Find The Exact Values Of The Six Trigonometric Functions Of Angle 0, If 9.-3 Is A Terminal Point

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Understanding how to determine the six trigonometric functions—sine, cosine, tangent, cosecant, secant, and cotangent—for a given angle is fundamental in trigonometry. When given a terminal point on the coordinate plane, such as (9, -3), we can use this information to find these functions for the corresponding angle 0, which is typically measured from the positive x-axis in standard position. In this article, we will explore the step-by-step process to find the exact values of the six trigonometric functions for angle 0, given the terminal point (9, -3).

Understanding the Terminal Point and Its Significance

What Is a Terminal Point?

In the context of angles and the coordinate plane, a terminal point refers to a point on the terminal side of an angle when it is positioned in standard position. Standard position means the vertex is at the origin (0,0), and the initial side lies along the positive x-axis. The terminal side is the ray that makes the given angle with the initial side.

Given the terminal point (9, -3), this point lies somewhere in the fourth quadrant, since x is positive and y is negative. This information helps us determine the angle's exact value and the corresponding trigonometric functions.

Relevance of the Terminal Point (9, -3)

The terminal point directly relates to the angle's position in the coordinate plane and allows us to compute the radius (or hypotenuse in a right-angled triangle) and the trigonometric ratios.

Calculating the Radius (Hypotenuse)

Before calculating the six trigonometric functions, we need to find the hypotenuse, often denoted as 'r', which is the distance from the origin to the terminal point.

Using the Distance Formula

The distance from the origin to the terminal point (x, y) is given by the formula:

$$
r = \sqrt{x^2 + y^2}
$$

Applying the coordinates:

$$
r = \sqrt{(9)^2 + (-3)^2} = \sqrt{81 + 9} = \sqrt{90}
$$

Simplify $\sqrt{90}$:

$$
r = \sqrt{90} = \sqrt{9 \times 10} = 3\sqrt{10}
$$

Thus, the hypotenuse $r = 3\sqrt{10}$.

Determining the Six Trigonometric Functions

With the terminal point (9, -3) and the hypotenuse $r = 3\sqrt{10}$, we can now find the exact values of the six functions.

Sine ($\sin \theta$)

$$
\sin \theta = \frac{y}{r}
$$

$$
\sin \theta = \frac{-3}{3\sqrt{10}} = -\frac{1}{\sqrt{10}}
$$

\]

To rationalize the denominator:

\[
\sin \theta = -\frac{1}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = -
\frac{\sqrt{10}}{10}
\]

Exact value:

\[
\boxed{\sin \theta = -\frac{\sqrt{10}}{10}}
\]

Cosine (\(\cos \theta\))

\[
\cos \theta = \frac{x}{r}
\]

\[
\cos \theta = \frac{9}{3\sqrt{10}} = \frac{3}{\sqrt{10}}
\]

Rationalize:

\[
\cos \theta = \frac{3}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} =
\frac{3\sqrt{10}}{10}
\]

Exact value:

\[
\boxed{\cos \theta = \frac{3\sqrt{10}}{10}}
\]

Tangent (\(\tan \theta\))

\[
\tan \theta = \frac{y}{x}
\]

$$
\tan \theta = \frac{-3}{9} = -\frac{1}{3}
$$

Exact value:

$$
\boxed{\tan \theta = -\frac{1}{3}}
$$

Cosecant ($\csc \theta$)

Cosecant is the reciprocal of sine:

$$
\csc \theta = \frac{1}{\sin \theta}
$$

$$
\csc \theta = \frac{1}{-\frac{\sqrt{10}}{10}} = -\frac{10}{\sqrt{10}} = -\sqrt{10}
$$

Exact value:

$$
\boxed{\csc \theta = -\sqrt{10}}
$$

Secant ($\sec \theta$)

