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Understanding derivatives is fundamental in calculus, as they provide valuable insights into the behavior of functions, including rates of change, slopes of curves, and local extrema. In this comprehensive guide, we will explore how to find the first and second derivatives of the quadratic function:

\[G(x) = -8x^2 + 28x + 6x - 59 \]

(Note: The original expression appears to contain a typo with "-8x?"—assuming it is meant to be "-8x^2". If the actual function differs, please clarify. For this article, we'll proceed with the standard quadratic form.)

Understanding Derivatives in Calculus

Derivatives measure how a function changes as its input changes. They are essential in various applications such as physics, engineering, economics, and more. The first derivative, denoted as G'(x), indicates the slope of the tangent line to the function at any point x, revealing whether the function is increasing or decreasing.

The second derivative, denoted as G''(x), provides information about the concavity of the function and the nature of its critical points (whether they are minima, maxima, or points of inflection).

Step-by-Step: Finding the First Derivative G'(x)

1. Recall the Power Rule for Differentiation

The power rule states that for any term \(ax^n \), its derivative is \(a \times n \times x^{n-1} \).

Mathematically:

$$ \frac{d}{dx}[ax^n] = a \times n \times x^{n-1} $$

2. Differentiate Each Term of G(x)

Given:

$$ G(x) = -8x^2 + 28x + 6x - 59 $$

Let's simplify the function first:

$$ G(x) = -8x^2 + (28x + 6x) - 59 = -8x^2 + 34x - 59 $$

Now, differentiate term-by-term:

- Derivative of $-8x^2$:

$$ \frac{d}{dx}(-8x^2) = -8 \times 2 \times x^{2-1} = -16x $$

- Derivative of $34x$:

$$ \frac{d}{dx}(34x) = 34 \times 1 \times x^{1-1} = 34 \times 1 \times x^{0} = 34 $$

- Derivative of constant -59:

$$ \frac{d}{dx}(-59) = 0 $$

3. Write the First Derivative G'(x)

Putting it all together:

$$ G'(x) = -16x + 34 $$

This is the first derivative of the function G(x).

Finding the Second Derivative G''(x)

1. Differentiate G'(x)

Given:

$$ G'(x) = -16x + 34 $$

Differentiate term-by-term:

- Derivative of $-16x$:

$$ \frac{d}{dx}(-16x) = -16 \times 1 = -16 $$

- Derivative of constant 34:

$$ \frac{d}{dx}(34) = 0 $$

2. Write the Second Derivative G''(x)

Thus:

$$ G''(x) = -16 $$

This indicates that the second derivative is a constant, which tells us the concavity of the original function.

Summary of Derivatives

- **Original Function:** $G(x) = -8x^2 + 34x - 59$
- **First Derivative:** $G'(x) = -16x + 34$
- **Second Derivative:** $G''(x) = -16$

Interpreting the Derivatives

1. Significance of G'(x)

- When $G'(x) > 0$, the function $G(x)$ is increasing.
- When $G'(x) < 0$, $G(x)$ is decreasing.
- Critical points occur where $G'(x) = 0$, indicating potential maxima or minima.

Setting $G'(x) = 0$:

$$ -16x + 34 = 0 $$

$$ -16x = -34 $$

$$ x = \frac{34}{16} = \frac{17}{8} = 2.125 $$

This critical point at $x = 2.125$ can be analyzed further to determine whether it is a maximum or minimum.

2. Significance of G''(x)

- Since $G''(x) = -16$ (a negative constant), the function $G(x)$ is concave downward everywhere.
- The critical point at $x = 2.125$ is a local maximum because the concavity is downward.

Applications of Derivatives in Analyzing Functions

Understanding how to find and interpret derivatives allows you to analyze the behavior of functions comprehensively. Here are some applications:

- **Finding Critical Points:** Points where $G'(x) = 0$ help identify local maxima and minima.

- **Determining Concavity:** The second derivative tells whether the graph of G(x) is concave up or down.

- **Optimization Problems:** Derivatives help in solving problems involving maximum profit, minimum cost, etc.

- **Graph Sketching:** Derivatives provide information about the shape and features of the graph.

Additional Tips for Derivative Calculations

- Always simplify the function before differentiating.
- Use the power rule for polynomial terms.
- Remember that constants differentiate to zero.
- For more complex functions involving products, quotients, or compositions, apply the product rule, quotient rule, or chain rule respectively.
- Confirm your critical points by evaluating the second derivative or using the first derivative test.

