Find The Exact Value Of Each Expression. (Enter Your Answer In Radians.)(a) Sin1(3/2)b) Cos1(1/2)

Find The Exact Value Of Each Expression. (Enter Your Answer In Radians.)(a) Sin−1(3/2) b) Cos−1(1/2)

Understanding inverse trigonometric functions is essential for solving many mathematical problems, especially those involving angles and their ratios. In this article, we will explore how to find the exact values of the expressions $\sin^{-1}\left(\frac{3}{2}\right)$ and $\cos^{-1}\left(\frac{1}{2}\right)$, with answers expressed in radians. We will delve into the properties of inverse sine and cosine functions, their domains and ranges, and the methods for calculating their values accurately.

Inverse Trigonometric Functions: An Overview

Inverse trigonometric functions, also known as arc functions, are the inverses of the basic trigonometric functions. They are used to determine the angle when the value of a trigonometric function is known.

What Are Inverse Sine and Cosine?

- Inverse Sine ($\sin^{-1} x$ or $\arcsin x$): It gives the angle θ whose sine is x, i.e., $\sin \theta = x$.
- Inverse Cosine ($\cos^{-1} x$ or $\arccos x$): It provides the angle θ such that $\cos \theta = x$.

Domains and Ranges

Understanding the domains and ranges of these functions is crucial:

Function	Domain	Range
$\sin^{-1} x$	$[-1, 1]$	$[- \pi/2, \pi/2]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$

Note that these ranges are chosen to make the inverse functions single-valued (principal values).

Evaluating $\sin^{-1}\left(\frac{3}{2}\right)$

Is $\sin^{-1}\left(\frac{3}{2}\right)$ Defined?

The first step is to examine whether $\sin^{-1}\left(\frac{3}{2}\right)$ is defined within the domain.

- Since $\sin^{-1} x$ is only defined for $x \in [-1, 1]$,
- and $\frac{3}{2} = 1.5$ which is outside the interval $[-1, 1]$,

Conclusion: $\sin^{-1}\left(\frac{3}{2}\right)$ is undefined in the real number system.

Implication

Because the value $\frac{3}{2}$ exceeds the maximum value of the sine function (1), there is no real angle θ such that $\sin \theta = \frac{3}{2}$. In other words, this expression does not have a real solution.

Complex Number Perspective

In the realm of complex analysis, inverse sine can be extended to complex numbers. The formula for $\arcsin z$ for complex z involves complex logarithms:

$$
\arcsin z = -i \ln \left(i z + \sqrt{1 - z^2}\right)
$$

However, for the scope of real-valued functions and standard mathematical contexts, the answer is:

$\sin^{-1}\left(\frac{3}{2}\right)$ is undefined in real numbers.

Evaluating $\cos^{-1}\left(\frac{1}{2}\right)$

Is $\cos^{-1}\left(\frac{1}{2}\right)$ Defined?

Yes. Since $\frac{1}{2}$ lies within the domain $[-1, 1]$, the inverse cosine function is defined.

Find the Exact Value

Recall the standard angles and their cosine values:

Angle (radians)	Degrees	$\cos \theta$
$\pi/3$	60°	1/2
$2\pi/3$	120°	-1/2

Given that $\cos \theta = \frac{1}{2}$, the principal value of $\arccos\left(\frac{1}{2}\right)$ in $[0, \pi]$ is:

$$
\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}
$$

Final Answer:

$$
\boxed{\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3} \text{ radians}}
$$

Summary of Results

Expression	Value	Explanation
$\sin^{-1}\left(\frac{3}{2}\right)$	Undefined in real numbers	Outside the domain of $\arcsin$
$\cos^{-1}\left(\frac{1}{2}\right)$	$\boxed{\frac{\pi}{3}}$ radians	Standard angle with cosine 1/2

Additional Insights and Applications

Understanding the limitations and applications of inverse trigonometric functions is vital for both theoretical and practical purposes.

Applications of Inverse Sine and Cosine

- Solving triangles: Finding angles when side ratios are known.
- Engineering and physics: Calculating angles of incidence, reflection, and other phenomena involving oscillations and waves.
- Mathematical modeling: Inverting functions to find parameters or inputs based on outputs.

