Find The Integral. (remember To Use Absolute Values Where Appropriate.) 47x 175 (x 5)(x2 8x 17) Dx

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Understanding how to evaluate integrals is fundamental in calculus, especially when dealing with expressions that involve polynomial functions and absolute values. The integral in question appears to be somewhat complex at first glance: ∫ 47x 175 (x 5)(x^2 8x 17) dx. To accurately compute this integral, it is crucial to interpret each component carefully, consider the role of absolute values where applicable, and apply the appropriate calculus techniques such as expansion, substitution, and integration rules.

This article provides a comprehensive, step-by-step guide on how to evaluate this integral, including the importance of absolute values, the algebraic manipulations involved, and the integration process, all structured for clarity and SEO optimization.

Understanding the Integral Expression

Before diving into the calculation, let's analyze the integral's components:

Original Integral:

$$
\int 47x \times 175 \times (x - 5) \times (x^2 - 8x - 17) \, dx
$$

Note: The original notation seems to omit multiplication signs; for clarity, we interpret:

- "47x" as $47x$
- "175" as 175
- "(x 5)" as $(x - 5)$
- "(x2 8x 17)" as $(x^2 - 8x - 17)$

Key Points:

- The integral involves polynomial factors.
- The presence of absolute values may be necessary if the integrand involves expressions that could be negative over certain intervals.
- The goal is to simplify the integrand, possibly expand the product, and then integrate term-by-term.

Role of Absolute Values in Integrals

When integrating functions that involve absolute values, the main concern is the sign of the expression inside the absolute value:

- Absolute value ensures the integrand remains non-negative.
- To evaluate integrals involving absolute values, one must determine the intervals over which the expression inside the absolute value is positive or negative.
- The integral is then split into segments, and the absolute value is replaced with its equivalent positive or negative form accordingly.

In this context:

- If any component of the integrand is an absolute value, identify the points where the expression inside changes sign.
- Evaluate the integral separately over these intervals to account for the sign change.

Note: In the given expression, there is no explicit absolute value notation, but the instruction suggests considering absolute values where appropriate, possibly because the expression could involve negative values over some intervals.

Step-by-Step Solution Approach

To compute the integral, follow these steps:

1. Rewrite the integrand clearly

Express the integrand fully expanded:

$$
47x \times 175 \times (x - 5) \times (x^2 - 8x - 17)
$$

Calculate the constant:

$$
47 \times 175 = 8225
$$

So the integrand is:

$$
8225 \times x \times (x - 5) \times (x^2 - 8x - 17)
$$

2. Expand the polynomial expression

First, expand $(x - 5) \times (x^2 - 8x - 17)$:

$$
(x - 5)(x^2 - 8x - 17) = x(x^2 - 8x - 17) - 5(x^2 - 8x - 17)
$$

Calculate each term:

$$
x \times x^2 = x^3
$$
$$
x \times (-8x) = -8x^2
$$
$$
x \times (-17) = -17x
$$

and

$$
-5 \times x^2 = -5x^2
$$
$$
-5 \times (-8x) = 40x
$$
$$
-5 \times (-17) = 85
$$

Now, combine all:

$$
x^3 - 8x^2 - 17x - 5x^2 + 40x + 85
$$

Simplify:

$$
x^3 - (8x^2 + 5x^2) + (-17x + 40x) + 85 = x^3 - 13x^2 + 23x + 85
$$

The entire integrand becomes:

$$
8225 \times x \times (x^3 - 13x^2 + 23x + 85)
$$

3. Multiply by x

Now, multiply x into the expanded polynomial:

$$
x \times x^3 = x^4
$$
$$
x \times (-13x^2) = -13x^3
$$
$$
x \times 23x = 23x^2
$$
$$
x \times 85 = 85x
$$

Thus, the integrand simplifies to:

$$
8225 \times (x^4 - 13x^3 + 23x^2 + 85x)
$$

Or, explicitly:

$$
\boxed{
\text{Integrand} = 8225x^4 - 8225 \times 13 x^3 + 8225 \times 23 x^2 + 8225 \times 85 x
}
$$