Secant is the reciprocal of cosine:

$$
\sec \theta = \frac{1}{\cos \theta}
$$

$$
\sec \theta = \frac{1}{\frac{3\sqrt{10}}{10}} = \frac{10}{3\sqrt{10}} = \frac{10}{3\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{10 \sqrt{10}}{3 \times 10} = \frac{\sqrt{10}}{3}
$$

Exact value:

\[
\boxed{\sec \theta = \frac{\sqrt{10}}{3}}
\]

Cotangent (\(\cot \theta\))

Cotangent is the reciprocal of tangent:

\[
\cot \theta = \frac{1}{\tan \theta} = -3
\]

Exact value:

\[
\boxed{\cot \theta = -3}
\]

Summary of the Exact Values

Function	Exact Value
\(\sin \theta\)	\(-\frac{\sqrt{10}}{10}\)
\(\cos \theta\)	\(\frac{3\sqrt{10}}{10}\)
\(\tan \theta\)	\(-\frac{1}{3}\)
\(\csc \theta\)	\(-\sqrt{10}\)
\(\sec \theta\)	\(\frac{\sqrt{10}}{3}\)
\(\cot \theta\)	\(-3\)

Understanding the Significance of the Results

The signs of these functions are consistent with the terminal point being in the fourth quadrant, where cosine is positive and sine is negative. The tangent, cotangent, secant, and cosecant functions also carry signs that reflect this quadrant placement.

Knowing these exact values is crucial in solving many trigonometric problems, especially those involving unit circles, right triangles, and coordinate geometry.

Additional Tips for Calculating Trigonometric Functions from a Terminal Point

- Always start by calculating the hypotenuse \(r\) using the distance formula.
- Use the definitions of sine, cosine, and tangent in terms of x, y, and r to find the basic ratios.
- Recall that the signs of the trigonometric functions depend on the quadrant where the terminal point is located.
- Rationalize denominators when expressing functions to obtain exact simplified forms.
- Use reciprocal relationships to find the other functions easily.

Conclusion

By analyzing the terminal point (9, -3), we have successfully calculated the exact values of all six trigonometric functions for the corresponding angle. These calculations reinforce the importance of understanding the relationships between coordinate points, the hypotenuse, and the basic trigonometric ratios. Mastery of these concepts enables precise solutions to a wide range of problems in trigonometry and coordinate geometry.

Whether you're studying for exams or applying trigonometry in real-world scenarios, knowing how to find these functions from a terminal point is an essential skill. Practice with different points and angles to strengthen your understanding and improve your problem-solving efficiency in trigonometry.

Frequently Asked Questions

What are the six trigonometric functions of angle θ when the terminal point is (9, -3)?

The six functions are: $\sin \theta = -3/\sqrt{90}$, $\cos \theta = 9/\sqrt{90}$, $\tan \theta = -3/9$, $\csc \theta = -\sqrt{90}/3$, $\sec \theta = \sqrt{90}/9$, $\cot \theta = 9/(-3)$.

How do you find the exact values of the trigonometric functions given the terminal point (9, -3)?

First, calculate the radius $r = \sqrt{(x^2 + y^2)} = \sqrt{(9^2 + (-3)^2)} = \sqrt{(81 + 9)} = \sqrt{90}$. Then, use the definitions: $\sin \theta = y/r$, $\cos \theta = x/r$, $\tan \theta = y/x$, and their reciprocals.

What is the value of sin θ for the point (9, -3)?

$\sin \theta = y/r = -3/\sqrt{90}$, which simplifies to $-3\sqrt{90}/90$ or $-\sqrt{10}/10$.

What is the value of cos θ given the point (9, -3)?

$\cos \theta = x/r = 9/\sqrt{90}$, which simplifies to $3\sqrt{10}/10$.

How is tan θ calculated from the point (9, -3)?

$\tan \theta = y/x = -3/9 = -1/3$.

What are the reciprocal functions csc θ and sec θ for the point (9, -3)?