Conclusion

Calculating the first and second derivatives of a function like $G(x) = -8x^2 + 34x - 59$ provides powerful insights into the function's behavior. The first derivative, $G'(x) = -16x + 34$, indicates where the function increases or decreases and helps locate critical points. The second derivative, $G''(x) = -16$, reveals the concavity, confirming that the critical point at $x = 2.125$ is a local maximum due to the negative second derivative.

Mastering these derivative calculations is essential for analyzing functions, solving optimization problems, and graphing curves accurately. Whether you are a student learning calculus or a professional applying these concepts, understanding how to find and interpret derivatives is a fundamental skill that opens doors to more advanced mathematical analysis.

If you have any specific questions or need further explanations on derivatives, differentiation techniques, or applying these concepts to real-world problems, feel free to ask!

Frequently Asked Questions

What is the first derivative G'(x) of the function G(x) = -8x^3 + 28x^2 + 6x - 59?

G'(x) = -24x^2 + 56x + 6

How do you find the second derivative G''(x) of the function G(x = -8x^3 + 28x^2 + 6x - 59?

Differentiate G'(x) = -24x^2 + 56x + 6 to get G''(x) = -48x + 56

What is the significance of the first derivative G'(x) in analyzing the function G(x)?

G'(x) indicates the slope of the tangent line to G(x) at any point, helping identify critical points and increasing/decreasing intervals.

Why is calculating the second derivative G''(x) important in understanding the behavior of G(x)?

G''(x) helps determine the concavity of the function and identify points of inflection.

At which points does the function G(x) have potential local maxima or minima based on G'(x)?

Solve G'(x) = 0 ⇒ -24x^2 + 56x + 6 = 0 to find critical points indicating potential extrema.

How can the second derivative test be applied to the critical points of G(x)?

Evaluate G''(x) at critical points: if G''(x) > 0, G(x) has a local minimum; if G''(x) < 0, it has a local maximum.

What is the overall shape of the graph of G(x) based on its derivatives?

Since G'(x) is quadratic and G''(x) is linear, the graph has points of inflection and changes in concavity, indicating a curve with possible peaks and valleys.

Can the derivatives of G(x) help in optimizing the function? If so, how?

Yes. Critical points from G'(x) help find extrema, and the second derivative test confirms whether they are maxima or minima, useful for optimization.

Additional Resources

Find The First And Second Derivatives Of The Function $G(x) = -8x^3 + 28x^2 + 6x - 59$: A Comprehensive Analysis

Understanding the behavior of a function often begins with examining its derivatives. Derivatives provide vital insights into the rate of change, concavity, and the overall shape of the graph. In this article, we undertake a detailed exploration of the function:

$$G(x) = -8x^3 + 28x^2 + 6x - 59$$

with the goal of accurately computing its first and second derivatives, denoted as $G'(x)$ and $G''(x)$ respectively. This case study exemplifies the systematic approach needed for derivative calculation, interpretation, and application in various mathematical and real-world contexts.

Introduction to Derivatives and Their Significance

Derivatives are fundamental concepts in calculus that quantify how a function changes as its input varies. The first derivative, $G'(x)$, indicates the slope of the tangent line to the graph at any point, revealing whether the function is increasing or decreasing. The second derivative, $G''(x)$, measures the concavity or convexity, informing us about the function's curvature and potential points of inflection.

In applied mathematics, physics, engineering, and economics, derivatives are tools to optimize solutions, analyze stability, and predict future behavior. The process of finding derivatives involves rules such as the power rule, product rule, quotient rule, and chain rule, depending on the complexity of the function.

Step-by-Step Derivation of $G'(x)$

Given the function:

$$G(x) = -8x^3 + 28x^2 + 6x - 59$$

the goal is to compute its first derivative $G'(x)$.

Applying the Power Rule

The power rule states that for $f(x) = a x^n$, the derivative is:

$$f'(x) = a n x^{n-1}$$

Applying this to each term:

1. Term: $-8x^3$

- Derivative: $-8 \times 3 x^{3-1} = -24x^2$

2. Term: $28x^2$

- Derivative: $28 \times 2 x^{2-1} = 56x$

3. Term: $6x$

- Derivative: $6 \times 1 x^{1-1} = 6 x^{0} = 6$

4. Term: -59

- Derivative: Constant term's derivative is zero.