Important Tips

- Always check the domain before evaluating inverse functions.
- Remember that inverse sine is limited to $[- \pi/2, \pi/2]$, which corresponds to angles in the first and fourth quadrants.
- Inverse cosine ranges from 0 to π, covering the first and second quadrants.

Complex Analysis Note

When dealing with complex numbers, the inverse sine function extends beyond the real domain, involving complex logarithms and square roots. This allows for solutions to equations like $\sin \theta = \frac{3}{2}$, but such solutions are generally outside the scope of basic real-valued trigonometry.

Conclusion

In conclusion, the key takeaway from evaluating these inverse trigonometric expressions is understanding their domains and ranges:

- The expression $\sin^{-1}\left(\frac{3}{2}\right)$ has no real solution because $\frac{3}{2}$ exceeds the maximum sine value of 1.
- The expression $\cos^{-1}\left(\frac{1}{2}\right)$ equals $\frac{\pi}{3}$ radians because it corresponds to the standard angle where cosine is 1/2.

Mastering these concepts enhances problem-solving skills in mathematics, physics, engineering, and related fields, as inverse trigonometric functions are foundational in analyzing and interpreting various phenomena involving angles and ratios.

Remember: Always verify the domain of your inverse trigonometric functions before attempting to evaluate or interpret their values, and be aware of the difference between real and complex solutions when necessary.

Frequently Asked Questions

Find the exact value of $\sin^{-1}(3/2)$.

The value of $\sin^{-1}(3/2)$ is undefined because the sine function's range is [-1, 1], and 3/2 exceeds this range.

Find the exact value of $\sin^{-1}(1/2)$ in radians.

$\sin^{-1}(1/2) = \pi/6$ radians.

What is the exact value of $\cos^{-1}(1/2)$ in radians?

$\cos^{-1}(1/2) = \pi/3$ radians.

Calculate the exact value of $\sin^{-1}(3/2)$ in radians.

Since 3/2 is outside the domain [-1, 1], $\sin^{-1}(3/2)$ is undefined in real numbers.

Express $\cos^{-1}(1/2)$ in radians.

$\cos^{-1}(1/2) = \pi/3$ radians.

Find the value of $\sin^{-1}(1/2)$ in radians.

$\sin^{-1}(1/2) = \pi/6$ radians.

Is $\sin^{-1}(3/2)$ a valid real number? Why or why not?

No, because 3/2 is outside the range of sine's output [-1, 1], making $\sin^{-1}(3/2)$ undefined in real numbers.

What is the exact radian measure for $\cos^{-1}(1/2)$?

$\pi/3$ radians.

Find the exact value of $\sin^{-1}(1/2)$.

$\pi/6$ radians.

Evaluate $\cos^{-1}(1/2)$ in radians.

$\pi/3$ radians.

Additional Resources

Trigonometry: Exact Values of Inverse Trigonometric Functions in Radians

In the realm of mathematics, especially trigonometry, understanding the precise values of inverse trigonometric functions is fundamental. These functions, often denoted as $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$, are pivotal for solving problems involving angles and distances. This article delves into calculating the exact values of two specific inverse trigonometric expressions: $\sin^{-1}\left(\frac{3}{2}\right)$ and

$\cos^{-1}\left(\frac{1}{2}\right)$, providing a comprehensive analysis suitable for students, educators, and enthusiasts seeking clarity and precision in their mathematical toolkit.

Understanding Inverse Trigonometric Functions

Inverse trigonometric functions serve as the reverse operations of basic trigonometric functions. While $\sin \theta$, $\cos \theta$, and $\tan \theta$ take an angle and return a ratio, their inverses do the opposite: they take a ratio and return the angle.

Key Points:
- The domain of $\sin^{-1} x$ is $[-1, 1]$.
- The domain of $\cos^{-1} x$ is $[-1, 1]$.
- The ranges are:
- $\sin^{-1} x$: $[- \frac{\pi}{2}, \frac{\pi}{2}]$.
- $\cos^{-1} x$: $[0, \pi]$.

Understanding these domains and ranges is crucial because they determine the possible values these functions can take and specify the principal value branch for each inverse function.

Analyzing the Expressions

Let's examine each expression separately, exploring their definitions, constraints, and methods for finding their exact values.

