Find The Length And Breadth Of The Rectangle Whose Area Is 6x^2-29x+30. Also Find Its Perimeter.

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Understanding the dimensions of a rectangle given its area is a fundamental concept in geometry, especially in algebraic contexts. When the area of a rectangle is expressed as a quadratic polynomial, such as $6x^2 - 29x + 30$, it becomes a challenging yet intriguing problem to determine the length and breadth. These dimensions are typically represented as algebraic expressions or functions of the variable x, and solving such problems involves factoring quadratic expressions, understanding the properties of rectangles, and applying formulas for perimeter.

In this article, we will explore how to find the length and breadth of a rectangle when its area is given as $6x^2 - 29x + 30$. We will also discuss how to compute its perimeter based on the dimensions derived. The process involves several steps, including factoring the quadratic polynomial, interpreting the factors as potential dimensions, and verifying the solutions. Additionally, we will cover the importance of such problems in algebra and geometry, their applications, and tips for solving similar problems efficiently.

Let's begin by understanding the context and significance of the problem.

Understanding the Problem: Area of a Rectangle as a Quadratic Expression

A rectangle's area is calculated as the product of its length and breadth:
$$ \text{Area} = \text{Length} \times \text{Breadth} $$

In typical problems, the length and breadth are constants or simple algebraic expressions. However, in advanced algebraic problems, these dimensions can be expressed as functions of a variable, such as x, leading to the area being a quadratic polynomial:
$$ \text{Area} = 6x^2 - 29x + 30 $$

The challenge lies in expressing the length and breadth as algebraic expressions such that their product equals the given quadratic polynomial. Usually, this involves factoring the quadratic into two linear factors, which then can be interpreted as the dimensions of the rectangle.

Why is this problem important?

- It enhances understanding of quadratic factorization.
- It offers insights into how algebraic expressions can represent geometric dimensions.
- It develops problem-solving skills in algebra and geometry.

- It has practical applications in designing objects and understanding algebraic relationships in real-world scenarios.

Next, we will move to the core process of solving the problem: factoring the quadratic expression.

Factoring the Quadratic Expression: $6x^2 - 29x + 30$

The primary step in solving for the dimensions of the rectangle is to factor the quadratic expression into two binomials:
$$ 6x^2 - 29x + 30 = (ax + b)(cx + d) $$
where a, b, c, and d are constants to be determined.

Step 1: Identify the coefficients

- Quadratic coefficient (a): 6
- Linear coefficient (b): -29
- Constant term (c): 30

Step 2: Find two numbers that multiply to $6 \times 30 = 180$ and sum to -29

We need two numbers, m and n, such that:
$$ m \times n = 180 $$
$$ m + n = -29 $$

Since the sum is negative and the product positive, both numbers are negative.

Possible factor pairs of 180:
- (-1, -180)
- (-2, -90)
- (-3, -60)
- (-4, -45)
- (-5, -36)
- (-6, -30)
- (-9, -20)
- (-10, -18)
- (-12, -15)

Check sums:
- -1 + -180 = -181
- -2 + -90 = -92
- -3 + -60 = -63
- -4 + -45 = -49
- -5 + -36 = -41
- -6 + -30 = -36
- -9 + -20 = -29 ← This matches our target sum!
- -10 + -18 = -28

- -12 + -15 = -27

So, the pair is -9 and -20.

Step 3: Rewrite the middle term using these numbers

Express the quadratic as:
$$ 6x^2 - 9x - 20x + 30 $$

Step 4: Factor by grouping

Group the terms:
$$ (6x^2 - 9x) + (-20x + 30) $$

Factor out common factors:
$$ 3x(2x - 3) - 10(2x - 3) $$

Now, factor out the common binomial:
$$ (2x - 3)(3x - 10) $$

Result:

The quadratic polynomial factors as:
$$ 6x^2 - 29x + 30 = (2x - 3)(3x - 10) $$

This is a crucial step because it allows us to interpret the dimensions of the rectangle.

Determining the Length and Breadth of the Rectangle

Having factored the quadratic into two linear factors, the next step is to interpret these factors as potential expressions for the length and breadth of the rectangle.

Possible dimensions:

- Length $L = 2x - 3$
- Breadth $B = 3x - 10$

or vice versa. Since the order does not matter for dimensions, both arrangements are possible.

Important considerations:

- Both dimensions should be positive real numbers for the rectangle to be valid physically.
- Therefore, the values of x should satisfy:
$$ 2x - 3 > 0 \Rightarrow x > \frac{3}{2} $$
$$ 3x - 10 > 0 \Rightarrow x > \frac{10}{3} \approx 3.33 $$

Thus, for both dimensions to be positive, the variable x must be greater than $\frac{10}{3}$.

Summary of conditions:

- For a valid rectangle with positive dimensions:
$$ x > \frac{10}{3} $$
- Dimensions:
$$ L = 2x - 3 $$
$$ B = 3x - 10 $$

Next, we will calculate the perimeter based on these dimensions.

Calculating the Perimeter of the Rectangle

The perimeter P of a rectangle is given by the formula:
$$ P = 2 \times (\text{Length} + \text{Breadth}) $$

Using the dimensions derived:
$$ P = 2 \times [(2x - 3) + (3x - 10)] $$

Simplify the sum inside the brackets:
$$ P = 2 \times (2x - 3 + 3x - 10) = 2 \times (5x - 13) $$

Thus,
$$ P = 10x - 26 $$

Key points:

- The perimeter is expressed as a linear function of x.
- For $x > \frac{10}{3}$, the perimeter will be positive and valid.

Example Calculation:

Suppose $x = 4$ (which is greater than 3.33):

- Length:
$$ L = 2(4) - 3 = 8 - 3 = 5 $$
- Breadth:
$$ B = 3(4) - 10 = 12 - 10 = 2 $$
- Perimeter:
$$ P = 10(4) - 26 = 40 - 26 = 14 $$

Check:
$$ P = 2 \times (L + B) = 2 \times (5 + 2) = 2 \times 7 = 14 $$
which matches the calculated value.

This confirms the correctness of the formula.

Summary and Final Remarks

- The quadratic area expression $6x^2 - 29x + 30$ factors into $(2x - 3)(3x - 10)$.
- The dimensions of the rectangle can be expressed as:
$$ \text{Length} = 2x - 3 $$
$$ \text{Breadth} = 3x - 10 $$
- For the dimensions to be positive, x must satisfy:
$$ x > \frac{10}{3} $$
- The perimeter of the rectangle as a function of x:
$$ P = 10x - 26 $$
- Example: For $x=4$, the dimensions are 5 and 2, and the perimeter is 14.

Applications and Further Insights

Understanding how to derive dimensions from a quadratic area expression has practical applications in various fields:

- Design and Architecture: Calculating dimensions of components based on area constraints.
- Engineering: Optimizing material usage while maintaining area specifications.
- Mathematics Education: Enhancing algebraic manipulation and geometric interpretation skills.
- Real-World Problem Solving: Modeling physical systems where dimensions depend on variables.

Additionally, this problem underscores the importance of algebraic factoring and quadratic analysis in solving geometric problems. Recognizing factorization patterns and interpreting algebraic expressions as physical dimensions are essential skills in advanced mathematics.

Tips for Solving Similar Problems