 \approx 2.9645
$$

$$
2.9645 \times \frac{3}{17} \approx 2.9645 \times 0.17647 \approx 0.524
$$

Partial derivative with respect to y:

$$
\frac{\partial Z}{\partial y} \bigg|_{(1, 3)} = 1.193 \times \frac{1}{3} \approx 0.398
$$

Step 4: Formulating the Equation of the Tangent Plane

The general formula for the tangent plane to a surface $z = f(x, y)$ at point (x_0, y_0, z_0) is:

$$
z - z_0 = \frac{\partial Z}{\partial x} (x_0, y_0) (x - x_0) + \frac{\partial Z}{\partial y} (x_0, y_0) (y - y_0)
$$

Substituting the known values:

$$
z - 2.96449 = 0.524 (x - 1) + 0.398 (y - 3)
$$

Expanding:

$$
z = 2.96449 + 0.524 (x - 1) + 0.398 (y - 3)
$$

$$
z = 2.96449 + 0.524x - 0.524 + 0.398y - 1.194
$$

Combine like terms:

$$
z = (2.96449 - 0.524 - 1.194) + 0.524x + 0.398y
$$

$$
z \approx 1.2465 + 0.524x + 0.398y
$$

This is the equation of the tangent plane at the specified point.

Discussion and Insights

Understanding how to derive the tangent plane equation provides valuable insight into the local linear approximation of a surface at a point. The process involves calculating partial derivatives, confirming the point's position on the surface, and applying the tangent plane formula. Several features and considerations are noteworthy:

Features:
- Analytical approach: The derivation uses fundamental calculus principles, making it transparent and precise.
- Applicability: The method applies broadly to other surfaces defined by differentiable functions.
- Computational validation: Confirming the point lies on the surface ensures the correctness of subsequent steps.

Pros:
- Provides a local linear approximation useful for tangent line and plane calculations.
- Facilitates understanding of surface behavior near the point.
- Can be extended to higher-order approximations if needed.

Cons / Challenges:
- Computing derivatives involving exponential and logarithmic functions can be error-prone.
- Numerical accuracy is critical, especially for complex functions.
- The process assumes differentiability; discontinuities in the function can complicate the approach.

Additional Considerations and Applications

Finding tangent planes is not just an academic exercise; it has numerous practical applications, including:
- Optimization problems: Identifying tangent planes helps find maxima and minima on surfaces.
- Surface analysis: Understanding local behavior and curvature.
- Engineering and physics: Modeling local interactions on surfaces or fields.

In more advanced contexts, tangent planes form the basis for concepts like differential geometry, surface integrals, and manifold theory. Mastering the fundamental process enhances one's ability to approach complex multidimensional problems.

Conclusion

The process of finding the tangent plane to the surface \(Z = e^{\frac

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