$\csc \theta = r/y = \sqrt{90}/(-3) = -\sqrt{90}/3 = -\sqrt{10}$, $\sec \theta = r/x = \sqrt{90}/9 = \sqrt{10}/1 = \sqrt{10}$.

What is cot θ for the terminal point (9, -3)?

$\cot \theta = x/y = 9/(-3) = -3$.

In which quadrant does the terminal point (9, -3) lie, and what does that imply for the signs of the trigonometric functions?

The point (9, -3) lies in the fourth quadrant, where $\cos \theta > 0$, $\sin \theta < 0$, $\tan \theta < 0$, $\csc \theta < 0$, $\sec \theta > 0$, and $\cot \theta < 0$.

How do you simplify the exact values of the

trigonometric functions for the point (9, -3)?

Express the values in simplest radical form: sin θ = -√10/10, cos θ = 3√10/10, tan θ = -1/3, csc θ = -√10, sec θ = √10, cot θ = -3.

Additional Resources

Find The Exact Values Of The Six Trigonometric Functions Of Angle 0, If 9.-3 Is A Terminal Point

Understanding how to find the exact values of the six trigonometric functions of an angle 0, given a terminal point (9, -3), is a fundamental skill in trigonometry. This process not only reinforces core concepts like the unit circle, coordinate geometry, and the Pythagorean theorem but also enhances problem-solving skills used in various mathematical and engineering contexts. In this comprehensive guide, we will explore step-by-step how to determine sine, cosine, tangent, cosecant, secant, and cotangent of the given angle, using the provided terminal point as the primary reference.

Introduction to the Problem

Suppose you are given a point (9, -3) which lies on the terminal side of an angle θ in standard position. The task is to find the exact values of all six trigonometric functions corresponding to that angle.

Why Is This Important?

- Coordinate Geometry & Trigonometry Connection: The point on the terminal side provides the necessary coordinates to apply trigonometric definitions.
- Application in Real-World Problems: These calculations are essential in fields like physics, engineering, and computer graphics.
- Understanding the Unit Circle: Even though the point isn't on the unit circle, converting it into a related form or understanding its position helps deepen comprehension of the circle's properties.

Step 1: Understanding the Coordinates and the Terminal Point

Given the point (9, -3):

- x-coordinate (adjacent side): 9
- y-coordinate (opposite side): -3

This point is located in the fourth quadrant because:

- x > 0
- y < 0

The position on the coordinate plane influences the signs of the trigonometric functions.

Step 2: Computing the Radius (r)

The radius r is the distance from the origin (0, 0) to the point (9, -3). This is essential because the six trigonometric functions are often expressed in terms of x, y, and r.

Using the Distance Formula:

$$
r = \sqrt{x^2 + y^2}
$$

Substituting the values:

$$
r = \sqrt{(9)^2 + (-3)^2} = \sqrt{81 + 9} = \sqrt{90}
$$

Simplify:

$$
r = \sqrt{90} = \sqrt{9 \times 10} = 3 \sqrt{10}
$$

Note: The exact value of r is $3\sqrt{10}$.

Step 3: Recall the Definitions of the Six Trigonometric Functions

Using the coordinate point and radius, the six basic functions are:

Function	Formula	Sign (based on quadrant)	Description
sin θ	y / r	Negative (since y < 0 in QIV)	Sine of θ
cos θ	x / r	Positive (since x > 0)	Cosine of θ
tan θ	y / x	Negative (since y < 0, x > 0)	Tangent of θ
csc θ	r / y	Negative	Cosecant of θ
sec θ	r / x	Positive	Secant of θ
cot θ	x / y	Negative	Cotangent of θ

Step 4: Calculating Each Trigonometric Function

4.1 Sine (sin θ)

$$
\sin θ = \frac{y}{r} = \frac{-3}{3 \sqrt{10}} = - \frac{3}{3 \sqrt{10}} = - \frac{1}{\sqrt{10}}
$$