Calculate the constants:

- $8225 \times 13 = 106,925$
- $8225 \times 23 = 189,175$
- $8225 \times 85 = 698,125$

So the integrand is:

$$
8225x^4 - 106,925x^3 + 189,175x^2 + 698,125x
$$

Calculating the Integral

Now, integrate term-by-term:

$$
\int 8225x^4 \, dx = 8225 \times \frac{x^5}{5} = \frac{8225}{5} x^5 = 1645 x^5
$$

$$
\int -106,925x^3 \, dx = -106,925 \times \frac{x^4}{4} = -26,731.25 x^4
$$

$$
\int 189,175x^2 \, dx = 189,175 \times \frac{x^3}{3} = 63,058.33 x^3
$$

$$
\int 698,125x \, dx = 698,125 \times \frac{x^2}{2} = 349,062.5 x^2
$$

Adding the constant of integration C:

$$
\boxed{
\int 47x \times 175 \times (x - 5) \times (x^2 - 8x - 17) \, dx = 1645 x^5 - 26,731.25 x^4 + 63,058.33 x^3 + 349,062.5 x^2 + C
}
$$

Considering Absolute Values in the Integral

While the integral has been evaluated assuming the integrand's sign remains consistent over the interval, in cases where the expression inside absolute values could change sign, special attention is necessary:

- Step 1: Find the roots of the polynomial expression inside the absolute value.
- Step 2: Determine the sign of the polynomial in each interval divided by these roots.
- Step 3: Split the integral at these roots.
- Step 4: Remove absolute value signs by considering the sign in each interval.
- Step 5: Integrate over each interval separately, adjusting the integrand accordingly.

Example:

Suppose the integrand involved $|x^2 - 8x - 17|$. To evaluate:

1. Find roots of $x^2 -8x - 17=0$:

$$
x = \frac{8 \pm \sqrt{64 + 68}}{2} = \frac{8 \pm \sqrt{132}}{2} = \frac{8 \pm 2\sqrt{33}}{2} = 4 \pm \sqrt{33}
$$

2. Determine the sign of the quadratic in each interval:

- For $x < 4 - \sqrt{33}$, the quadratic is positive or negative depending on the leading coefficient.

- Repeat for other intervals.

3. Rewrite the integral accordingly, replacing $|x^2 - 8x - 17|$ with $\pm (x^2 - 8x - 17)$ depending on the interval.

Additional Tips for Integrating Polynomial Expressions

- Always check if the polynomial can be factored; factoring simplifies integration.
- Use substitution when dealing with composite functions.
- Remember to include the constant of integration when indefinite integrals are involved.
- For definite integrals, evaluate the antiderivative at the bounds and subtract.

Summary and Final Remarks

Evaluating complex integrals like \(\int 47x \times 175 (x - 5)(x^2 -

Frequently Asked Questions

How do I interpret the integral of 47x + 175 over the interval from (x - 5) to (x^2 - 8x + 17)?

You need to evaluate the definite integral of the function $47x + 175$ with respect to x, considering the limits of integration as $(x - 5)$ and $(x^2 - 8x + 17)$. Remember to include absolute values where the integrand or limits are negative to ensure correct area calculations.

Why should I consider absolute values when finding this integral?

Absolute values are necessary when the integrand or the limits of integration can be negative, to correctly compute the area under the curve regardless of the sign of the function or limits.

How do I handle the variable limits (x - 5) and (x^2 - 8x + 17) when integrating?

Since the limits are expressed in terms of x, you need to treat them as bounds for the integral with respect to x. If the limits depend on x, you may need to clarify whether the integral is with respect to a different variable or interpret it as a definite integral between fixed bounds. If variables are involved, consider substitution or evaluate the integral over specific bounds.

Can you show the step-by-step solution for the integral of 47x + 175 from (x - 5) to (x^2 - 8x + 17)?

Certainly! First, find the indefinite integral of 47x + 175, which is (47/2)x^2 + 175x + C. Next, evaluate this expression at the upper limit (x^2 - 8x + 17) and the lower limit (x - 5), then subtract. Remember to consider absolute values if the limits or integrand are negative within the interval.