Constructing $G'(x)$

Combining the derivatives:

$$
\boxed{
G'(x) = -24x^2 + 56x + 6
}
$$

This first derivative encapsulates the rate at which $G(x)$ changes with respect to x, and is fundamental for analyzing critical points, increasing/decreasing intervals, and potential maxima or minima.

Calculating the Second Derivative $G''(x)$

The second derivative involves differentiating $G'(x)$:

$$
G'(x) = -24x^2 + 56x + 6
$$

Applying the power rule again:

1. Term: $-24x^2$

- Derivative: $-24 \times 2 x^{2-1} = -48x$

2. Term: $56x$

- Derivative: $56 \times 1 x^{1-1} = 56$

3. Constant term: 6

- Derivative: 0

Thus, the second derivative:

$$
\boxed{
G''(x) = -48x + 56
}
$$

This second derivative provides insights into the concavity of the function $G(x)$, which is valuable for understanding the nature of critical points and the curvature of the graph.

Analysis and Interpretation of Derivatives

Understanding the derivatives' implications offers a comprehensive view of the function's behavior.

Critical Points and Increasing/Decreasing Intervals

Critical points occur where $G'(x) = 0$:

$$
-24x^2 + 56x + 6 = 0
$$

Dividing through by -2 for simplicity:

$$
12x^2 - 28x - 3 = 0
$$

Applying the quadratic formula:

$$
x = \frac{28 \pm \sqrt{(-28)^2 - 4 \times 12 \times (-3)}}{2 \times 12}
$$

Calculating discriminant:

$$
\Delta = 784 - 4 \times 12 \times (-3) = 784 + 144 = 928
$$

Thus,

$$
x = \frac{28 \pm \sqrt{928}}{24}
$$

Simplifying $\sqrt{928}$:

$$
\sqrt{928} \approx 30.464
$$

Finally,

$$
x \approx \frac{28 \pm 30.464}{24}
$$

- For the plus sign:

$$
x \approx \frac{28 + 30.464}{24} = \frac{58.464}{24} \approx 2.436
$$

- For the minus sign:

$$
x \approx \frac{28 - 30.464}{24} = \frac{-2.464}{24} \approx -0.103
$$

These critical points suggest potential local maxima or minima, depending on the concavity at those points.

Concavity and Points of Inflection

The second derivative:

$$
G''(x) = -48x + 56
$$

- When $G''(x) > 0$, the function is concave upward (convex).
- When $G''(x) < 0$, the function is concave downward (concave).

Set $G''(x) = 0$:

$$
-48x + 56 = 0 \Rightarrow x = \frac{56}{48} = \frac{14}{12} = \frac{7}{6} \approx 1.167
$$

This point is a potential inflection point where the concavity changes.

Implications and Applications

The detailed derivation and analysis of the derivatives of $G(x)$ serve multiple purposes:

- Optimization: Identifying critical points helps find local maxima and minima, which is valuable in applications like economics (maximizing profit) or physics (finding equilibrium points).
- Curve Sketching: Understanding concavity and inflection points aids in accurately sketching the graph of $G(x)$, revealing its shape and behavior.
- Predictive Modeling: Derivatives inform models about how a system responds to changes in input variables, essential in engineering and scientific research.

Summary of Key Results

Derivative	Expression	Significance
First Derivative $G'(x)$	$-24x^2 + 56x + 6$	Rate of change, critical points
Second Derivative $G''(x)$	$-48x + 56$	Concavity, inflection points

The critical points at $x \approx -0.103$ and $x \approx 2.436$ are of particular interest for further analysis. The inflection point at $x \approx 1.167$ marks a change in concavity, influencing the function's shape.

Conclusion

The process of finding the first and second derivatives of the function $G(x) = -8x^3 + 28x^2 + 6x - 59$ exemplifies the systematic application of calculus principles. Through careful application of the power rule, quadratic formula, and analytical interpretation, we have unveiled the dynamic characteristics of the function. These derivatives not only clarify the function's increasing/decreasing behavior and curvature but also lay the groundwork for optimization, graphing, and real-world modeling.

Such comprehensive derivative analysis underscores the importance of calculus in both theoretical pursuits and practical applications, reinforcing its position as a cornerstone of mathematical sciences. Whether for academic study, research, or engineering design, mastering the derivation and interpretation of derivatives remains a vital skill.

Note: For detailed graphing or numerical analysis, computational tools such as graphing calculators or software (e.g., WolframAlpha, Desmos) can further illustrate the function's behavior based on the derivatives calculated herein.

Find The First And Second Derivatives Of The Function G X 8x 28x2 6x 59 G X G X

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