Rationalizing the denominator:

$$
\sin θ = - \frac{1}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = - \frac{\sqrt{10}}{10}
$$

Result:

$$
\boxed{
\sin θ = - \frac{\sqrt{10}}{10}
}
$$

4.2 Cosine (cos θ)

$$
\cos θ = \frac{x}{r} = \frac{9}{3 \sqrt{10}} = \frac{3}{\sqrt{10}}
$$

Rationalize:

$$
\cos θ = \frac{3}{\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} = \frac{3 \sqrt{10}}{10}
$$

Result:

$$
\boxed{
\cos θ = \frac{3 \sqrt{10}}{10}
}
$$

4.3 Tangent (tan θ)

$$
\tan θ = \frac{y}{x} = \frac{-3}{9} = - \frac{1}{3}
$$

Result:

\[
\boxed{
\tan θ = - \frac{1}{3}
}
\]

4.4 Cosecant (csc θ)

\[
\csc θ = \frac{r}{y} = \frac{3 \sqrt{10}}{-3} = - \sqrt{10}
\]

Result:

\[
\boxed{
\csc θ = - \sqrt{10}
}
\]

4.5 Secant (sec θ)

\[
\sec θ = \frac{r}{x} = \frac{3 \sqrt{10}}{9} = \frac{\sqrt{10}}{3}
\]

Result:

\[
\boxed{
\sec θ = \frac{\sqrt{10}}{3}
}
\]

4.6 Cotangent (cot θ)

\[
\cot θ = \frac{x}{y} = \frac{9}{-3} = -3
\]

Result:

\[

$$
\boxed{
\cot θ = -3
}
$$

Summary of Exact Values

Function	Exact Value	Sign (based on quadrant)
sin θ	$-\frac{\sqrt{10}}{10}$	Negative in QIV
cos θ	$\frac{3 \sqrt{10}}{10}$	Positive in QIV
tan θ	$-\frac{1}{3}$	Negative in QIV
csc θ	$-\sqrt{10}$	Negative in QIV
sec θ	$\frac{\sqrt{10}}{3}$	Positive in QIV
cot θ	-3	Negative in QIV

Additional Insights and Applications

Understanding the Sign Conventions

Since the point is in the fourth quadrant,:

- Sine and cosecant are negative.
- Cosine and secant are positive.
- Tangent and cotangent are negative.

This aligns with the signs obtained from the coordinate analysis.

Visualizing the Angle

While the exact angle θ isn't explicitly calculated here, the terminal point's coordinates provide a clear geometric picture:

- The angle's terminal side passes through (9, -3).
- The inclination is measured from the positive x-axis.
- The magnitude of the point's position vector determines the radius r.

Practical Uses

- Calculations involving oblique angles in physics.
- Engineering problems involving direction and magnitude.
- Trigonometric modeling in computer graphics and animation.

Final Remarks

By starting with the terminal point (9, -3), we've carefully derived all six exact trigonometric function values for the corresponding angle θ. The key steps involved:

- Calculating the radius using the distance formula.
- Applying fundamental definitions of the trigonometric functions.
- Rationalizing denominators for exact algebraic expressions.
- Considering the quadrant to determine the sign of each function.

This method exemplifies a systematic approach to solving trigonometric problems involving arbitrary points, reinforcing the importance of coordinate geometry and the Pythagorean theorem in trigonometry.

Summary

- The radius (r) is $3\sqrt{10}$.
- The exact values of the functions are:
- $\sin \theta = -\sqrt{10} / 10$
- $\cos \theta = 3\sqrt{10} / 10$
- $\tan \theta = -1/3$
- $\csc \theta = -\sqrt{10}$
- $\sec \theta = \sqrt{10} / 3$
- $\cot \theta = -3$

These precise calculations not only solidify understanding but also serve as a foundation for tackling more complex trigonometric problems involving angles and coordinate points in multiple quadrants.

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