What is the significance of the quadratic expression (x^2 - 8x + 17) in this integral?

The quadratic (x^2 - 8x + 17) serves as the upper limit of the integral. Its shape and sign affect whether you need to consider absolute values, especially if it becomes negative within certain x-values. Understanding its roots or vertex can help determine the behavior of the integral.

Should I evaluate the integral for specific values of x or as a general expression?

It depends on the context. If you need a specific numerical result, substitute a particular value of x before evaluating the integral. For a general expression, keep x as a variable and simplify the indefinite integral accordingly.

How do I incorporate absolute values into the integral when the limits or integrand change sign?

Identify where the integrand or limits are negative. Divide the integral into sections where the function is positive or negative, then take the absolute value of each section's integral before summing, ensuring the total area is positive.

Is there a way to simplify the process of integrating functions involving absolute values?

Yes. You can analyze the points where the expression inside the absolute value equals zero to split the integral into regions. Then, evaluate the integral in each region without absolute values, applying the absolute value where necessary, and sum the results.

What precautions should I take when calculating this integral to ensure accuracy?

Ensure that you correctly handle the limits, especially if they depend on variables. Pay attention to signs of the integrand and bounds, incorporate absolute values where appropriate, and verify each step to avoid sign errors or misinterpretations of the limits.

Additional Resources

Find The Integral. (Remember To Use Absolute Values Where Appropriate.) 47x 175 (x 5)(x2 8x 17) Dx

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Introduction: The Significance of Integration in Mathematics

Integration is a fundamental concept in calculus that serves as the backbone for understanding areas, volumes, and accumulative quantities. Whether in physics, engineering, economics, or pure mathematics, the ability to evaluate integrals accurately is essential. Among the many types of integrals, indefinite integrals—also known as antiderivatives—are particularly significant because they help reconstruct functions from their derivatives.

The integral in question, ∫ [47x + 175] (x + 5)(x^2 + 8x + 17) dx, presents an intriguing problem that combines polynomial expressions with the necessity of proper algebraic manipulation and the use of absolute values when handling definite integrals across different intervals. This article will thoroughly analyze this integral, breaking down its components, exploring the techniques required for evaluation, and emphasizing the importance of absolute values in the context of integrals involving potentially negative values.

Understanding the Integral Components

Before delving into the calculation, it is crucial to understand each component of the integral:

Integral Expression:

$$
\int \left[47x + 175\right] (x + 5)(x^2 + 8x + 17) \, dx
$$

Key Elements:

1. The Polynomial Coefficients:
- The linear term $(47x + 175)$
- The quadratic expression $(x^2 + 8x + 17)$
2. Linear Factor:
- $((x + 5))$

3. The Entire Expression:
- The product of the linear term $(47x + 175)$ with the quadratic and linear factors.

Step 1: Simplifying the Expression

The first step toward integrating this function is algebraic simplification, making the integrand more manageable.

Expanding the Polynomial Products

The integrand contains the product of three expressions:

$$
(47x + 175) \times (x + 5) \times (x^2 + 8x + 17)
$$

Given its structure, it is often easier to handle such products by grouping and expanding step-by-step.

Step 1.1: Expand $(x + 5) \times (x^2 + 8x + 17)$

Let's first expand $(x + 5)(x^2 + 8x + 17)$:

$$
(x + 5)(x^2 + 8x + 17) = x \times (x^2 + 8x + 17) + 5 \times (x^2 + 8x + 17)
$$

Calculating each term:

- $x \times x^2 = x^3$
- $x \times 8x = 8x^2$
- $x \times 17 = 17x$

- $5 \times x^2 = 5x^2$
- $5 \times 8x = 40x$
- $5 \times 17 = 85$

Adding all these:

$$
x^3 + 8x^2 + 17x + 5x^2 + 40x + 85
$$

Combine like terms:

- x^3
- $8x^2 + 5x^2 = 13x^2$
- $17x + 40x = 57x$
- Constant term: 85

Result:

$$
x^3 + 13x^2 + 57x + 85
$$

Step 1.2: Multiply the result by $(47x + 175)$

Now, the integrand becomes:

$$
(47x + 175) \times (x^3 + 13x^2 + 57x + 85)
$$

We can distribute term-by-term:

\[
47x \times (x^3 + 13x^2 + 57x + 85) + 175 \times (x^3 + 13x^2 + 57x + 85)
\]

Calculating each:

- \(47x \times x^3 = 47x^4\)
- \(47x \times 13x^2 = 611x^3\)
- \(47x \times 57x = 2,679x^2\)
- \(47x \times 85 = 3,995x\)

- \(175 \times x^3 = 175x^3\)
- \(175 \times 13x^2 = 2,275x^2\)
- \(175 \times 57x = 9,975x\)
- \(175 \times 85 = 14,875\)

Now, sum all these:

\[
\begin{aligned}
& 47x^4 \\
& + (611x^3 + 175x^3) = 786x^3 \\
& + (2,679x^2 + 2,275x^2) = 4,954x^2 \\
& + (3,995x + 9,975x) = 13,970x \\
& + 14,875
\end{aligned}
\]

Final expanded integrand:

\[
47x^4 + 786x^3 + 4,954x^2 + 13,970x + 14,875
\]

Step 2: Integrating the Polynomial Expression

The integral reduces to:

\[
\int \left(47x^4 + 786x^3 + 4954x^2 + 13970x + 14875\right) dx
\]

Term-by-Term Integration

Using the power rule for integration:

\[

\int x^n dx = \frac{x^{n+1}}{n+1} + C
\]

we get:

\[
\begin{aligned}
\int 47x^4 dx &= 47 \times \frac{x^5}{5} = \frac{47}{5} x^5 \\
\int 786x^3 dx &= 786 \times \frac{x^4}{4} = \frac{786}{4} x^4 = \frac{393}{2} x^4 \\
\int 4954x^2 dx &= 4954 \times \frac{x^3}{3} = \frac{4954}{3} x^3 \\
\int 13970x dx &= 13970 \times \frac{x^2}{2} = 6985 x^2 \\
\int 14875 dx &= 14875 x
\end{aligned}
\]

Resulting Antiderivative:

\[
\boxed{
\frac{47}{5} x^5 + \frac{393}{2} x^4 + \frac{4954}{3} x^3 + 6985 x^2 + 14875 x + C
}
\]

Step 3: Recognizing the Role of Absolute Values

While the indefinite integral is straightforward, the mention of "remember to use absolute values where appropriate" is particularly significant when evaluating definite integrals over specific intervals, especially those where the integrand might change sign.

When Are Absolute Values Necessary?

- Sign Changes of the Integrand: If the integrand or the function being integrated takes on negative values over an interval, the definite integral's value (which corresponds to the net area) must be carefully interpreted.
- Area Calculations: To compute the total area (a scalar quantity), the integrand's negative portions must be converted to positive by applying absolute values before integrating or summing.

Practical Example:

Suppose we are asked to evaluate the definite integral from \(a\) to \(b\). If the integrand crosses the x-axis within \([a, b]\), then the total area is:

\[
\int_a^b |f(x)| dx
\]

which differs from:

\[

\left| \int_a^b f(x) dx \right|
\]

because the latter accounts for cancellation of negative and positive areas, whereas the former sums absolute magnitudes.

In the context of the polynomial expressions involved, it's essential to analyze the roots of the integrand to determine where the function may be negative, and accordingly, incorporate absolute values when computing total areas or when definite integrals are specified across such intervals.

Step 4: Final Notes on the Integration Process and Application

Handling Definite Integrals

If the problem specified limits of integration, say from \(x = x_1\) to \(x = x_2\), the process would involve:

1. Finding the roots of the integrand to identify intervals of sign change.
2. Splitting the integral at roots.
3. Applying absolute values to the integrand in each interval if calculating total area.
4. Evaluating the antiderivative at each boundary point and summing appropriately.

Significance of Absolute Values

Failure to consider absolute values when necessary can lead to misinterpretation of areas, especially in physical applications like calculating the total distance traveled, where negative velocities or positions imply direction but do not reduce

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