1. $\sin^{-1}\left(\frac{3}{2}\right)$

Initial Observation:

- The argument $\frac{3}{2}$ is approximately 1.5.
- Since the domain of $\sin^{-1} x$ is $[-1, 1]$, the value $\frac{3}{2}$ exceeds this range.

Implication:

- The inverse sine function is not defined for values outside $[-1, 1]$.
- Therefore, $\sin^{-1}\left(\frac{3}{2}\right)$ does not exist within the real numbers.

Conclusion:

- The exact value cannot be determined in the real domain because the input is outside the permissible range.
- If considering complex analysis, the inverse sine function can be extended to complex numbers, but this involves more advanced concepts beyond the scope of typical trigonometry.

Summary:

Expression	Domain Constraint	Result	Explanation
$\sin^{-1}\left(\frac{3}{2}\right)$	$\frac{3}{2} \notin [-1,1]$	No real solution	Not defined in real numbers

---

2. $\cos^{-1}\left(\frac{1}{2}\right)$

Understanding the Argument:

- The value $\frac{1}{2}$ is within the domain $[-1,1]$.

Recall the Cosine Function:

- $\cos \theta = \frac{1}{2}$ when $\theta = \pm \frac{\pi}{3} + 2k\pi$, for integer k.

Principal Value for $\cos^{-1}$:

- The range of $\cos^{-1}$ is $[0, \pi]$, so the principal value must be within this interval.

Finding the Exact Value:

- The angle θ in $[0, \pi]$ satisfying $\cos \theta = \frac{1}{2}$ is $\theta = \frac{\pi}{3}$.

Therefore:

$$
\boxed{
\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}
}
$$

Implication:

- The exact value is $\frac{\pi}{3}$ radians, a well-known special angle in trigonometry.

Summary of Results

Expression	Exact Value	Notes
$\sin^{-1}\left(\frac{3}{2}\right)$	Not defined in real numbers	Argument outside $[-1,1]$
$\cos^{-1}\left(\frac{1}{2}\right)$	$\frac{\pi}{3}$	Standard angle in radians

---

Practical Implications and Applications

Understanding these inverse functions' exact values is not just an academic exercise—it's vital in fields such as engineering, physics, computer graphics, and navigation. For example:

- In physics, inverse trigonometric functions help determine angles from known ratios, such as in projectile motion or wave analysis.
- In engineering, they are essential for signal processing and control systems.
- In computer science, many algorithms rely on inverse functions to interpret ratios and convert between coordinate systems.

Knowing the exact value of $\cos^{-1} \left(\frac{1}{2}\right)$ as $\frac{\pi}{3}$ simplifies calculations and enhances understanding, especially when dealing with angles in radians.

Final Thoughts and Recommendations

- Always verify whether the argument of an inverse trigonometric function falls within the domain $[-1,1]$. If not, the value either doesn't exist in the real number system or requires complex analysis.
- Recognize standard angles and their sine, cosine, and tangent values to quickly determine inverse values.
- In educational settings, practice converting between degrees and radians and memorizing key inverse values for common ratios.

Summary

- $\sin^{-1}\left(\frac{3}{2}\right)$ is undefined in the real domain due to domain restrictions.
- $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$ radians, a fundamental value derived from common angles.

By understanding these principles, students and professionals can confidently evaluate inverse trigonometric functions, ensuring accuracy and efficiency in mathematical problem-solving.

Note: For those interested in exploring complex values or additional inverse functions, advanced mathematical frameworks and complex analysis are recommended.

Find The Exact Value Of Each Expression Enter Your Answer In Radians A Sin1 3 2 B Cos1 1 2

Find The Exact Value Of Each Expression Enter Your Answer In Radians A Sin1 3 2 B Cos1 1 2

Related Articles

- Did Dorothea Dix Have A Positive Or Negative Impact On The Reforms Era? Explain In 2 Sentences.
- Sujet: Prfriez Vous Jouer Dans La Rue Avec Vos Camarades Ou Rester La Maison Surfer Sur Internet ?
- HELP Please What Is This Question? 6358 Was Rounded To The Nearest One. What Is The Lower Bound?

Back to Home